

14. P(6, 8), Q(4, 0) এবং R(0, 0) শীর্ষবিন্দু বিশিষ্ট ত্রিভুজের ক্ষেত্রফল-

- (A) 32 Sq. units (B) 16 Sq. units
(C) 12 Sq. units (D) 24 Sq. units

Solve

$$\Delta PQR = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 4 & 0 & 1 \\ 6 & 8 & 1 \end{vmatrix} = \frac{1}{2} \times 32 = 16 \text{ square units}$$

Shortcut: একটি বিন্দু (0, 0) হলে,

$$\text{ক্ষেত্রফল} = \left| \frac{1}{2}(x_1y_2 - x_2y_1) \right| = \left| \frac{1}{2}(0 - 32) \right| = 16$$

15. a এর মান কত হলে, $\frac{1}{2}\hat{i} + \frac{1}{3}\hat{j} + a\hat{k}$ ভেক্টরটি একটি একক ভেক্টর হবে-

- (A) $\pm \frac{2}{3}$ (B) $\pm \frac{\sqrt{15}}{6}$ (C) $\pm \frac{7}{6}$ (D) $\pm \frac{\sqrt{23}}{6}$

Solve প্রশ্নমতে,

$$\left| \frac{1}{2}\hat{i} + \frac{1}{3}\hat{j} + a\hat{k} \right| = 1 \Rightarrow \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + a^2} = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{9} + a^2 = 1 \Rightarrow a^2 = 1 - \frac{1}{4} - \frac{1}{9} = \frac{23}{36} \Rightarrow a = \pm \frac{\sqrt{23}}{6}$$

16. ABC ত্রিভুজের BC, CA, এবং AB বাহুর মধ্যবিন্দুগুলো যথাক্রমে D, E এবং F হলে-

- (A) $\vec{AD} = \vec{AB} + \vec{BC}$ (B) $\vec{DA} = \vec{DF} + \vec{DE}$
(C) $\vec{AD} = \vec{AB} + \vec{AC}$ (D) $\vec{AD} = \vec{BE} + \vec{CF}$

Solve $2\vec{AD} = \vec{AB} + \vec{AC}$

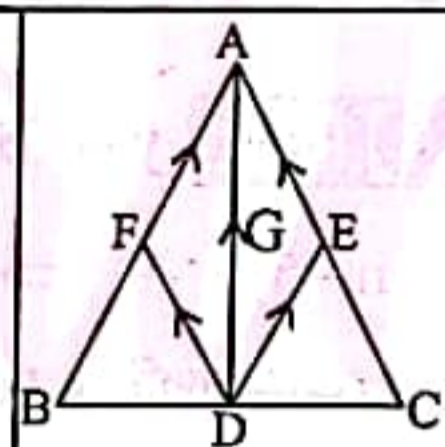
$$\Rightarrow \vec{AD} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AC}$$

$$\Rightarrow \vec{AD} = \vec{AF} + \vec{AE}$$

$$\Rightarrow \vec{DA} = \vec{FA} + \vec{EA}$$

$$\Rightarrow \vec{DA} = \vec{DE} + \vec{DF}$$

$$\Rightarrow \vec{DA} = \vec{DF} + \vec{DE}$$



[\because FA = DE; FA \parallel DE
EA = DF; EA \parallel DF]

Shortcut: $\vec{DF} + \vec{DE} = 2\vec{DG}$

$$\vec{DF} + \vec{DE} = \vec{DA}$$

17. $\sin 65^\circ + \cos 65^\circ$ এর মান-

- (A) $2\cos 20^\circ$ (B) $\sqrt{2} \cos 20^\circ$
(C) $\sqrt{2} \sin 20^\circ$ (D) $2\sin 20^\circ$

Solve $\sin 65^\circ + \cos 65^\circ$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin 65^\circ + \frac{1}{\sqrt{2}} \cos 65^\circ \right)$$

$$= \sqrt{2} (\cos 65^\circ \cos 45^\circ + \sin 65^\circ \sin 45^\circ)$$

$$= \sqrt{2} \cos(65^\circ - 45^\circ) = \sqrt{2} \cos 20^\circ$$

18. $3x + 5y = 2$, $2x + 3y = 0$, $ax + by + 1 = 0$ সমবিন্দুগামী

হলে, a এবং b এর সম্পর্ক -

- (A) $4a - 6b = 1$ (B) $4a - 6b = 2$
(C) $6a - 4b = 1$ (D) $6a - 4b = 2$

Solve $3x + 5y = 2$ ---- (i)

$$2x + 3y = 0$$
 ---- (ii)

$$ax + by + 1 = 0$$
 ---- (iii)

(i) ও (ii) ছেদ বিন্দু (-6, 4)

(iii) নং এ বসাই $-6a + 4b + 1 = 0 \Rightarrow 6a - 4b = 1$

বিকল্প,

$$\begin{vmatrix} 3 & 5 & 2 \\ 2 & 3 & 0 \\ a & b & -1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 3+2a & 5+2b & 0 \\ 2 & 3 & 0 \\ a & b & -1 \end{vmatrix} = 0$$

$$\Rightarrow 9 + 6a - 10 - 4b = 0 \Rightarrow 6a - 4b = 1$$

19. $5x^2 + 15x - 10y - 4 = 0$ পরাবৃত্তের নিয়ামকের সমীকরণ-

- (A) $40x + 81 = 0$ (B) $2x + 3 = 0$
(C) $40y + 81 = 0$ (D) $40y + 41 = 0$

Solve $5x^2 + 15x - 10y - 4 = 0$

$$\Rightarrow 5(x^2 + 3x) = 10y + 4$$

$$\Rightarrow 5 \left(x^2 + 2 \cdot \frac{3}{2}x + \frac{9}{4} \right) = 10y + 4 + \frac{45}{4}$$

$$\Rightarrow 5 \left(x + \frac{3}{2} \right)^2 = 10y + \frac{61}{4}$$

$$\Rightarrow 5 \left(x + \frac{3}{2} \right)^2 = 10 \left(y + \frac{61}{40} \right)$$

$$\Rightarrow \left(x + \frac{3}{2} \right)^2 = 4 \cdot \frac{1}{2} \left(y + \frac{61}{40} \right) \Rightarrow a = \frac{1}{2}$$

নিয়ামকের সমীকরণ, $y + \frac{61}{40} + a = 0 \Rightarrow y + \frac{61}{40} + \frac{1}{2} = 0$

$$\Rightarrow 40y + 61 + 20 = 0 \Rightarrow 40y + 81 = 0$$

20. ABC ত্রিভুজের $\cos A + \cos C = \sin B$ হলে, $\angle C$ এর মান-

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Solve $\cos A + \cos C = \sin B$

$$\Rightarrow 2\cos \frac{C+A}{2} \cos \frac{C-A}{2} = 2\sin \frac{B}{2} \cos \frac{B}{2}$$

$$\Rightarrow \cos \left(\frac{\pi}{2} - \frac{B}{2} \right) \cos \frac{C-A}{2} = \sin \frac{B}{2} \cos \frac{B}{2}$$

$$\Rightarrow \sin \frac{B}{2} \cos \frac{C-A}{2} = \sin \frac{B}{2} \cos \frac{B}{2}$$

$$\because \sin \frac{B}{2} \neq 0 \Rightarrow \cos \frac{C-A}{2} = \cos \frac{B}{2}$$

$$\Rightarrow C - A = B \Rightarrow B + A = C$$

$$\Rightarrow A + B + C = 2C \Rightarrow \pi = 2C \Rightarrow C = \frac{\pi}{2}$$

