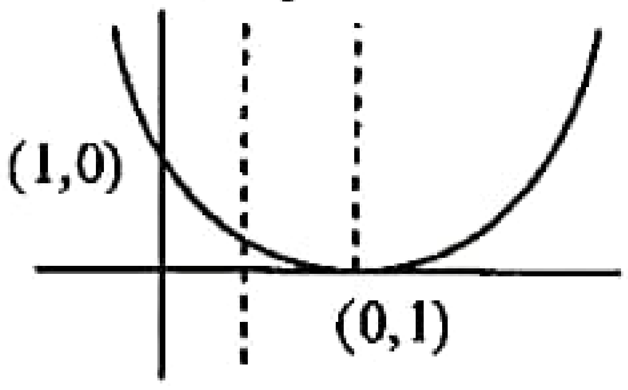
অধ্যায়- ১০ : যোগজীকরণ

Written

01. $x = b$ রেখাটি $y = (1-x)^2, y = 0$ এবং $x = 0$ দ্বারা আবদ্ধ ক্ষেত্রকে $R_1(0 \le x \le b)$ এবং $R_2(b \le x \le 1)$ অংশদ্বয়ে এমনভাবে বিভক্ত করে যেন $R_1 - R_2 = \frac{1}{4}$ হয়, b এর মান কত? **[BUET'18-19]**

সমাধানঃ



$$R_1 = \int_0^b y\, dx = \int_0^b (1-x)^2 dx = \left[-\frac{(1-x)^3}{3}\right]_0^b = -\frac{(1-b)^3}{3} + \frac{1}{3}$$

$$R_2 = \int_b^1 y\, dx = \int_b^1 (1-x)^2 dx = \left[-\frac{(1-x)^3}{3}\right]_b^1 = \frac{(1-b)^3}{3}$$

$$R_1 - R_2 = -\frac{2(1-b)^3}{3} + \frac{1}{3} = \frac{1}{4} \Rightarrow -\frac{2}{3}(1-b)^3 = -\frac{1}{12}$$

$$\Rightarrow (1-b)^3 = \frac{1}{8} \Rightarrow 1 - b = \frac{1}{2} \Rightarrow b = \frac{1}{2} \text{ (Ans.)}$$

02. মান নির্ণয় করঃ (i) $\int_{-\infty}^{0} xe^{-x^2} dx$ (ii) $\int_0^{\infty} xe^{-x^2} dx$ **[RUET'18-19]**

সমাধানঃ (i) let, $x^2 = z \Rightarrow 2xdx = dz \Rightarrow xdx = \frac{dz}{2}$

x	$-\infty$	0
z	∞	0

$$\therefore \int_{\infty}^{0} \frac{e^{-z}}{2} dz = -\frac{1}{2}[e^{-z}]_{\infty}^{0} = \frac{1}{2}[e^{-z}]_0^{\infty} = \frac{1}{2}(0-1) = -\frac{1}{2} \text{(Ans.)}$$

(ii) $$\int_0^{\infty} \frac{e^{-z}}{2} dz = -\frac{1}{2}[e^{-z}]_0^{\infty} = -\frac{1}{2}[0-1] = \frac{1}{2} \text{ (Ans.)}$$

03. $\int_0^3 \frac{xe^x}{(x+1)^2} dx$ এর মান নির্ণয় কর। **[BUTEX'18-19]**

সমাধানঃ $$\int_0^3 \frac{xe^x}{(x+1)^2} dx = \int_0^3 \frac{e^x(x+1-1)}{(x+1)^2} dx = \int_0^3 e^x\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right) dx = \left[\frac{e^x}{x+1}\right]_0^3 = \frac{e^3}{4} - 1 \text{ (Ans.)}$$

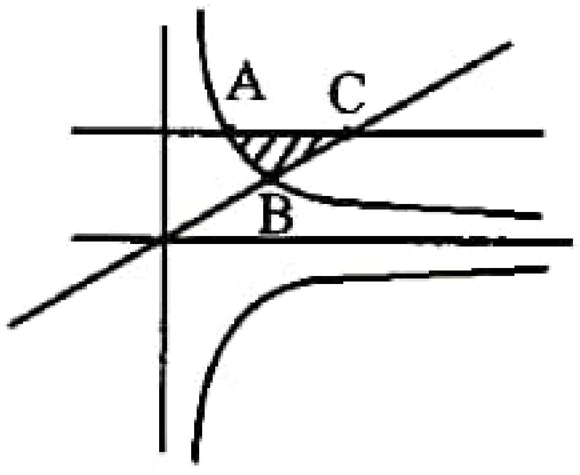
04. দেখাও যে, $\int_{-1}^{1} x^3 \cos x\, dx = 0$। **[BUTEX'18-19]**

সমাধানঃ ধরি, $f(x) = x^3 \cos x$

এখানে, $f(-x) = -x^3 \cos x = -f(x)$ ∴ ফাংশনটি অযুগ্ম তাই $\int_{-1}^{1} x^3 \cos x dx = 0$

05. $x = \frac{1}{y^2}$, $x = y$ এবং $y = 2$ রেখাগুলির দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। ক্ষেত্রটির চিত্র অংকন কর। **[BUET'17-18]**

সমাধান: $x = \frac{1}{y^2} \Rightarrow y = \pm\frac{1}{\sqrt{x}}$



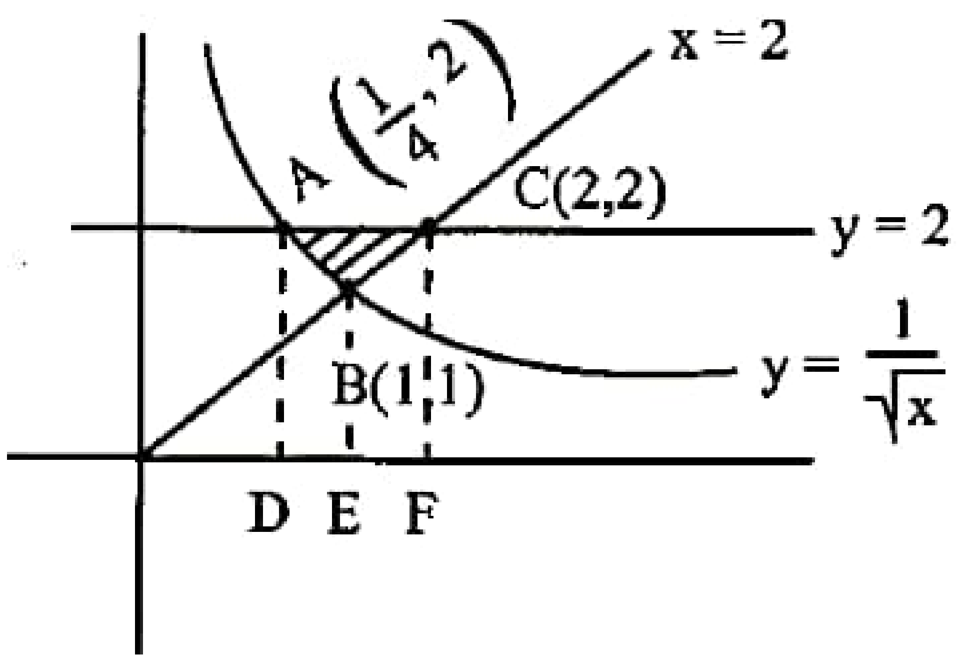
আবদ্ধ ক্ষেত্রের ক্ষেত্রফল = ADEFC ক্ষেত্রের ক্ষেত্রফল – ADEB ক্ষেত্রের ক্ষেত্রফল – BEFC ক্ষেত্রের ক্ষেত্রফল

$$= \int_{\frac{1}{4}}^{2} 2 \cdot dx - \int_{\frac{1}{4}}^{1} \frac{1}{\sqrt{x}} \cdot dx - \int_1^2 x \cdot dx$$

$$= 2 \cdot [x]_{\frac{1}{4}}^{2} - 2 \cdot [\sqrt{x}]_{\frac{1}{4}}^{1} - \left[\frac{x^2}{2}\right]_1^2$$

$$= 2 \cdot \left(2 - \frac{1}{4}\right) - 2 \cdot \left(\sqrt{1} - \sqrt{\frac{1}{4}}\right) - \frac{1}{2} \cdot (2^2 - 1^2)$$

$$= 2 \cdot \frac{7}{4} - 2 \cdot \frac{1}{2} - \frac{1}{2} \cdot 3 = 1 \text{ বর্গ একক (Ans.)}$$



বিকল্প পদ্ধতিঃ $x = \frac{1}{y^2}$ ও $x = y$ এর ছেদবিন্দু, $\frac{1}{y^2} = y \Rightarrow y^3 = 1 \Rightarrow y = 1 \therefore x = \frac{1}{y^2} = 1$

সুতরাং, আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= \int_1^2 \left(y - \frac{1}{y^2}\right) dy = \left[\frac{y^2}{2} + \frac{1}{y}\right]_1^2 = \left(2 + \frac{1}{2} - \frac{1}{2} - 1\right)$ বর্গ একক $= 1$ বর্গ একক।

06. যোজিতফল নির্ণয় কর: $\int_0^{\ln 2} \frac{e^x dx}{1+e^x}$ **[RUET'17-18]**

সমাধান: [Let, $1 + e^x = z; e^x.dx = dz$, If $x = 0,\ z = 2$; if $x = \ln 2; z = 3$]

$\int_0^{\ln 2} \frac{e^x dx}{1+e^x} = \int_2^3 \frac{dz}{z} = [\ln z]_2^3 = \ln 3 - \ln 2 = \ln\frac{3}{2}$ (Ans.)

07. মান নির্ণয় কর: $\int \frac{e^{m\tan^{-1}x}}{(1+x^2)^2} dx$ **[BUET'16-17]**

সমাধান: ধরি, $\tan^{-1}x = \theta \ \therefore x = \tan\theta \ \therefore dx = \sec^2\theta\, d\theta$

$\therefore I = \int \frac{e^{m\theta}}{(1+\tan^2\theta)^2}\sec^2\theta\, d\theta = \int \frac{e^{m\theta}}{\sec^2\theta} d\theta = \int e^{m\theta}\cos^2\theta\, d\theta = \frac{1}{2}\int e^{m\theta}(1+\cos 2\theta)d\theta$

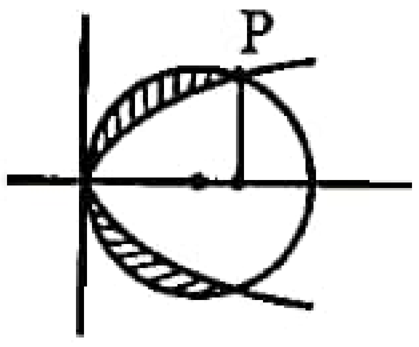
$= \frac{1}{2}\int e^{m\theta}d\theta + \frac{1}{2}\int e^{m\theta}\cos 2\theta d\theta \ \left[\because \int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)\right]$

$= \frac{1}{2m}e^{m\theta} + \frac{1}{2}\cdot\frac{e^{m\theta}}{m^2+2^2}\{m\cos 2\theta + 2\sin m\theta\} + c$

$= \frac{1}{2m}e^{m\tan^{-1}x} + \frac{e^{m\tan^{-1}x}}{m^2+4}\{m\cos(2\tan^{-1}x) + 2\sin(m\tan^{-1}x)\} + c$

08. $y^2 = ax$ এবং $x^2 + y^2 = 4ax$ রেখাদ্বয়ের অন্তর্বর্তী এলাকার ক্ষেত্রফল নির্ণয় কর। **[BUET'16-17]**

সমাধান:



ছেদবিন্দু নির্ণয়: $x^2 + ax = 4ax \Rightarrow x = 0, 3a \ \therefore y = 0, \pm\sqrt{3}a \ \therefore P \equiv (3a, \sqrt{3}a)$

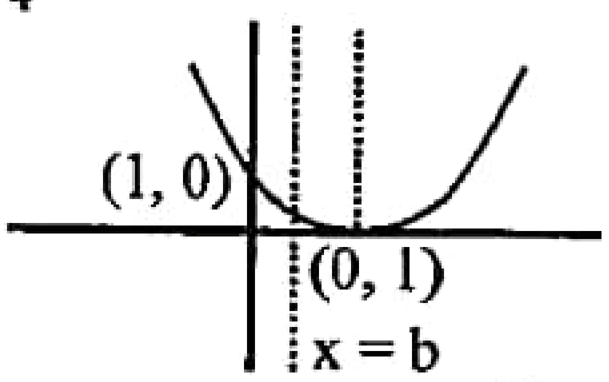
$\therefore$ ক্ষেত্রফল $= 2 \times \int_0^{3a}(\sqrt{4ax - x^2} - \sqrt{ax})\, dx = 2 \times \int_0^{3a}\left(\sqrt{4a^2 - (x-2a)^2} - \sqrt{ax}\right) dx$

$= 2 \times \left[\left(\frac{x-2a}{x}\right)\sqrt{4ax - x^2} + 2q^2\sin^{-1}\left(\frac{x-2a}{2a}\right) - \frac{2}{3}\sqrt{a}x^{\frac{3}{2}}\right]_0^{3a} = 2 \times \left(\frac{a}{2} \times \sqrt{3}a + 2a^2 \times \frac{\pi}{6} - 2\sqrt{3}a^2 + \pi a^2\right)$

$= \left(\frac{2}{3}\pi - 3\sqrt{3} + 2\pi\right)a^2 = \left(\frac{8}{3}\pi - 3\sqrt{3}\right)a^2$

09. $x = b$ রেখাটি $y = (1-x)^2, y = 0$ এবং $x = 0$ দ্বারা আবদ্ধ ক্ষেত্রকে $R_1(0 \le x \le b)$ এবং $R_2(b \le x \le 1)$ অংশদ্বয়ে বিভক্ত করে যেখানে $R_1 - R_2 = \frac{1}{4}$ । b এর মান নির্ণয় কর। **[RUET'15-16]**

সমাধান: $y = (1-x)^2$



$R_1 = \int_0^b ydx = \int_0^b(1-x)^2dx = \left[-\frac{(1-x)^3}{3}\right]_0^b = -\frac{(1-b)^3}{3} + \frac{1}{3}$

$R_2 = \int_b^1 ydx = \int_b^1(1-x)^2dx = \left[-\frac{(1-x)^3}{3}\right]_b^1 = \frac{(1-b)^3}{3}$

$\therefore R_1 - R_2 = -\frac{2(1-b)^3}{3} + \frac{1}{3} = \frac{1}{4} \Rightarrow -\frac{2}{3}(1-b)^3 = -\frac{1}{12} \Rightarrow (1-b)^3 = \frac{1}{8} \Rightarrow 1 - b = \frac{1}{2} \Rightarrow b = \frac{1}{2}$ (Ans.)

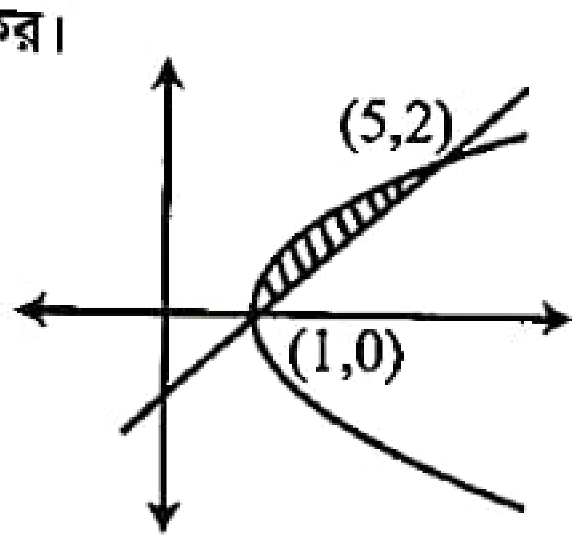
10. $y^2 = x - 1$ পরাবৃত্ত এবং $2y = x - 1$ সরলরেখা দিয়ে আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। **[BUET'14-15]**

সমাধান: $y^2 = x - 1; 2y = x - 1 \Rightarrow y^2 = 2y \Rightarrow y = 0,2 \therefore x = 1,5$

$\therefore$ ছেদবিন্দুদ্বয় $(1,0)$ ও $(5,2)$

$\therefore \Delta = \int_1^5(y_1 - y_2)dx = \int_1^5 \sqrt{x-1} - \left(\frac{x}{2} - \frac{1}{2}\right)dx$

$= \left[\frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{4} + \frac{x}{2}\right]_1^5 = \frac{19}{12} - \frac{1}{4} = \frac{4}{3}$ বর্গ একক (Ans.)

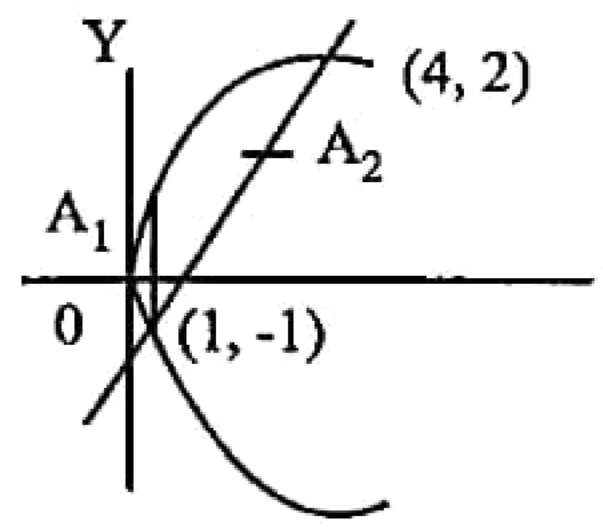


11. যোগজ নির্ণয় কর: $\int \frac{x^2+1}{x^4+1}dx$ **[BUET'14-15]**

সমাধান: $\int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+(\sqrt{2})^2}dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+c$ (Ans.)

12. x এর সাপেক্ষে যোগজ করে $x = y^2$ এবং $y = x-2$ রেখা দুটো দিয়ে আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। **[BUET'10-11,12-13]**

সমাধান: $x = y^2, y = x-2 \quad \therefore x = (x-2)^2 \quad \therefore x = 4$ or, 1



Area, $A_1 = 2\int_0^1 \sqrt{x}\,dx = 2.\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1 = \frac{4}{3}$

Area, $A_2 = \int_1^4 (y_1-y_2)\,dx = \int_1^4 [\sqrt{x}-(x-2)]\,dx$

$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_1^4 - \frac{1}{2}[x^2]_1^4 + 2[x]_1^4 = \frac{2}{3}(8-1) - \frac{1}{2}(16-1) + 2(4-1) = \frac{19}{6}$

$\therefore A = A_1 + A_2 = \frac{4}{3}+\frac{19}{6} = 4.5$ sq. units (Ans.)

13. (a) মান নির্ণয় কর: $\int_0^1 2x^3e^{-x^2}dx$ **[RUET'12-13]**

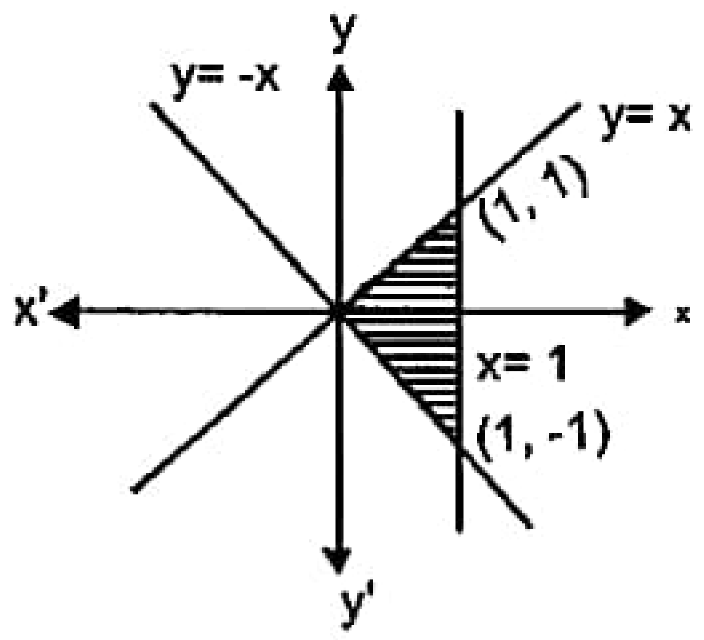
সমাধান: Let, $I = \int 2x^3e^{-x^2}dx \; \therefore I = \int 2x.x^2e^{-x^2}dx$ | Let, $x^2 = z \Rightarrow 2xdx = dz$

x	0	1
z	0	1

$= \int ze^{-z}dz = -ze^{-z} - \int\left\{\frac{d}{dz}(z)\int e^{-z}\right\}dz = -ze^{-z} - \int\left(-e^{-z}\right)dz = -ze^{-z} - e^{-z}$

$\therefore [I]_0^1 = \left\{-1.e^{-1} - e^{-1} - \left(-0.e^{-0} - e^{-0}\right)\right\} = 1-2e^{-1}$

(b) $y^2 = x^2$ এবং $x = 1$ দ্বারা সীমাবদ্ধ ক্ষেত্রের মান নির্ণয় কর।



সমাধান: $y^2 = x^2 \Rightarrow y = \pm x$

এবং $x = 1$ রেখাদ্বয়ের ছেদবিন্দু ত্রয় (0,0), (1,1), (1,-1)

$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1$ বর্গ একক

14. (a) মান নির্ণয় করঃ $\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ **[RUET'11-12]**

সমাধান: এটি একটি জোড় ফাংশন। এর ক্ষেত্রে π থেকে $\frac{\pi}{2}$ এবং $\frac{\pi}{2}$ থেকে 0 লিমিটের মধ্যে একই ক্ষেত্রফল হবে।

$I = \int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx = 2\int_0^{\pi/2} \frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx = 2\int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2\tan^2 x}$

ধরি, $\tan x = z \Rightarrow \sec^2 x\,dx = dz$; $x = \frac{\pi}{2}$ হলে, $z = \infty$, $x = 0$ হলে, $z = 0$

$2\int_0^{\infty} \frac{dz}{a^2+b^2z^2} = \frac{2}{a^2}\int_0^{\infty} \frac{dz}{1+\frac{b^2z^2}{a^2}} = 2\frac{1}{ab}\left[\tan^{-1}\left(\frac{bz}{a}\right)\right]_0^{\infty} = \frac{2}{ab}\left[\tan^{-1}\left(\frac{b\infty}{a}\right) - \tan^{-1}\left(\frac{b.0}{a}\right)\right]$

$= \frac{2}{ab}[\tan^{-1}\infty - 0] = \frac{2}{ab}.\frac{\pi}{2} = \frac{\pi}{ab}$

(b) মান নির্ণয় কর ঃ $\int_0^1 \ln(x^2+1)\,dx$

সমাধান: $\int_0^1 \ln(x^2+1)dx$; $\int \ln(x^2+1)dx = \ln(x^2+1).\int dx - \int\left(\frac{2x}{x^2+1}\int dx\right)dx$

$= x\ln(x^2+1) - 2.\int\frac{x^2}{x^2+1}dx = x\ln(x^2+1) - 2.\int\frac{x^2+1-1}{x^2+1}dx$

$= x\ln(x^2+1) - 2.\int\left(1-\frac{1}{x^2+1}\right)dx = x\ln(x^2+1) - 2(x-\tan^{-1}x)+c$

লিমিট বসিয়ে, $1.\ln(1^2+1)-2(1-\tan^{-1}1)-0+2(0-\tan^{-1}0) = \ln 2 - 2\left(1-\frac{\pi}{4}\right) = \ln 2 - 2 + \frac{\pi}{2}$

15. $\int_{\pi/3}^{\pi/2}\frac{dx}{1+\sin x-\cos x}$ এর মান নির্ণয় কর। **[BUET'11-12]**

সমাধান: $\int\frac{dx}{1+\sin x-\cos x} = \int\frac{dx}{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}-\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$

$= \int\frac{\left(1+\tan^2\frac{x}{2}\right)dx}{1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}-1+\tan^2\frac{x}{2}} = \int\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+2\tan\frac{x}{2}}$

ধরি, $z=\tan\frac{x}{2}$

$dz=\frac{1}{2}\sec^2\frac{x}{2}dx$

$\frac{1}{z(z+1)}=\frac{1}{z}-\frac{1}{z+1}$

$x=\frac{\pi}{3}$ হলে $z=\frac{1}{\sqrt{3}}$

$x=\frac{\pi}{2}$ হলে $z=1$

$= \int\frac{\frac{1}{2}\sec^2\frac{x}{2}dx}{\tan\frac{x}{2}\left(\tan\frac{x}{2}+1\right)} = \int\frac{dz}{z(z+1)} = \int\frac{dz}{z}-\int\frac{dz}{z+1} = \ln(z)-\ln(z+1)=\ln\frac{z}{z+1}$

$\therefore \int_{\pi/3}^{\pi/2}\frac{dx}{1+\sin x-\cos x} = \left[\ln\frac{z}{z+1}\right]_{1/\sqrt{3}}^{1} = \ln\frac{1}{2}-\ln\frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}+1} = \ln\frac{1}{2}-\ln\frac{1}{1+\sqrt{3}} = \ln(\sqrt{3}+1)-\ln 2 = \ln\frac{\sqrt{3}+1}{2}$

16. (a) যোজিত ফল নির্ণয় কর ঃ $\int x^3e^{x^2}dx$ **[RUET'11-12]**

সমাধান: ধরি, $x^2=z \Rightarrow 2xdx=dz \Rightarrow dx=\frac{dz}{2x}$

$\int x^3e^z.\frac{dz}{2x}=\frac{1}{2}\int ze^z dz. = \frac{1}{2}\left[z\int e^z dz-\int(1.\int e^z dz)\,dz\right]$

$=\frac{1}{2}ze^z-\frac{1}{2}e^z = \frac{1}{2}e^z(z-1)=\frac{1}{2}e^{x^2}(x^2-1)+c$

(b) যোজিত ফল নির্ণয় করঃ $\int \sin^{-1}\sqrt{\left(\frac{x}{x+a}\right)}\,dx$

সমাধান: $I=\int \sin^{-1}\sqrt{\frac{x}{x+a}}\,dx \;= x\sin^{-1}\sqrt{\frac{x}{x+a}}-\int\frac{1}{\sqrt{1-\frac{x}{x+a}}}.\frac{1}{2}.\frac{\sqrt{x+a}}{\sqrt{x}}.\frac{x+a-x}{(x+a)^2}.x\,dx$

$=x\sin^{-1}\sqrt{\frac{x}{x+a}}-\int\frac{\sqrt{x+a}}{\sqrt{a}}.\frac{1}{2}.\frac{\sqrt{x+a}}{\sqrt{x}}.\frac{a}{(x+a)^2}.x\,dx \;= x\sin^{-1}\sqrt{\frac{x}{x+a}}-\frac{\sqrt{a}}{2}\int\frac{\sqrt{x}}{x+a}dx$

$\int\frac{\sqrt{x}}{x+a}dx$; ধরি, $\sqrt{x}=z \Rightarrow x=z^2 \Rightarrow dx=2zdz$

$\int\frac{2z^2dz}{z^2+a}=2\int\frac{z^2+a-a}{z^2+a}dz=2\int\left(1-\frac{a}{z^2+a}\right)dz$

$=2z-2a.\frac{1}{\sqrt{a}}\tan^{-1}\frac{z}{\sqrt{a}}=2z-2\sqrt{a}\tan^{-1}\frac{z}{\sqrt{a}}$

$=2\sqrt{x}-2\sqrt{a}\tan^{-1}\sqrt{\frac{x}{a}} \quad \therefore I=x\sin^{-1}\sqrt{\frac{x}{x+a}}-\frac{\sqrt{a}}{2}(2\sqrt{x}-2\sqrt{a}\tan^{-1}\sqrt{\frac{x}{a}})+c$

17. মান নির্ণয় কর: $\int_0^4 y\sqrt{4-y}\,dy$ **[BUET'10-11]**

সমাধান: $I=\int_0^4 y\sqrt{4-y}\,dy$

ধরি, $y=4\sin^2\theta \Rightarrow dy=8\sin\theta\cos\theta\,d\theta$

যখন, $y=0$ তখন $\theta=0$, যখন $y=4$ তখন $\theta=\frac{\pi}{2}$

$\therefore I=\int_0^{\frac{\pi}{2}}4\sin^2\theta\sqrt{4-4\sin^2\theta}\;.8\sin\theta\cos\theta\,d\theta \;=64\int_0^{\frac{\pi}{2}}\sin^3\theta\cos^2\theta\,d\theta$

ধরি, $\cos\theta=z \Rightarrow -\sin\theta\,d\theta=dz$

যখন, $\theta=0$ তখন $z=1$; $\theta=\frac{\pi}{2}$ তখন $z=0$

$\therefore I=-64\int_1^0(1-z^2)z^2\,dz=64\int_0^1(z^2-z^4)dz=64\left[\frac{z^3}{3}-\frac{z^5}{5}\right]_0^1=64\left\{\frac{1}{3}-\frac{1}{5}-0\right\}=\frac{128}{15}$ (Ans.)

18. মান নির্ণয় করঃ (a) $\int_0^1\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ (b) $\int_0^{\frac{\pi}{2}}(1+\cos x)^2\sin x\,dx$ **[RUET'07-08, BUTex'10-11]**

সমাধান: (a) ধরি, $\sin^{-1}x=z \Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dz$; when $x=0,\; z=0$; when $x=1,\; z=\frac{\pi}{2}$

$\therefore\int_0^1\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\int_0^{\pi/2}z\,dz=\left[\frac{z^2}{2}\right]_0^{\pi/2}=\frac{\pi^2}{8}$ (Ans.)

(b) ধরি, $1+\cos x=z \Rightarrow -\sin x\,dx=dz$; When $x=0,\; z=2$; When $x=\frac{\pi}{2},\; z=1$

$\therefore\;\int_0^{\frac{\pi}{2}}(1+\cos x)^2\sin x\,dx=\int_2^1 z^2(-dz)=\left[-\frac{z^3}{3}\right]_2^1=-\frac{1}{3}+\frac{8}{3}=\frac{7}{3}$ (Ans.)

19. x এর সাপেক্ষে নিম্নের ফাংশনটি ইন্টিগ্রেট কর : $\frac{e^x(x^2+1)}{(x+1)^2}$ [BUET'02-03,06-07,RUET'10-11]

সমাধান: $\int \frac{e^x(x^2+1)}{(x+1)^2}dx = \int e^x \frac{x^2-1+2}{(x+1)^2}dx$

$= e^x\left\{\frac{x^2-1}{(x+1)^2}+\frac{2}{(x+1)^2}\right\}dx = \int e^x\left\{\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right\}dx$

$= \int e^x\left\{\frac{(x-1)}{(x+1)}+\frac{d}{dx}\left(\frac{x-1}{x+1}\right)\right\}dx. = e^x\frac{(x-1)}{(x+1)}+c$ $[\because \int e^x\{f(x)+f'(x)\}dx = e^x f(x)+c]$ (Ans.)

20. মান নির্ণয় কর : $\int_1^{\sqrt{3}} x\cot^{-1}x\,dx$ [BUET'09-10]

সমাধান: $\int x\cot^{-1}x\,dx = \cot^{-1}x \int x\,dx - \int\left(\frac{d}{dx}(\cot^{-1}x)\int x\,dx\right)dx$

$= \frac{x^2}{2}\cot^{-1}x - \int -\frac{1}{1+x^2}.\frac{x^2}{2}dx + c_1 = \frac{x^2\cot^{-1}x}{2}+\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx+c_1$

$= \frac{x^2\cot^{-1}x}{2}+\frac{1}{2}\int dx - \frac{1}{2}\int\frac{dx}{1+x^2}+c_1 = \frac{x^2\cot^{-1}x}{2}+\frac{x}{2}-\frac{1}{2}\tan^{-1}x+c$

$\int_1^{\sqrt{3}} x\cot^{-1}x\,dx = \left[\frac{x^2\cot^{-1}x}{2}+\frac{x}{2}-\frac{1}{2}\tan^{-1}x\right]_1^{\sqrt{3}} = \frac{3}{2}.\frac{\pi}{6}+\frac{\sqrt{3}}{2}-\frac{1}{2}.\frac{\pi}{3}-\frac{1}{2}.\frac{\pi}{4}-\frac{1}{2}+\frac{1}{2}.\frac{\pi}{4} = \frac{\sqrt{3}-1}{2}+\frac{\pi}{12}$

21. মান নির্ণয় কর : $\int_1^{\sqrt{3}} x\tan^{-1}x\,dx$ [BUTex'09-10]

সমাধান: $\int_1^{\sqrt{3}} x\tan^{-1}x\,dx$

এখন, $\int x\tan^{-1}x\,dx = \tan^{-1}x\int x\,dx - \int\left\{\frac{d}{dx}\tan^{-1}x\int x\,dx\right\}dx = \frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$

$= \frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx = \frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int dx+\frac{1}{2}\int\frac{dx}{1+x^2} = \frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+c$

$\therefore \int_1^{\sqrt{3}} x\tan^{-1}x = \left[\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{\tan^{-1}x}{2}\right]_1^{\sqrt{3}} = \frac{3}{2}\tan^{-1}\sqrt{3}-\frac{\sqrt{3}}{2}+\frac{\tan^{-1}\sqrt{3}}{2}-\frac{1}{2}\tan^{-1}1+\frac{1}{2}-\frac{\tan^{-1}1}{2}$

$= \frac{3}{2}\times\frac{\pi}{3}-\frac{\sqrt{3}}{2}+\frac{\pi}{3\times2}-\frac{\pi}{8}+\frac{1}{2}-\frac{\pi}{8} = \frac{1-\sqrt{3}}{2}+\frac{\pi}{2}+\frac{\pi}{6}-\frac{\pi}{4} = \frac{1-\sqrt{3}}{2}+\frac{5\pi}{12}$ (Ans.)

22. মান নির্ণয় কর : $\int_{\frac{1}{2}}^{1}\frac{dx}{x\sqrt{4x^2-1}}$ [BUET'04-05,CUET'07-08,BUTex'09-10]

সমাধান: $\int_1^2\frac{\frac{1}{2}dy}{\frac{y}{2}\sqrt{y^2-1}}$ [Let, $2x = y$ $\therefore x = \frac{y}{2}$ $\therefore dx = \frac{dy}{2}$]

x	1	1/2
y	2	1

$= \int_1^2\frac{dy}{y\sqrt{y^2-1}} = [\sec^{-1}y]_1^2 = [\sec^{-1}2-\sec^{-1}1] = \frac{\pi}{3}-0 = \frac{\pi}{3}$ (Ans.)

23. (a) যোজিত ফল নির্ণয় করঃ $\int \frac{dx}{a\cos x - b\sin x}$. **[CUET'09-10]**

সমাধান: $\int \frac{dx}{a\cos x - b\sin x} = \int \frac{dx}{a\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) - b\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}} = \int \frac{\sec^2\frac{x}{2}dx}{a\left(1-\tan^2\frac{x}{2}\right) - 2b\tan\frac{x}{2}}$,

[Let, $\tan\frac{x}{2} = z \Rightarrow \sec^2\frac{x}{2}.dx = 2dz$]

$= \int \frac{2dz}{a(1-z^2)-2bz} = -\frac{1}{a}\int \frac{2dz}{z^2+2\frac{b}{a}z-1} = -\frac{1}{a}\int \frac{2dz}{\left(z+\frac{b}{a}\right)^2 - \frac{b^2}{a^2} - 1} = -\frac{1}{a}\int \frac{2dz}{\left(z+\frac{b}{a}\right)^2 - \left(\frac{b^2+a^2}{a^2}\right)}$

$= -\frac{1}{a}\int \frac{2dz}{\left(z+\frac{b}{a}\right)^2 - \left(\frac{\sqrt{a^2+b^2}}{a}\right)^2} = -\frac{1}{a} \times \frac{2}{2\times\frac{\sqrt{a^2+b^2}}{a}} \ln\left|\frac{z+\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}}{z+\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}}\right| + c$

$= \frac{1}{\sqrt{a^2+b^2}} \ln\left|\frac{az+b+\sqrt{a^2+b^2}}{az+b-\sqrt{a^2+b^2}}\right| + c = \frac{1}{\sqrt{a^2+b^2}} \ln\left|\frac{a\tan\frac{x}{2}+b+\sqrt{a^2+b^2}}{a\tan\frac{x}{2}+b-\sqrt{a^2+b^2}}\right| + c$

(b) যোজিত ফল নির্ণয় করঃ $\int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)\,(2+\sin x)}$

সমাধান: $\int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$

ধরি, $1+\sin x = z$ বা, $\cos x dx = dz$; $x = \frac{\pi}{2}$ হলে, $z = 2$; $x = 0$ হলে, $z = 1$

$\therefore \int_1^2 \frac{dz}{z(z+1)} = \int_1^2 \frac{dz}{z} - \int_1^2 \frac{dz}{z+1} = \left[\ln|z| - \ln|z+1|\right]_1^2 = \left[\ln\left|\frac{z}{z+1}\right|\right]_1^2$

$= \ln\frac{2}{3} - \ln\frac{1}{2} = \ln\frac{2}{3} + \ln 2 = \ln\frac{2\times 2}{3} = \ln\frac{4}{3}$ (Ans.)

24. মান নির্ণয় কর ঃ (a) $\int_0^1 \tan^{-1}x\, dx$ (b) $\int_{-\ln 2}^{0} \frac{e^{-x}\, dx}{1+e^{-x}}$ **[RUET'09-10]**

সমাধান: (a) $\int_0^1 \tan^{-1}x\, dx$

এখন, $\int \tan^{-1}x\, dx = \tan^{-1}x \int dx - \int\left(\frac{d(\tan^{-1}x)}{dx}\int dx\right)dx = x\tan^{-1}x - \int \frac{x}{1+x^2}\, dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + c$

$\therefore \int_0^1 \tan^{-1}x\, dx = \left[x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)\right]_0^1 = [\tan^{-1}1] - \left[\frac{1}{2}\ln 2 + 0 - \frac{1}{2}\ln 1\right] = \frac{\pi}{4} - \frac{1}{2}\ln 2$

(b) $\int_{-\ln 2}^{0} \frac{e^{-x}\,dx}{1+e^{-x}} = \int_{-\ln 2}^{0} \frac{-(-e^{-x})\,dx}{1+e^{-x}} = -[\ln(1+e^{-x})]_{-\ln 2}^{0} = [\ln(1+e^{-x})]_{0}^{-\ln 2}$

$= \ln(1+e^{\ln 2}) - \ln(1+e^{0}) = \ln(1+2) - \ln(1+1) = \ln\frac{3}{2}$ (Ans.)

25. যোগজটির মান নির্ণয় করঃ $\int_{0}^{\pi/4} \frac{\sin 2x\,dx}{\sin^4 x + \cos^4 x}$ **[BUET'08-09]**

সমাধান: $= \int_{0}^{\pi/4} \frac{\frac{\sin 2x\,dx}{\cos^4 x}}{\frac{\sin^4 x + \cos^4 x}{\cos^4 x}} = \int_{0}^{\pi/4} \frac{2\tan x \sec^2 x}{\tan^4 x + 1}\,dx = \int_{0}^{1} \frac{2z dz}{z^4 + 1},$

[Let, $\tan x = z$, $\sec^2 x\ dx = dz$, $x = 0$, $z = 0$, $x = \frac{\pi}{4}$, $z = 1$]

$= \int_{0}^{1} \frac{du}{1+u^2}$ [$z^2 = u$, $2zdz = du$, $z = 0$, $u = 0$, $z = 1$, $u = 1$] $= [\tan^{-1} u]_{0}^{1} = \left[\frac{\pi}{4} - 0\right] = \frac{\pi}{4}$ (Ans.)

26. যোজিত ফল নির্ণয় কর ঃ $\int_{0}^{\pi/2} \sin^2 x \sin 3x dx$ **[RUET'03-04, CUET'08-09]**

সমাধান: $\frac{1}{2}\int_{0}^{\pi/2} 2\sin^2 x \sin 3x\,dx = \frac{1}{2}\int_{0}^{\pi/2} (1-\cos 2x)\sin 3x\ dx$

$= \frac{1}{2}\int_{0}^{\pi/2} \sin 3x\ dx - \frac{1}{2}\int_{0}^{\pi/2} \sin 3x.\cos 2x\,dx = \frac{1}{6}[-\cos 3x]_{0}^{\pi/2} - \frac{1}{4}\int_{0}^{\pi/2} 2\sin 3x \cos 2x\ dx$

$= \frac{1}{6}(-0+1) - \frac{1}{4}\int_{0}^{\pi/2} (\sin 5x + \sin x)dx$

$= \frac{1}{6} - \frac{1}{4}\left[\frac{1}{5}(-\cos 5x) - \cos x\right]_{0}^{\pi/2} = \frac{1}{6} + \frac{1}{4}\left[\frac{\cos 5x}{5} + \cos x\right]_{0}^{\pi/2} = \frac{1}{6} + \frac{1}{4}\left[0 - \frac{1}{5} + 0 - 1\right] = \frac{-2}{15}$

27. a) যোজিত ফল নির্ণয় কর : $\int \frac{dx}{1+\sin x}$ **[BUET'03-04, RUET'08-09]**

সমাধান: $\int \frac{dx}{1+\sin x} = \int \frac{(1-\sin x)dx}{1-\sin^2 x} = \int \frac{(1-\sin x)dx}{\cos^2 x} = \int \sec^2 x dx - \int \tan x \sec x dx = \tan x - \sec x + c$

b) মান নির্ণয় কর : $\int_{0}^{1} \frac{xe^x dx}{(1+x)^2}$

সমাধান: $\int_{0}^{1} \frac{xe^x dx}{(1+x)^2} = \int_{0}^{1} e^x \frac{1+x-1}{(1+x)^2} dx = \int_{0}^{1} e^x \left\{\frac{1+x}{(1+x)^2} - \frac{1}{(1+x)^2}\right\} dx$

$= \int_{0}^{1} e^x \left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\} dx = \int_{0}^{1} e^x \{f(x) + f'(x)\} dx$ [where $f(x) = \frac{1}{1+x}$, $f'(x) = \frac{-1}{(1+x)^2}$]

$= [e^x f(x)]_{0}^{1} = \left[e^x/(1+x)\right]_{0}^{1} = \left[\frac{e}{(1+1)} - \frac{e^0}{(1+0)}\right] = \frac{e}{2} - 1$ (Ans.)

28. যোগজটির মান নির্ণয় কর। $\int_0^{\pi/6}\frac{dx}{1-\tan^2 x}$ [BUET'07-08]

সমাধান: Let, $I=\int\frac{dx}{1-\tan^2 x}=\int\frac{dx}{1-\frac{\sin^2 x}{\cos^2 x}}=\int\frac{\cos^2 x\,dx}{\cos^2 x-\sin^2 x}$

$=\frac{1}{2}\int\frac{(1+\cos 2x)dx}{\cos 2x}=\frac{1}{2}\int(\sec 2x+1)dx=\frac{1}{2}\left[\frac{1}{2}\ell n(\sec 2x+\tan 2x)+x\right]+c$

Now, $\int_0^{\pi/6}\frac{dx}{1-\tan^2 x}=\left[\frac{1}{4}\ell n(\sec 2x+\tan 2x)+\frac{x}{2}\right]_0^{\pi/6}$

$=\left[\frac{1}{4}\ell n\left(2+\sqrt{3}\right)+\frac{\pi}{12}\right]-(0+0)=\frac{\pi}{12}+\frac{1}{4}\ell n\left(2+\sqrt{3}\right)$ (Ans.)

29. মান নির্ণয় কর ঃ $\int_1^{e^2}\frac{dx}{x(1+\ln x)^2}$ [CUET'07-08]

সমাধান: $I=\int_1^{e^2}\frac{dx}{x(1+\ln x)^2}$

ধরি, $(1+\ln x)=z$ বা, $\frac{1}{x}dx=dz$

$x=1$ হলে $z=1$; $x=e^2$ হলে $z=3$ $\therefore I=\int_1^3\frac{dz}{z^2}=\left[-\frac{1}{z}\right]_1^3=\frac{-1}{3}+1=\frac{2}{3}$ (Ans.)

30. যোজিত ফল নির্ণয় করঃ $\int\frac{x+1}{3x^2-x-2}dx$ [RUET'07-08]

সমাধান: $\int\frac{x+1}{3x^2-x-2}dx=\int\frac{x+1}{3x^2-3x+2x-2}dx=\int\frac{x+1}{(x-1)(3x+2)}dx$

$=\int\left(\frac{2}{(x-1)5}+\frac{-\frac{2}{3}+1}{\left(-\frac{2}{3}-1\right)(3x+2)}\right)dx=\int\left(\frac{2}{5(x-1)}-\frac{1}{5(3x+2)}\right)dx$

$=\frac{2}{5}\ln(x-1)-\frac{1}{5}\frac{\ln(3x+2)}{3}+c=\frac{2}{5}\ln(x-1)-\frac{1}{15}\ln(3x+2)+c$ (Ans.)

31. মান নির্ণয় কর $\int_0^1 y\sqrt{(1-y)}dy$ [BUTex'07-08]

সমাধান: ধরি, $1-y=z$ $\therefore -dy=dz$ বা, $dy=-dz$

y	1	0
z	0	1

$\therefore\int_1^0(1-z)z^{\frac{1}{2}},(-dz)=-\int_1^0 z^{\frac{1}{2}}dz+\int_1^0 z^{\frac{3}{2}}dz=-\left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^0+\left[\frac{z^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_1^0$

$=-\frac{2}{3}\left[z^{\frac{3}{2}}\right]_1^0+\frac{2}{5}\left[z^{\frac{5}{2}}\right]_1^0=\frac{2}{3}-\frac{2}{5}=\frac{4}{15}$ (Ans.)

32. যোজিত ফল নির্ণয় কর ঃ (a) $\int e^x \cos x\, dx$ **[RUET'04-05, KUET'06-07]**

সমাধান: Let, $I=\int e^x \cos x\, dx = e^x\int \cos x\, dx - \int\left(\frac{d}{dx} e^x \int \cos x\, dx\right) dx$

$= e^x \sin x - \int e^x \sin x\, dx = e^x \sin x - e^x \int .\sin x\, dx + \int\left(\frac{d}{dx} e^x \int \sin x\, dx\right) dx$

$= e^x \sin x + e^x \cos x - \int e^x \cos x\, dx \quad \Rightarrow I = \frac{1}{2} e^x(\sin x + \cos x) + C.$ (Ans)

(b) $\int \frac{dx}{x^2-3x+2}$

সমাধান: $\int \frac{dx}{x^2-3x+2} = \int \frac{dx}{x^2 - 2.x\frac{3}{2} + \left(\frac{3}{2}\right)^2 - \frac{1}{4}} = \int \frac{dx}{\left(x-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$

$= \frac{1}{2.\frac{1}{2}} \ln\left(\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right) + C = \ln\left(\frac{2x-4}{2x-2}\right) + C = \ln\left(\frac{x-2}{x-1}\right) + C.$ (Ans.)

33. মান নির্ণয় কর ঃ (a) $\int_0^a \sqrt{a^2-x^2}\,dx$ (b) $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}\,dx$ **[CUET'05-06, RUET'06-07]**

সমাধান: (a) ধরি, $x = a\sin\theta \quad \therefore dx = a\cos\theta\, d\theta \quad \therefore$ যখন, $x=0,\ \theta=0$ এবং $x=a,\ \theta=\frac{\pi}{2}$

$\therefore \int_0^a \sqrt{a^2-x^2}\,dx = \int_0^{\pi/2}\sqrt{a^2(1-\sin^2\theta)}.a\cos\theta d\theta = \int_0^{\pi/2} a^2\cos^2\theta d\theta.$

$= \frac{a^2}{2}\int_0^{\pi/2}(1+\cos 2\theta)\, d\theta = \frac{a^2}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = \frac{1}{4}\pi a^2$

(b) $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}\,dx$

সমাধান: ধরি, $\sin x = t \quad \therefore \cos x\, dx = dt.$ when. $x = 0.\ t = 0$ and when $x = \frac{\pi}{2};\ t=1$

$\therefore \int_0^{\pi/2} \cos^3 x\sqrt{\sin x}\,dx = \int_0^{\pi/2}\cos^2 x\sqrt{\sin x}.\cos x dx = \int_0^1 (1-t^2)\sqrt{t}\,dt$

$= \int_0^1 (t^{1/2} - t^{5/2})dt = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1 = \frac{2}{3} - \frac{2}{7} = \frac{14-6}{21} = \frac{8}{21}$ (Ans.)

34. যোজিত ফল নির্ণয় কর ঃ $\int \frac{adx}{(\sqrt{x^2+a^2})^3}$ **[RUET'06-07]**

সমাধান: $\int \frac{adx}{(\sqrt{x^2+a^2})^3}$; let, $x = a\tan\theta \quad \therefore dx = a\sec^2\theta\, d\theta$

$= \int \frac{a^2\sec^2\theta\, d\theta}{\{a^2(1+\tan^2\theta)\}^{3/2}} = \int \frac{a^2\sec^2\theta\, d\theta}{a^3\sec^3\theta} = \frac{1}{a}\int\cos\theta d\theta = \frac{1}{a}\sin\theta + c = \frac{1}{a}\sin\left[\tan^{-1}\left(\frac{x}{a}\right)\right] + c$

35. (i) মান নির্ণয় কর : $\int_0^{\frac{\pi}{4}} \frac{\cos x dx}{\sqrt{2-\sin^2 x}}$ (ii) যোগজ নির্ণয় কর : $\int\left(1+\cos^2\frac{x}{2}\right)dx$ **[BUTex'06-07]**

সমাধান: (i) $\int_0^{\pi/4} \frac{\cos x}{\sqrt{2-\sin^2 x}}dx$

$= \int_0^{1/\sqrt{2}} \frac{dm}{\sqrt{2-m^2}} \qquad = \int_0^{1/\sqrt{2}} \frac{dm}{\sqrt{(\sqrt{2})^2-m^2}}$

$= \left[\sin^{-1}\frac{m}{\sqrt{2}}\right]_0^{1/\sqrt{2}} = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ (Ans.)

let, $\sin x = m$
$dm = \cos x dx$

x	0	$\pi/4$
m	0	$\frac{1}{\sqrt{2}}$

(ii) $\int\left(1+\cos^2 x/2\right)dx \Rightarrow \frac{1}{2}\int\left(2+2\cos^2 x/2\right)dx = \frac{1}{2}\int(2+1+\cos x)\,dx$

$\Rightarrow \frac{1}{2}\int(3+\cos x)dx = \frac{3}{2}x+\frac{1}{2}\sin x + c$ Ans.

36. $y = x^2$ এবং $x = y^2$ পরাবৃত্ত দুইটি দ্বারা সীমাবদ্ধ এলাকার ক্ষেত্রফল নির্ণয় কর। **[BUTex'05-06]**

সমাধান: $f_1(x) = y = x^2$ এবং $f_2(x) = y = \sqrt{x}$; $y = x^2$ ------------(i) ; $x = y^2$ ------------(ii)

(i) ও (ii) হতে, $x = x^4 \qquad (x^3-1)x = 0$

$x = 0 \quad x^3 = 1 \qquad x = 1$

$x = 0$ হলে, $y = 0$ এবং $x = 1$, $y = 1$; $(x, y) = (0, 0)$ ও $(1, 1)$

ক্ষেত্রফল $= \int_0^1 [f_2(x) - f_1(x)]dx = \int_0^1(\sqrt{x}-x^2)dx$

$= \int_0^1\sqrt{x}\,dx - \int_0^1 x^2\,dx = \frac{2}{3}\left[x^{3/2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}-\frac{1}{3} = \frac{1}{3}$ বর্গ একক (Ans.)

37. মান নির্ণয় কর : (a) $\int\frac{\cos x\ dx}{3+\cos^2 x}$ (b) $\int_0^{\pi} x\sin^2 x\,dx$ **[KUET'05-06]**

সমাধান: (a) $\int\frac{\cos x\ dx}{3+\cos^2 x} = \int\frac{\cos x\ dx}{4-\sin^2 x}$; $\int\frac{d(\sin x)dx}{2^2-(\sin x)^2} = \frac{1}{4}\ln\left|\frac{2+\sin x}{2-\sin x}\right| + C$ (Ans.)

সমাধান: (b) $I = \int x\sin^2 x dx = \int\frac{x}{2}(1-\cos 2x)\,dx = \int\frac{x}{2}dx - \int\frac{x\cos 2x}{2}dx = \frac{x^2}{4}-\frac{1}{2}\int x\cos 2x$

Again, $= \int x\cos 2x = x\int\cos 2x dx - \int\left(\frac{dx}{dx}\int\cos 2x\,dx\right)dx$

$= \frac{x\sin 2x}{2} - \int\frac{\sin 2x}{2}dx = \frac{x\sin 2x}{2}+\frac{1}{2}\frac{\cos 2x}{2} \quad \therefore \left[I = \frac{x^2}{4}-\frac{1}{2}\left\{\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}\right\}\right]_0^{\pi}$

$= \left[\frac{x^2}{4}-\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}\right]_0^{\pi} = \left[\frac{\pi^2}{4}-0-\frac{1}{8}\right]+\frac{1}{8} = \frac{\pi^2}{4}$ (Ans.)

38. (a) যোজিত ফল নির্ণয় কর : $\int\frac{xdx}{\sqrt{1-x}}$ **[KUET'03-04, CUET'04-05, RUET'05-06]**

সমাধান: $\int\frac{xdx}{\sqrt{1-x}} = \int\frac{(1-z^2)(-2zdz)}{z} = -2\int(1-z^2)dz = -2\int dz + 2\int z^2\,dz$

$= -2z+2\frac{z^3}{3}+c = -2\sqrt{1-x}+\frac{2}{3}\left(\sqrt{1-x}\right)^3 + c$ [Ans.]

$1-x = z^2$
$\therefore dx = -2zdz$

(b) যোজিত ফল নির্ণয় কর: $\int x \sin^{-1} x^2 dx$

সমাধান: $\frac{1}{2}\int \sin^{-1}z\,dz$

$=\frac{1}{2}\left[\sin^{-1} z\int dz-\int\left\{\frac{d}{dz}(\sin^{-1} z)\int dz\right\}dz\right]$

$=\frac{1}{2}\left[z\sin^{-1} z-\int\frac{z}{(\sqrt{1-z^2})}dz\right]$ $=\frac{1}{2}\left[z\sin^{-1} z+1/2\int\frac{dt}{\sqrt{t}}\right]$

$=\frac{1}{2}\left[x^2\sin^{-1} x^2+\sqrt{t}\right]+c$ $=\frac{1}{2}\left[x^2\sin^{-1} x^2+\sqrt{1-x^4}\right]+c$ (Ans.)

Let, $x^2 = z \Rightarrow xdx = \frac{1}{2}dz$

Let, $1 - z^2 = t$

$\Rightarrow -2zdz = dt$

$\therefore zdz = -\frac{1}{2}dt$

39. (i) $\frac{dy}{dx}$ নির্ণয় কর, যেখানে $y = x^{x^x}$ (ii) মান নির্ণয় কর ঃ $\int e^x \sec x\,(1+\tan x)\,dx$ **[BUTex'05-06]**

সমাধান: (i) $y = x^{x^x}$; $\ln y = x^x \ln x$; $\frac{1}{y}.\frac{dy}{dx} = x^x.\frac{1}{x}+\ln x.\frac{d}{dx}x^x$

Again, $x^x = m \Rightarrow \ln m = x\ln x \Rightarrow \frac{1}{m}.\frac{dm}{dx} = x.\frac{1}{x}+\ln x$ $=1+\ln x$

$\Rightarrow\frac{dm}{dx}=m(1+\ln x)=x^x(1+\ln x)$ $\therefore\frac{d}{dx}x^x = x^x(1+\ln x)$ $\therefore\frac{dy}{dx}=x^{x^x}.x^x\left[\ln(x)\{\ln(x)+1\}+\frac{1}{x}\right]$ (Ans.)

(ii) $\int e^x \sec x(1+\tan x)dx$ $[\because\int e^x[f(x)+f'(x)]dx=e^x f(x)+c]$

$=\int e^x(\sec x+\sec x\tan x)\,dx$ $=e^x\sec x+c$ (Ans.)

40. যোগজ নির্ণয় কর ঃ $\int x^2(\ln x)^2 dx$ **[BUET'05-06]**

সমাধান: $\int x^2(\ln x)^2 dx.$ $=(\ln x)^2\frac{x^3}{3}-\int\frac{2\ln x}{x}.\frac{x^3}{3}dx.$ $=\frac{x^3}{3}(\ln x)^2-\frac{2}{3}\int \ln x.x^2dx$

$=\frac{x^3}{3}(\ln x)^2-\frac{2}{3}\left[\frac{x^3}{3}\ln x-\int\frac{1}{x}\frac{x^3}{3}dx\right]=\frac{x^3}{3}(\ln x)^2-\frac{2x^3}{9}\ln x+\frac{2}{3}.\frac{1}{3}.\frac{x^3}{3}+c.$

$=\frac{x^3}{3}(\ln x)^2-\frac{2}{9}x^3(\ln x)+\frac{2x^3}{27}+c$ (Ans.)

41. যোগজ নির্ণয় কর ঃ $\int e^x \sec x(1+\tan x)dx$ **[BUET'04-05]**

সমাধান: $\int e^x \sec x(1+\tan x)dx$

$=\sec x\int e^x dx-\int\left\{\frac{d}{dx}(\sec x)\int e^x dx\right\}dx+\int e^x\sec x\tan x dx$

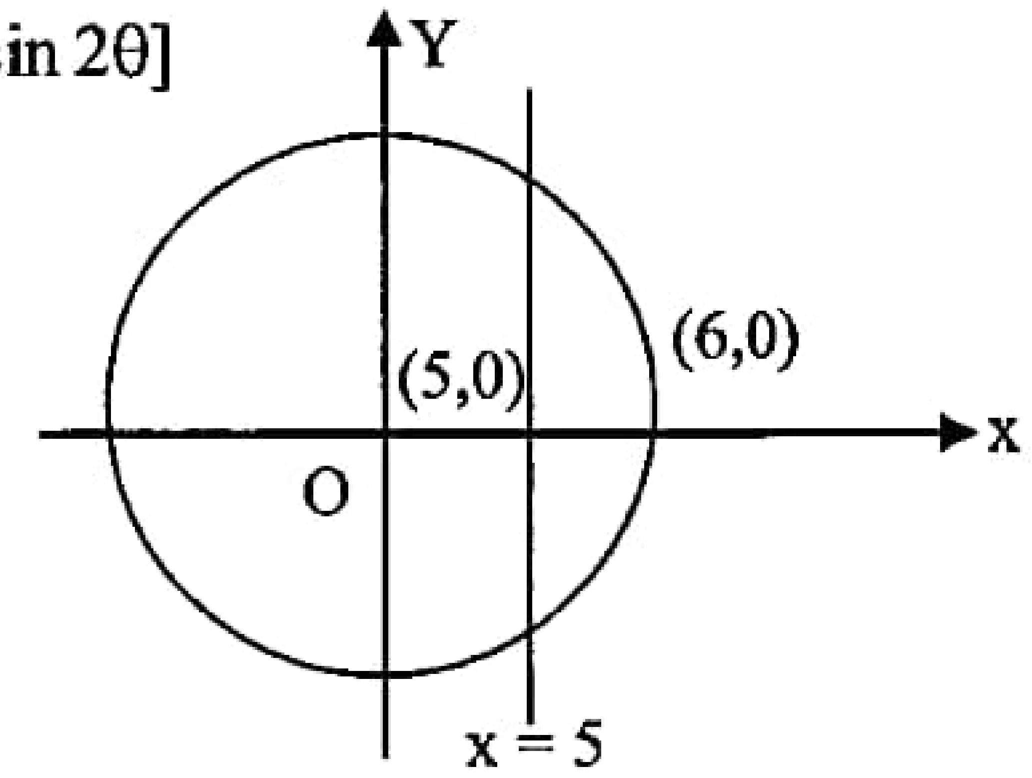
$=e^x\sec x-\int\sec x\tan x.e^x dx+\int e^x\sec x\tan x dx+c_1=e^x\sec x+c$ (Ans.)

42. $x^2+y^2=36$ একটি বৃত্ত এবং $x=5$ সরলরেখা দ্বারা আবদ্ধ ক্ষুদ্রতর ক্ষেত্রটির ক্ষেত্রফল নির্ণয় কর। [CUET'04-05]

সমাধান: ক্ষেত্রফল $=2\int_5^6 y\,dx \;=2\int_5^6 \sqrt{36-x^2}\,dx$

$x=6\sin\theta$. then, $dx=6\cos\theta d\theta$; $x=5.\ \theta=\sin^{-1}\frac{5}{6}$; $x=6$; $\theta=\pi/2$

$\therefore\ 2\int y dx=2\int 36\cos^2\theta d\theta=36\int(1+\cos 2\theta)\,d\theta=36[\theta+\frac{1}{2}\sin 2\theta]$



$\therefore 2\int_5^6 y dx=36\left[\theta+\frac{1}{2}\sin 2\theta\right]_{\sin^{-1}5/6}^{\pi/2}$

$=36\left[\pi/2+0-\sin^{-1}\left(\frac{5}{6}\right)-\frac{1}{2}\sin\left(2\sin^{-1}\frac{5}{6}\right)\right]$

$=36\left[\pi/2-\sin^{-1}(5/6)-\frac{1}{2}\sin\left\{2\sin^{-1}\left(\frac{5}{6}\right)\right\}\right]$ Ans.

43. (a) যোজিত ফল নির্ণয় কর : $\int_1^2 \frac{dx}{x^2\sqrt{4-x^2}}$ [BUTex'02-03, RUET'04-05]

X	θ
1	$\frac{\pi}{6}$
2	$\frac{\pi}{2}$

$x=2\sin\theta$

$\therefore dx=2\cos\theta\,d\theta$

সমাধান: $\int_{\pi/6}^{\pi/2}\frac{2\cos\theta\,d\theta}{4\sin^2\theta\ 2\cos\theta}=\frac{1}{4}\int_{\pi/6}^{\pi/2}\text{cosec}^2\theta\,d\theta$

$=\frac{1}{4}[-\cot\theta]_{\pi/6}^{\pi/2}=\frac{1}{4}\left(-\cot\frac{\pi}{2}+\cot\frac{\pi}{6}\right)=\frac{1}{4}(0+\sqrt{3})=\frac{\sqrt{3}}{4}$. (Ans.)

(b) $x^2+y^2=r^2$ দ্বারা গঠিত বৃত্তের ক্ষেত্রফল নির্ণয় কর।

সমাধান: ক্ষেত্রফল $=4\int_0^r y dx=4\int_0^r\sqrt{r^2-x^2}\,dx$

$y=\sqrt{r^2-x^2}$, ধরি, $x=r\sin\theta$

$\therefore dx=r\cos\theta\ d\theta$

X	θ
0	0
r	$\frac{\pi}{2}$

$=4\int_0^{\pi/2} r^2\cos^2\theta\,d\theta=2r^2\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta$

$=2r^2\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi/2}=2r^2\left[\left(\frac{\pi}{2}+\frac{1}{2}.0\right)-(0+0)\right]=\pi r^2$. (Ans.)

44. মান নির্ণয় কর : (i) $\int_0^\pi\sqrt{1+\cos x}\,dx$ (ii) $\int_0^1\frac{\cos^{-1}x}{\sqrt{1-x^2}}\,dx$ [BUTex'04-05]

সমাধান: (i) $\int_0^\pi\sqrt{1+\cos x}\,dx=\int_0^\pi\sqrt{2\cos^2\frac{x}{2}}\,dx=\sqrt{2}\int_0^\pi\cos\frac{x}{2}dx=2\sqrt{2}\left[\sin\frac{x}{2}\right]_0^\pi$

$=2\sqrt{2}\,\left|\sin\pi/2-\sin 0\right|=2\sqrt{2}$ Ans.

সমাধান: ii) $\int_0^1 \frac{\cos^{-1}x}{\sqrt{1-x^2}}dx$, Let, $\cos^{-1}x = m$; $x=0$ হলে, $m=\frac{\pi}{2}$; $x=1$ হলে, $m=0$

$$\Rightarrow -\frac{1}{\sqrt{1-x^2}}dx = dx \Rightarrow \frac{dx}{\sqrt{1-x^2}} = -dx$$

$$\therefore \int_{\pi/2}^{0} -m\,dm = -\frac{1}{2}[m^2]_{\pi/2}^{0} = -\frac{1}{2}\left[0^2 - \frac{\pi^2}{4}\right] = \frac{\pi^2}{8} \quad \text{(Ans.)}$$

45. যোজিত ফল নির্ণয় কর : $\int \sin(\ln x)dx$ **[KUET'04-05]**

সমাধান: $\int \sin(\ln x)dx = \sin(\ln x).\int dx - \int\left(\frac{d}{dx}(\sin(\ln x))\int dx\right)dx$

$$= \sin(\ln x)x - \int \frac{\cos(\ln x)}{x}.x\,dx = x\sin(\ln x) - [x\cos(\ln x) + \int \sin(\ln x)dx]$$

$$2\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x) \quad \therefore \int \sin(\ln x)dx = \frac{1}{2}\{x\sin(\ln x) - x\cos(\ln x)\} + c$$

46. নিচের ফাংশনগুলির যোজিত ফল নির্ণয় কর: **[KUET'04-05]**

(a) $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$ (b) $\int \frac{dx}{5+4x-x^2}$

সমাধান: (a) $\int \frac{\sqrt{\tan x}\ dx}{\frac{\sin x \cos x}{\cos^2 x} \times \cos^2 x} = \int \frac{\sqrt{\tan x}.\sec^2 x\,dx}{\tan x}$

$$\int \frac{\sec^2 x}{\sqrt{\tan x}}dx\ [\tan x = y \ \Rightarrow \sec^2 x dx = dy] \ = \int \frac{dy}{\sqrt{y}} = 2\sqrt{y} + c = 2\sqrt{\tan x} + c \ \text{(Ans.)}$$

(b) $\int \frac{dx}{5+4x-x^2} = \int \frac{-dx}{x^2-4x-5} = \int \frac{-dx}{x^2-5x+x-5} = \int \frac{-dx}{(x-5)(x+1)}$

$$= -\int \frac{dx}{(x-5)6} - \int \frac{dx}{(-6)(x+1)} = -\frac{1}{6}\ln(x-5) + \frac{1}{6}\ln(x+1) + c \quad \text{(Ans.)}$$

47. মান নির্ণয় কর : $\int \frac{dx}{e^{2x}-3e^x}$ **[KUET'04-05]**

সমাধান: ধরি, $e^x = z \Rightarrow e^x.\,dx = dz \Rightarrow dx = \frac{dz}{z}$

$\int \frac{dz}{z(z^2-3z)} = \int \frac{dz}{z^2(z-3)}$ এখানে, $\frac{1}{z^2(z-3)} = \frac{A}{(z-3)} + \frac{B}{z^2} + \frac{C}{z}$; $A=\frac{1}{9}$; $B=-\frac{1}{3}$; $C=-\frac{1}{9}$

$$I = \int\left\{\frac{1}{9(z-3)} - \frac{1}{3z^2} - \frac{1}{9z}\right\}dz = \frac{1}{9}\int\frac{dz}{z-3} - \frac{1}{3}\int\frac{dz}{z^2} - \frac{1}{9}\int\frac{dz}{z} = \frac{1}{9}\ln(z-3) - \frac{1}{9}\ln z + \frac{1}{3z} + c$$

$$= \frac{1}{9}\ln(e^x-3) - \frac{1}{9}\ln(e^x) + \frac{1}{3e^x} + c = \frac{1}{9}\ln\left(e^x-3\right) - \frac{x}{9} + \frac{1}{3e^x} + c$$

48. $\int_2^3 \frac{dx}{(x-1)\sqrt{x^2-2x}}$ এর মান নির্ণয় কর : **[BUET'01-02,03-04]**

সমাধান: $\int_2^3 \frac{dx}{(x-1)\sqrt{x^2-2x}}$

Let, $x^2-2x=z^2$
$\therefore 2zdz=(2x-2)dx \therefore zdz=(x-1)dx$

$=\int_2^3 \frac{(x-1)dx}{(x^2-2x+1)\sqrt{x^2-2x}} = \int_0^{\sqrt{3}} \frac{zdz}{(z^2+1)\sqrt{z^2}} = \int_0^{\sqrt{3}} \frac{dz}{z^2+1} = \left[\frac{1}{1}\tan^{-1}z\right]_0^{\sqrt{3}}$

$=\tan^{-1}\sqrt{3}-\tan^{-1}0=\tan^{-1}\tan\frac{\pi}{3}=\frac{\pi}{3}$

বিকল্প: $\int_2^3 \frac{dx}{(x-1)\sqrt{x^2-2x}} = \int_2^3 \frac{d(x-1)}{(x-1)\sqrt{(x-1)^2-1}} = \left[\sec^{1}(x-1)\right]_2^3 = \sec^{-1}2-\sec^{-1}1=\frac{\pi}{3}-0=\frac{\pi}{3}$

49. মান নির্ণয় কর : $\int_0^{\pi/2} \sqrt{\cos x}\sin^3 xdx$ **[KUET'03-04]**

সমাধান: $\int_0^{\pi/2} \sqrt{\cos x}\sin^3 xdx = \int_0^{\pi/2} \sqrt{\cos x}\sin^2 x\sin xdx = \int_0^{\pi/2} \sqrt{\cos x}(1-\cos^2 x)\sin xdx$

$\therefore -\int_1^0 \sqrt{z}(1-z^2)dz = -\int_1^0 z^{\frac{1}{2}}dz + \int_1^0 z^{5/2}dz$

[Let, $\cos x = z$ বা, $-\sin xdx = dz$ for, $x=0, z=1$ $x=\frac{\pi}{2}, z=0$]

$=-\frac{2}{3}\left[z^{3/2}\right]_1^0+\frac{2}{7}\left[z^{7/2}\right]_1^0 = 2/3 - 2/7 = \frac{8}{21}$ (Ans.)

50. মান নির্ণয় কর : (a) $\int_0^1 x^3\sqrt{1+3x^4}\, dx$ **[RUET'03-04]**

সমাধান: (a) let, $1+3x^4=z \Rightarrow x^3dx=\frac{1}{12}dz$; $x=1, z=4$ $x=0, z=1$

$\therefore I=\frac{1}{12}\int_1^4\sqrt{z}dz = \frac{1}{12}\cdot\frac{2}{3}\left[z^{3/2}\right]_1^4 = \frac{1}{18}\left[4^{3/2}-1^{3/2}\right]=\frac{7}{18}$ (Ans.)

51. মান নির্ণয় কর : (i) $\int_0^{\pi/2} (1+\cos x)^2\sin x\, dx$ (ii) $\int_{-2}^5 \frac{7x\, dx}{\sqrt{(x^2+3)}}$ **[BUTex'03-04]**

সমাধান:(i) $\int_0^{\pi/2} (1+\cos x)^2 \sin x\, dx = \int_2^1 z^2(-dz)$

$=-\left[\frac{z^3}{3}\right]_2^1 = -\left[\frac{1}{3}-\frac{8}{3}\right]=\frac{8}{3}-\frac{1}{3}=\frac{7}{3}$ (Ans.)

Let, 1 + cosx = z

$\therefore$ sinx dx = – dz

x	0	π/2
z	2	1

(ii) $\int_{-2}^5 \frac{7x\, dx}{\sqrt{(x^2+3)}} = \frac{7}{2}\int_7^{28}\frac{dz}{\sqrt{z}} = \frac{7}{2}\left[2\sqrt{z}\right]_7^{28}$

$=7\left[\sqrt{28}-\sqrt{7}\right]=7\left(2\sqrt{7}-\sqrt{7}\right)=7\sqrt{7}$ (Ans.)

Let, $x^2+3=z \Rightarrow xdx=\frac{dz}{2}$

x	–2	5
z	7	28

52. মান নির্ণয় কর $\int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\sqrt{\sin x}} dx$ [CUET'03-04]

সমাধান: $\int \frac{\cos^3 x}{\sqrt{\sin x}} = \int \frac{\cos x\,(1-\sin^2 x)\,dx}{\sqrt{\sin x}} = \int \frac{(1-z^2)\,dz}{\sqrt{z}}$ | Let, $\sin x = z \therefore \cos x\,dx = dz$

$= \int \frac{1}{\sqrt{z}} dz - \int z^{\frac{3}{2}} dz = 2\sqrt{z} - \frac{2}{5} z^{\frac{5}{2}} = 2\sqrt{\sin x} - \frac{2}{5} \sin^{\frac{5}{2}} x$

$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\sqrt{\sin x}} dx = \left[2\sqrt{\sin x} - \frac{2}{5}\sin^{\frac{5}{2}} x\right]_0^{\pi/2} = 2 - \frac{2}{5} = \frac{8}{5}$ (Ans.)

53. যোজিত ফল নির্ণয় করঃ $\int \frac{xe^x}{(x+1)^2} dx$ [CUET'03-04]

সমাধান: $\int \frac{xe^x}{(x+1)^2} dx = \int e^x \frac{(x+1)-1}{(x+1)^2} dx = e^x . \frac{1}{x+1} + c$ (Ans.) | $[\int e^x \{f(x) + f'(x)\}dx = e^x f(x) + C]$

54. যোজিত ফল নির্ণয় ঃ $\int \sqrt{1+e^x}\, dx$ [RUET'03-04]

সমাধান: $\int \sqrt{1+e^x}\, dx = \int \sqrt{z^2} \frac{2zdz}{z^2-1} = \int \frac{2z^2 dz}{z^2-1}$ | ধরি, $1 + e^x = z^2 \quad e^x = z^2 - 1;\ e^x dx = 2zdz$

$= \int 2dz + \int \frac{2dz}{z^2-1} = 2z + 2 \times \frac{1}{2.1} \ln \frac{z-1}{z+1} + C = 2\left(\sqrt{1+e^x}\right) + \ln \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1} + C$

55. নির্দিষ্ট ইন্টিগ্র্যালটি নির্ণয় কর ঃ $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx$ [BUET'02-03]

সমাধান: $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx = \int_{\log 2}^{1} e^z \left(\frac{1}{z} - \frac{1}{z^2}\right) dz$ | Let, $\log x = z \Rightarrow x = e^z \Rightarrow dx = e^z dz$ | $x = 2$ হলে, $z = \log 2$; $x = e$ হলে, $z = 1$

$= \int_{\log 2}^{1} e^z \left\{\frac{1}{z} + \frac{d}{dz}\left(\frac{1}{z}\right)\right\} dz = \left[\frac{e^z}{z}\right]_{\log 2}^{1} = e - \frac{e^{\log 2}}{\log 2} = e - \frac{2}{\log 2}$ (Ans.)

56. সমাকলন কর ঃ $\int \frac{d\theta}{1+3\cos^2\theta}$. [BUTex'02-03]

সমাধান: $\int \frac{d\theta}{1+3\cos^2\theta} = \int \frac{\sec^2\theta}{\sec^2\theta + 3} d\theta = \int \frac{\sec^2\theta}{\tan^2\theta + 4} d\theta = \int \frac{dz}{z^2 + 2^2}$ [let $z = \tan\theta$]

$= \frac{1}{2}\tan^{-1}\frac{z}{2} + c = \frac{1}{2}\tan^{-1}\left(\frac{\tan\theta}{2}\right) + c$ (Ans.)

57. ক) সমাকলন কর ঃ $\int \frac{\sin x+\cos 2x}{1-\sin x}dx$ খ) মান নির্ণয় কর ঃ $\int_0^{\pi/2} \frac{dx}{\sin x+\cos x}$ [BUTex'01-02]

সমাধান: ক) $\int \frac{\sin x+\cos 2x}{1-\sin x}dx = \int \frac{\sin x+1-2\sin^2 x}{1-\sin x}dx = \int \frac{-2\sin^2 x+2\sin x-\sin x-1}{1-\sin x}dx$

$= \int \frac{-(2\sin x-1)(\sin x-1)}{-(\sin x-1)}dx = -2\cos x - x + c$ (Ans.)

খ) $\int_0^{\pi/2} \frac{dx}{\sin x+\cos x}dx = \int_0^{\pi/2} \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} = \int_0^{\pi/2} \frac{\sec^2\frac{x}{2}dx}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}} \therefore \int_0^1 \frac{2dx}{2z+1-z^2}$

[Let, $\tan\frac{x}{2} = z \Rightarrow \sec^2\frac{x}{2}dx = 2dz$; $x=\frac{\pi}{2}$ হলে $z=1$; $x=0$ হলে $z=0$]

$\therefore I = 2\int_0^1 \frac{dz}{(\sqrt{2})^2-(z-1)^2} = 2\frac{1}{2\sqrt{2}}\left[\ln\frac{\sqrt{2}+z-1}{\sqrt{2}-z+1}\right]_0^1 = \frac{1}{\sqrt{2}}\ln\frac{\sqrt{2}}{\sqrt{2}} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}$ (Ans.)

58. ইন্টিগ্রেট কর ঃ $\int \frac{x^2}{e^{x^3}-e^{-x^3}}dx$ সমাকলন ধ্রুবক [BUET'01-02]

সমাধান: $\frac{1}{3}\int \frac{3x^2}{e^{x^3}-e^{-x^3}}dx = \frac{1}{3}\int \frac{1}{e^z-e^{-z}}dz$ $[z=x^3;\ dx=3x^2\ dx] = \frac{1}{3}\int \frac{e^z dz}{e^{2z}-1}$

$= \frac{1}{6}\times\ln\left|\frac{e^z-1}{e^z+1}\right|+c$ [c সমাকলন ধ্রুবক] $= \frac{1}{6}\ln\left|\frac{e^{x^3}-1}{e^{x^3}+1}\right|+c$ (Ans.)

59. মান নির্ণয় কর ঃ $\int_0^a \frac{(a^2-x^2)}{(a^2+x^2)^2}dx$ [BUET'00-01]

সমাধান: মনে করি, $z=\frac{x}{a^2+x^2}$ $\therefore dz = \frac{a^2-x^2}{(a^2+x^2)^2}.dx.$ $x=0$ হলে, $z=0$

$x=a$ হলে, $z=\frac{1}{2a}$. $\int_0^{\frac{1}{2a}} dz = [z]_0^{\frac{1}{2a}} = \frac{1}{2a}$. (Ans.)

60. যোগজ নির্ণয় করঃ $\int \frac{dx}{(x^2+9)^2}$ [BUET'00-01]

সমাধান: $\int \frac{dx}{(x^2+9)^2}$ | let, $x=3\tan\theta \Rightarrow dx = 3\sec^2\theta d\theta$

$= \int \frac{3\sec^2\theta d\theta}{81(\sec^2\theta)^2} = \int \frac{d\theta}{27\sec^2\theta} = \frac{1}{54}\int 2\cos^2\theta d\theta = \frac{1}{54}\int(1+\cos 2\theta)d\theta$

$= \frac{1}{54}\left(\theta+\frac{1}{2}\sin 2\theta\right)+c = \frac{1}{54}\left(\tan^{-1}\frac{x}{3}+\frac{1}{2}.\frac{2.\frac{x}{3}}{1+\frac{x^2}{9}}\right)+c = \frac{1}{54}\left(\tan^{-1}\frac{x}{3}+\frac{3x}{9+x^2}\right)+c$ (Ans.)

MCQ

01. $\int \frac{dx}{\sqrt{(-2x^2+4x+1)}}$ এর মান কোনটি? **[KUET'18-19]**

(a) $\frac{1}{\sqrt{2}}\sin^{-1}\left\{\sqrt{\frac{2}{3}}\,(x-1)\right\}$ (b) $\frac{1}{5}\sin^{-1}x$ (c) $\frac{1}{\sqrt{3}}\sin^{-1}(x+1)$
(d) $\frac{1}{6}\cos^{-1}(x+1)$ (e) $\frac{1}{\sqrt{2}}\cos^{-1}(x-1)$

সমাধান: (a); $\int \frac{dx}{\sqrt{(-2x^2+4x+1)}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{-\left(x^2-2x+\frac{1}{2}-\frac{1}{2}-1\right)}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2-(x-1)^2}}$

$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x-1}{\sqrt{\frac{3}{2}}}\right) + c = \frac{1}{\sqrt{2}}\sin^{-1}\left\{\sqrt{\frac{2}{2}}(x-1)\right\} + c$

02. $\int_{-1}^{1}\frac{e^x dx}{1+2e^x}$ এর মান কোনটি? **[KUET'18-19]**

(a) $\ln(1+e)$ (b) $\frac{1}{2}\ln\frac{1+2e}{1+2e^{-1}}$ (c) $\frac{1}{3}\ln(1+2e)$ (d) $\frac{1}{4}\ln(1-2e)$ (e) $\frac{1}{3}\ln(1+3e)$

সমাধান: (b); using calculator.

03. $\int \frac{1}{e^{ax}+e^{-ax}}dx = ?$ **[SUST'18-19]**

(a) $\frac{1}{a}\tan^{-1}(e^{ax}) + c$ (b) $\frac{1}{a}\cot^{-1}(e^{ax}) + c$ (c) $\frac{1}{a}\cot^{-1}(1+e^{ax}) + c$
(d) $\frac{1}{a}\tan^{-1}(1+e^{ax}) + c$ (e) $\ln(e^{ax}+e^{-ax}) + c$

সমাধান: (a); $\int \frac{dx}{e^{ax}+e^{-ax}} = \int \frac{e^{ax}dx}{e^{2ax}+1} = \frac{1}{a}\int\frac{dz}{z^2+1}$ [$e^{ax} = z$ ধরে] $= \frac{1}{a}\tan^{-1}z + c = \frac{1}{a}\tan^{-1}(e^{ax}) + c$

04. $\int_0^a \sqrt{(a^2-x^2)}\,dx$ এর মান কোনটি? **[Ans: a] [KUET'17-18]**

(a) $\frac{\pi a^2}{4}$ (b) $\frac{\pi a^2}{3}$ (c) $\frac{\pi a^2}{5}$ (d) $\frac{\pi a^2}{7}$ (e) $3\pi a^2$

05. $\int \frac{6x-7}{4x^2-4x+5}dx$ এর মান হলো- **[KUET'17-18]**

(a) $\frac{3}{2}\log(4x-4x+5) + \frac{1}{2}\tan^{-1}\frac{2x-1}{2}$
(b) $3\log(4x^2-4x+5) + \tan^{-1}\frac{2x-1}{2} + c$
(c) $\frac{3}{2}\log(4x-4x+5) + 2\tan^{-1}\left(\frac{2x-1}{2}\right) + c$
(d) $\frac{3}{2}\log(4x^2-4x+5) + \tan^{-1}\frac{2x-1}{2} + c$
(e) $3\log(4x^2-4x+5) + \frac{1}{2}\tan^{-1}\frac{2x-1}{2} + c$

সমাধান: ((No correct answer)); let, $z = 4x^2-4x+5 \therefore dz = (8x-4)\,dx$

এখন $\int\frac{6x-7}{4x^2-4x+5}dx = \int\frac{\frac{6}{8}(8x-4)-4}{4x^2-4x+5}dx = \frac{3}{4}\int\frac{dz}{z} - 4\int\frac{dx}{4x^2-4x+5}$

$= \frac{3}{4}\ln z - \frac{4}{4}\int\frac{dx}{x^2-x+\frac{5}{4}} = \frac{3}{4}\ln|4x^2-4x+5| - \int\frac{dx}{x^2-2.x.\frac{1}{2}+\frac{1}{4}+1}$

$= \frac{3}{4}\ln|4x^2-4x+5| - \int\frac{dx}{\left(x-\frac{1}{2}\right)^2+1^2} = \frac{3}{4}\ln|\,4x^2-4x+5| - \tan^{-1}\left(\frac{2x-1}{2}\right)$

06. $\int_0^{n\frac{\pi}{2}}\cos^2\theta\, d(\tan\theta)$ এর মান কত? **[SUST'17-18]**

(a) $\tan\left(n\frac{\pi}{2}\right)$ (b) $\frac{1}{3}\cos^3\left(n\frac{\pi}{2}\right)$ (c) $\frac{1}{3}\sin^3\left(n\frac{\pi}{2}\right)$ (d) $n\pi$ (e) $n\frac{\pi}{2}$

সমাধান: (e); $\int_0^{\frac{n\pi}{2}} \cos^2\theta \cdot d(\tan\theta) = \int_0^{\frac{n\pi}{2}} \cos^2\theta \cdot \sec^2\theta \cdot d\theta = \int_0^{\frac{n\pi}{2}} d\theta = \frac{n\pi}{2}$

07. 3 একক ব্যাস ও 15 একক উচ্চতা বিশিষ্ট একটি সিলিন্ডার 3 একক ব্যাস বিশিষ্ট সুষম ও মসৃণ গোলক দ্বারা পূর্ণ করা হলো। সিলিন্ডারের কত অংশ ফাঁকা থাকবে? **[SUST'17-18]**

(a) $\frac{1}{15}$ (b) $\frac{2}{15}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ (e) 0

সমাধান: (c); সিলিন্ডারটি $\frac{15}{3} = 5$ টি গোলক দ্বারা পূর্ণ করতে হবে।

$\therefore$ ফাঁকা অংশ $= \frac{\frac{\pi d^2 h}{4} - 5\times\frac{4}{3}\pi\frac{d^3}{8}}{\frac{\pi d^2 h}{4}} = 1 - 5\times\frac{4}{3}\times\frac{\pi d^3}{8}\times\frac{4}{\pi d^2 h} = 1 - \frac{10}{3}\frac{d}{h} = 1 - \frac{10}{3}\times\frac{3}{15} = \frac{1}{3}$

08. $\int_0^{\frac{\pi}{4}} \frac{\sin 2\theta}{\sin^4\theta+\cos^4\theta} d\theta$ এর মান কোনটি? **[KUET'16-17]**

(a) $\frac{2\pi}{5}$ (b) $\frac{3\pi}{7}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{4}$ (e) $\frac{\pi}{3}$

সমাধান: (d); $\int_0^{\frac{\pi}{4}} \frac{\sin 2\theta}{\sin^4\theta+\cos^4\theta} d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{2\sin\theta\cos\theta}{\sin^4\theta+\cos^4\theta} d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{2\tan\theta\sec^2\theta}{\tan^4\theta+1} d\theta$

$= 2\int \frac{z}{1+z^4} dz$

$= \int_0^1 \frac{dx}{1+x^2}$

$= \tan^{-1}(x)|_0^1$

$= \frac{\pi}{4}$

Let, $z = \tan\theta$, $dt = \sec^2\theta\, d\theta$

θ	z
0	0
$\frac{\pi}{4}$	1

Let, $z^2 = x$, $2zdz = dx$

z	x
0	0
1	1

09. $\int \frac{x^2-1}{x^2-4} dx$ এর মান কোনটি? **[KUET'16-17]**

(a) $x + \frac{3}{4}\ln\left|\frac{x+2}{x-2}\right| + c$ (b) $x + \frac{3}{4}\ln\left|\frac{x-2}{x+2}\right| + c$ (c) $x + \frac{3}{2}\ln\left|\frac{x-2}{x+2}\right| + c$ (d) $x + \frac{3}{2}\ln\left|\frac{x+2}{x-2}\right| + c$ (e) $x + \frac{1}{2}\ln\left|\frac{x-2}{x+2}\right| + c$

সমাধান: (b); $\int \frac{x^2-1}{x^2-4}dx = \int \frac{x^2-4+3}{x^2-4}dx = \int dx + 3\int\frac{dx}{x^2-4} = x + \frac{3}{2.2}\ln\left|\frac{x-2}{x+2}\right| + c = x + \frac{3}{4}\ln\left|\frac{x-2}{x+2}\right| + c$

10. যদি $\int_0^4 f(x)dx = 6$ হয় তবে $\int_{-1}^3 f(x+1)dx$ এর মান কত? **[BUTex'16-17]**

(a) 5 (b) 7 (c) 0 (d) 6

সমাধান: (d); ধরি, $x + 1 = y$, তাহলে $dx = dy$

$\therefore \int_{-1}^3 f(x+1)dx = \int_0^4 f(y)dy = \int_0^4 f(x)dx = 6$

11. $\int \frac{\cos x - \cos 2x}{1-\cos x} dx = ?$ **[BUTex'16-17]**

(a) $2x + \sin x + c$ (b) $x + \sin x + c$ (c) $x + \sin 2x + c$ (d) $x + 2\sin x + c$

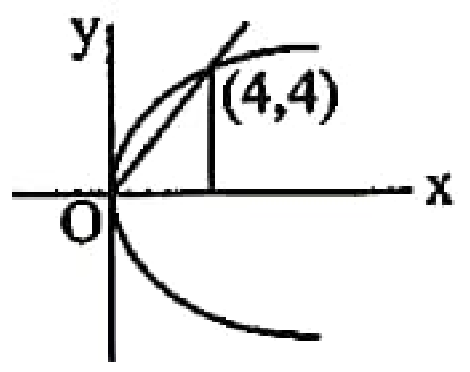
সমাধান: (d); $\int \frac{\cos x-\cos 2x}{1-\cos x}dx = \int \frac{2\sin\frac{x}{2}\sin\frac{3x}{2}}{2\sin^2\frac{x}{2}}dx = \int \frac{\sin\frac{3x}{2}}{\sin\frac{x}{2}}dx = \int \frac{3\sin\frac{x}{2} - 4\sin^3\frac{x}{2}}{\sin\frac{x}{2}}dx$

$= \int\left(3 - 4\sin^2\frac{x}{2}\right)dx = \int\{3 - 2(1-\cos x)\}dx = x + 2\sin x + c$

12. $y^2 = 4x$ বক্ররেখা এবং $y = x$ সরলরেখা দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল হবে? **[BUTex'16-17]**

(a) $\frac{3}{8}$ sq. units (b) 3 sq. units (c) $\frac{8}{3}$ sq. units (d) 8 sq. units

সমাধান: (c); $A = \int_0^4 2\sqrt{x}\,dx - \int_0^4 x\,dx = \left[\frac{4}{3}(x)^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4 = \left\{\frac{4}{3}\times 4^{\frac{3}{2}} - \frac{4^2}{2}\right\} = \frac{8}{3}$ sq. units



13. a এর মান কত হলে $\int_1^a\{2 + x\ln(x^2+5)\}\,dx + \int_1^a\{3 - x\ln(x^2+5)\}\,dx = 30$ [SUST'16-17]

(a) 2 (b) ln 3 (c) 4 (d) ln 5 (e) 7

সমাধান: (e); $\int_1^a\{2 + x\ln(x^2+5)\}\,dx + \int_1^a\{3 - x\ln(x^2+5)\}\,dx = 30$

$[5x]_1^a = 30$; $5a - 5 = 30 \Rightarrow a = 7$

14. $f\left(\frac{x}{x+1}\right) = x + 1$ হলে $\int f(x+3)dx = ?$ [SUST'16-17]

(a) $\ln x + c, x \neq 3$
(b) $\ln|x+4| + c, x \neq -4$
(c) $\ln|x+3| + c, x \neq -3$
(d) $-\ln|x+2| + c, x \neq -2$
(e) $\ln|x+1| + c, x \neq -1$

সমাধান: (d); $y = \frac{x}{x+1}, x = \frac{-y}{y-1}$ $\therefore f(y) = \frac{-y}{y-1} + 1$; $f(x) = \frac{x}{1-x} + 1$ $\therefore \int f(x+3)dx = -\ln|x+2| + c$

15. $\int x^x(1 + \log x)dx = ?$ [SUST'16-17]

(a) $x\log x + c$ (b) $c + x^x \log x$ (c) $\log(x^x + 1) + c$ (d) $x^x + c$ (e) $(x+1)\log x + c$

সমাধান: (d); $\frac{d}{dx}(x^x) = x^x(1 + \ln x)$ $\therefore \int x^x(1 + \ln x)dx = x^x + c$

16. a এর মান কত হলে $\int_{\sqrt5}^a \frac{2x}{x^2-4}dx = \ln 3a$? [SUST'15-16]

(a) $\sqrt6$ (b) $2\sqrt2$ (c) $2\sqrt3$ (d) 3 (e) 4

সমাধান: (e); $\int_{\sqrt5}^a \frac{2x}{x^2-4}dx = [\ln|(x^2-4)|]_{\sqrt5}^a = \ln\frac{(a^2-4)}{(\sqrt5)^2-4} = \ln(a^2-4)$

প্রশ্নমতে, $\ln(a^2-4) = \ln(3a)$ or, $a^2 - 4 = 3a$ or, $a^2 - 3a - 4 = 0$

সমাধান করে পাই, $a = 4$ অথবা, $a = (-1)$ কিন্তু, $a = (-1)$ হলে $\ln(3a) = \ln(-3)$ যা অসংজ্ঞায়িত

সুতরাং, a এর গ্রহণযোগ্য মান $= 4$ ।

17. $x = 0$ বিন্দুতে $f(x) = \ln(2x+1)$ এবং নিচের কোন বক্ররেখার স্পর্শকের ঢাল সমান হবে? [SUST'15-16]

(a) $f(x) = 1 + x + 2x^2$
(b) $f(x) = -2x - 2x^2 + 4x^3$
(c) $f(x) = x - \frac12 x^2 + \frac13 x^3$
(d) $f(x) = 2x - 2x^2 + \frac83 x^3$
(e) $f(x) = x - \frac{x^2}{2!} + \frac{x^3}{3!}$

সমাধান: (d); $f(x) = \ln(2x+1) \Rightarrow f'(x) = \frac{2}{2x+1}$; $x = 0$ বিন্দুতে ঢাল $= 2$

আবার, $f(x) = 2x - 2x^2 + \frac83 x^3$ $\therefore f'(x) = 2 - 4x + 8x^2$; $x = 0$ বিন্দুতে ঢাল $= 2$

অন্যকোন অপশনের ক্ষেত্রে $x = 0$ বিন্দুতে ফাংশনের ঢাল 2 হয় না।

18. $\int_{-1}^2 |x|dx$ এর মান কত? [SUST'15-16]

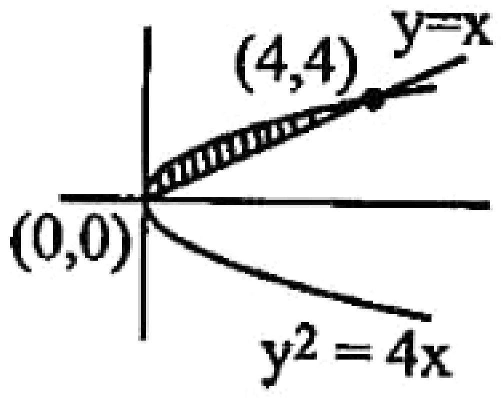
(a) $-\frac52$ (b) $\frac32$ (c) 2 (d) $\frac52$ (e) 3

সমাধান: (d); $\int_{-1}^2 |x|\,dx = \int_{-1}^0 -x\,dx + \int_{-1}^2 x\,dx = \left[-\frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^2 = \left(2 + \frac12\right) = \frac52$

19. $y^2 = 4x$ এবং $y = x$ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল কত? [BUTex'15-16]

(a) 3 (b) 8 (c) $\frac83$ (d) $\frac38$

সমাধান: (c);

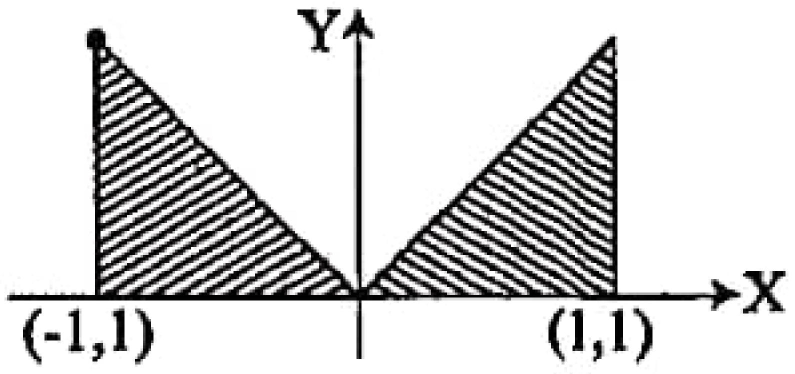


আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= \int_0^4(2\sqrt x - x)\,dx = \left[2x^{\frac32} \times \frac23 - \frac{x^2}{2}\right]_0^4 = \left(2 \times 8 \times \frac23 - \frac{16}{2}\right) = \frac83$ বর্গ একক

20. $\int_{-1}^1 |x|dx = ?$ [SUST'14-15, BUTex'15-16]

(a) 2 (b) -1 (c) 1 (d) 0

সমাধান: (c);



$\int_{-1}^{1}|x|dx$ = ছায়াকৃত অংশের ক্ষেত্রফল $= \frac{1}{2}\times 1\times 1+\frac{1}{2}\times 1\times 1 = 1$ একক

বিকল্প: $\int_{-1}^{1}|x|dx = -\int_{-1}^{0}x.dx+\int_{0}^{1}x.dx = -\left[\frac{x^2}{2}\right]+\left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}+\frac{1}{2} = 1$ বর্গ একক

21. $\int_0^{\infty}\frac{22dx}{x^2-14x+170}$ এর মান কত? [KUET'15-16]

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 22π (e) 11π

সমাধান: (No correct answer); $\int_0^{\infty}\frac{22dx}{x^2-14x+170}$

$= \int_0^{\infty}\frac{22dx}{(x-7)^2+121} = \int_0^{\infty}\frac{22\times 11\sec^2\theta\, d\theta}{121(\sec^2\theta)}$

$= \int_{\tan^{-1}\left(-\frac{7}{11}\right)}^{\frac{\pi}{2}}2d\theta = [2\theta]_{\tan^{-1}\left(-\frac{7}{11}\right)}^{\frac{\pi}{2}}$

$= \pi - 2\tan^{-1}\left(-\frac{7}{11}\right) = \pi + 2\tan^{-1}\frac{7}{11}$

Now, $x-7 = 11\tan\theta$
$dx = 11\sec^2\theta\, d\theta$
When, $x=\infty, \theta = \frac{\pi}{2}$
When, $x = 0, \theta = \tan^{-1}\left(-\frac{7}{11}\right)$

22. $\int\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}d\theta$ এর মান হলো- [KUET'15-16]

(a) $\log_e\cos\left(\theta+\frac{\pi}{4}\right)+c$ (b) $\log_e\sin\left(\theta-\frac{\pi}{4}\right)+c$
(c) $\log_e\sec\left(\theta+\frac{\pi}{4}\right)+c$ (d) $\log_e\text{cosec}\left(\theta+\frac{\pi}{4}\right)+c$ (e) $\log_e\sin 2\theta+c$

সমাধান: (c); $\int\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}d\theta = \int\frac{\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}d\theta = \int\frac{1+\sin 2\theta}{\cos 2\theta}d\theta = \int\sec 2\theta\, d\theta+\int\tan 2\theta\, d\theta$

$= \frac{1}{2}\ln(\sec 2\theta+\tan 2\theta) - \frac{1}{2}\ln\cos 2\theta + c' = \frac{1}{2}\ln\left(\frac{1+\sin 2\theta}{\cos^2 2\theta}\right)+c' = \frac{1}{2}\ln\frac{(\sin\theta+\cos\theta)^2}{(\cos\theta+\sin\theta)^2(\cos\theta-\sin\theta)^2}+c'$

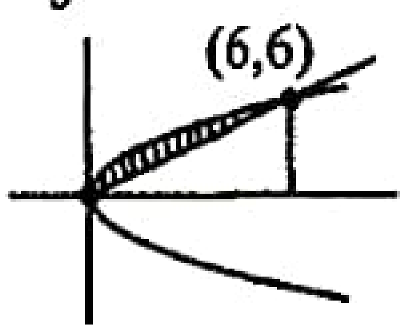
$= \ln\left(\frac{1}{\cos\theta-\sin\theta}\right)+c' = \ln\frac{1}{\sqrt{2}\left(\cos\theta\cos\frac{\pi}{4}-\sin\theta\sin\frac{\pi}{4}\right)}+c' = \ln\frac{1}{\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right)}+c'$

$= \ln\frac{\sec\left(\theta+\frac{\pi}{4}\right)}{\sqrt{2}}+c' = \ln\sec\left(\theta+\frac{\pi}{4}\right)+\ln\frac{1}{\sqrt{2}}+c' = \ln\sec\left(\theta+\frac{\pi}{4}\right)+c$

23. $y^2 = 6x$ পরাবৃত্ত ও $y = x$ সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল বের কর। [CUET'15-16]

(a) $\frac{256}{3}$ unit (b) $\frac{128}{3}$ unit (c) $\frac{28}{3}$ unit (d) $\frac{64}{3}$ unit

সমাধান: (সঠিক উত্তর নেই);



$\therefore$ ক্ষেত্রফল $= \int_0^6(\sqrt{6x}-x)\,dx = \left[\sqrt{6}\times\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}x^2\right]_0^6 = 6$

24. $\int_0^{\pi/2}\sin^5\theta\cos\theta\, d\theta$ এর মান হবে- [BUTex'15-16, CUET'10-11]

(a) 1/4 (b) 1/5 (c) 1/6 (d) None of these

সমাধান: $\int_0^{\pi/2}\sin^5\theta\cos\theta\, d\theta = \int_0^1 z^5\, dz = \left[\frac{z^6}{6}\right]_0^1 = \frac{1}{6}$

ধরি, $\sin\theta = z \quad \therefore \cos\theta\, d\theta = dz$; $\theta = 0$ হলে, $z = 0$; $\theta = \frac{\pi}{2}$ হলে, $z = 1$

25. $\int x^2[1+\ln(x^3+1)]dx = ?$ [SUST'14-15]

(a) $(x^3+1)\ln(x^3+1)$ (b) $\frac{1}{3}(x^3+1)\ln(x^3+1)$ (c) $\frac{x^3+1}{\ln(x^3+1)}$
(d) $\frac{3(x^3+1)}{x^3+1}$ (e) $\frac{\ln(x^3+1)}{x^3+1}$

সমাধান: (b); $\int x^2[1+\ln(x^3+1)]dx = \int x^2\,dx + \int x^2\ln(x^3+1)\,dx$
$= \frac{x^3}{3} + \frac{1}{3}\int \ln z\,dz\ [x^3+1=z;\ 3x^2dx = dz]$
$= \frac{1}{3}x^3 + \frac{1}{3}(z\ln z - z) = \frac{1}{3}x^3 + \frac{1}{3}\{(x^3+1)\ln|x^3+1| - (x^3+1)\}$
$= \frac{1}{3}\{(x^3+1)\ln|x^3+1| - 1\} + c' = \frac{1}{3}(x^3+1)\ln|x^3+1| - \frac{1}{3} + c'\ \left[c = c' - \frac{1}{3}\right]$

26. $\int_0^1 \frac{1-x}{1+x}dx$ এর মান কোনটি? **[KUET'14-15]**

(a) $3\ln3 + \frac{1}{2}$ (b) $2\ln2 - 1$ (c) $4\ln3 + 1$ (d) $\frac{1}{2}\ln3$ (e) $2\ln3 + 5$

সমাধান: (b); $\int_0^1 \frac{1-x}{1+x}dx = \int_0^1 \frac{dx}{1+x} - \int_0^1 \frac{x}{1+x}dx = [\ln(1+x)]_0^1 - \int_0^1 \frac{1+x-1}{1+x}dx$
$= \ln2 - \int_0^1 dx + \int_0^1 \frac{1}{1+x}dx = \ln2 + [\ln(1+x)]_0^1 - [a]_0^1 = 2.\ln2 - 1$

27. $\int \frac{dx}{x\sqrt{x^2-a^2}}$ এর মান কোনটি? **[KUET'14-15]**

(a) $\frac{1}{a}\sec^{-1}\frac{x}{a}$ (b) $\tan^{-1}x$ (c) $\cos^{-1}x$ (d) $\sin^{-1}x$ (e) $\text{cosec}^{-1}x$

সমাধান: (a); $\int \frac{dx}{x\sqrt{x^2-a^2}} = \int \frac{Yadx}{a\frac{x}{a}\sqrt{\left(\frac{x}{a}\right)^2 - 1^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

28. $\int_1^e \ln x dx$ এর মান- **[BUTex'14-15]**

(a) 1 (b) e (c) $e - 1$ (d) $1 - e$

সমাধান: (a); $\int_1^e \ln xdx = [x\ln x - x]_1^e = e.1 - e - 0 + 1 = 1$

29. $\int_0^1 2x^3 e^{-x^2}dx$ এর মান নির্ণয় কর। **[CUET'14-15]**

(a) $-\frac{2}{e} + 1$ (b) $-\frac{2}{e}$ (c) $-\frac{1}{e} + 1$ (d) None of them

সমাধান: (a); Let, $x^2 = z \therefore 2xdx = dz \therefore \int 2x^3e - x^2\,dx = \int ze^{-z}dz = z\int e^{-z} - \int\{\frac{d}{dz}(z)\int e^{-z}dz\}dz$
$= -ze^{-z} - \int(-e^{-z})dz = -ze^{-z} - e^{-z} + c = -(z+1)e^{-z}$
$\therefore \int_0^1 ze^{-z}dz = -[(z+1)e^{-z}]_0^1 = (2e^{-1} - 1) = 1 - \frac{2}{e} = -\frac{2}{e} + 1$

30. $\int_0^1 e^{-x^2}\,dx = ?$ **[RUET'14-15]**

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{2^{2k}(k!)^2}$ (b) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)k!}$ (c) $\sum_{k=1}^{\infty} \frac{(-1)^k \ln x}{\sqrt{k}k!}$ (d) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k!}$ (e) $\sum_{k=1}^{\infty} \frac{(-1)^k x^{-\frac{1}{3}}}{2\ln k}$

সমাধান: (b); $\int_0^1 e^{-x^2dx} = \int_0^1 \left(1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots\right)dx = \left(1 - \frac{1}{3} + \frac{1}{5\times2!} - \frac{1}{7\times3!} + \cdots\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)k!}$

31. $\int a^{a^{a^x}}.a^{a^x}.a^x dx = ?$ **[Ans: d][RUET'14-15]**

(a) a^{a^x} (b) $\frac{a^x}{\log a}$ (c) $\frac{a^{a^x}}{3}$ (d) $\frac{a^{a^{a^x}}}{(\log a)^3}$ (e) 1

সমাধান: $I = \int a^{a^{a^x}} \cdot a^{a^x} \cdot a^x dx$

ধরি, $a^x = z$

$a^x \ln a\, dx = dz \quad \therefore I = \int \frac{a^{a^z}.a^z}{\ln a}dz$

ধরি, $a^z = y$

$\therefore a^z \ln a\, dz = dy \therefore I = \int \frac{a^y\,dy}{\ln a.\ln a} = \frac{a^y}{(\ln a)^3} = \frac{a^{a^{a^x}}}{(\ln a)^3}$

[প্রশ্নে $\log a$ বলতে $\ln a$ কে বুঝানো হয়েছে]

32. বক্ররেখা $x = y^2$ এবং $y = x - 2$ রেখা দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল হবে- **[BUET'13-14]**

(a) $\frac{7}{3}$ (b) $\frac{9}{2}$ (c) $\frac{7}{2}$ (d) $\frac{11}{2}$

সমাধান: (b) ; Area $= \left|\int_{y_1}^{y_2}(y^2 - y - 2)dy\right| = \left|\int_{-1}^{2}(y^2 - y - 2)dy\right| = \left|-\frac{9}{2}\right| = \frac{9}{2}$ s.u

33. x-অক্ষ, y-অক্ষ, $y=\ln 5$ এবং $y=\ln x$ বক্ররেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল হবে- **[BUET'13-14]**

(a) ln 4 sq. unit (b) 5 sq. unit (c) 4 sq. unit (d) ln 5 sq. unit

সমাধান: (c) ; $y=\ln x \therefore x=e^y$ Area $=\int_{y_1}^{y_2} e^y dy=\int_0^{\ln 5} e^y dy = e^{\ln 5}-e^0=5-1=4$s.u

34. $2\int \sin\left(2e^{x^2}\right) xe^{x^2}dx$ এর মান হল ঃ **[BUET'13-14]**

(a) $\sin\left(2e^{x^2}\right)+c$ (b) $2\sin\left(2e^{x^2}\right)+c$ (c) $\cos^2\left(e^{x^2}\right)+c$ (d) $\sin^2\left(e^{x^2}\right)+c$

সমাধান: (d) ; $y=e^{x^2}$ ধরে, $I=\int \sin 2y\, dy = -\frac{\cos 2y}{2}+c = \frac{1-2\sin^2 y}{2}+c'$

$\sin^2 y-\frac{1}{2}+c'=\sin^2(e^{x^2})+c$ $\left[c=c'-\frac{1}{2}\right.$ ধরে$\left.\right]$

35. $\int_{\frac{1}{2}}^{1}\frac{dx}{\sqrt{x-x^2}}$ এর মান কত ? **[SUST'12-13, BUET'13-14]**

(a) $-\pi/4$ (b) $-\pi/2$ (c) $\pi/4$ (d) $\pi/2$ (e) π

সমাধান: $\int_{\frac{1}{2}}^{1}\frac{dx}{\sqrt{x-x^2}}=\int_{\frac{1}{2}}^{1}\frac{dx}{x\sqrt{\frac{1}{x}-1}}$

ধরি, $\frac{1}{x}=z^2 \therefore -\frac{1}{x^2}dx=2zdz \therefore \frac{1}{x}dx=-2xzdz=-2\frac{1}{z^2}zdz=-\frac{2dz}{z}$.

$\therefore \int\frac{dx}{x\sqrt{\frac{1}{x}-1}}=\int\frac{-\frac{2dz}{z}}{\sqrt{z^2-1}}=-2\int\frac{dz}{z\sqrt{z^2-1}}=-2\left[\sec^{-1}z\right]+c=-2\sec^{-1}\frac{1}{\sqrt{x}}+c=-2\cos^{-1}\sqrt{x}+c$

$\therefore \int_{\frac{1}{2}}^{1}\frac{dx}{\sqrt{x-x^2}}=\left[-2\cos^{-1}\sqrt{x}\right]_{\frac{1}{2}}^{1} = -2\cos^{-1}(1)+2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{2}$.

36. $\int_0^a\frac{dx}{\sqrt{x(a-x)}}$ এর মান- **[RUET'13-14]**

(a) $\frac{\pi}{a^2}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{2a^2}$ (e) None

সমাধান: (c) ; $\int_0^a\frac{dx}{\sqrt{x(a-x)}}=\int_0^a\frac{dx}{\sqrt{ax-x^2}}=\int_0^a\frac{dx}{\sqrt{\frac{a^2}{4}-\left(x^2-ax+\frac{a^2}{4}\right)}}$

$=\int_0^a\frac{dx}{\sqrt{\left(\frac{a}{2}\right)^2-\left(x-\frac{a}{2}\right)^2}}=\left[\sin^{-1}\frac{x-\frac{a}{2}}{\frac{a}{2}}\right]_0^a=\sin^{-1}(1)-\sin^{-1}(-1)=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi.$

37. $\int\frac{dx}{e^x+e^{-x}}=?$ **[RUET'13-14]**

(a) $\sin^{-1}e^x$ (b) $\tan^{-1}\frac{1}{e^x}$ (c) $\tan^{-1}e^x$ (d) $\cos^{-1}e^x$ (e) None

সমাধান: (c) ; $\int\frac{dx}{e^x+e^{-x}}=\int\frac{e^x dx}{(e^x)^2+1}=\int\frac{dz}{z^2+1}$ [ধরি, $z=e^x$] $=\tan^{-1}(z)=\tan^{-1}(e^x)$.

38. মান নির্ণয় কর $\int\frac{1}{\sqrt[3]{1-6x}}dx$ $\left[=\int(1-6x)^{-\frac{1}{3}}dx=\frac{(1-6x)^{-\frac{1}{3}+1}}{(-\frac{1}{3}+1)(-6)}=\frac{(1-6x)^{\frac{2}{3}}}{-4}\right]$ **[Ans: c][BUTex'13-14]**

(a) $\frac{1}{4}(1-6x)^{2/3}$ (b) $-\frac{1}{4}(1-6x)^{3/2}$ (c) $-\frac{1}{4}(1-6x)^{2/3}$ (d) $-\frac{1}{4}(6x-1)^{2/3}$

39. $\int\frac{1}{\sqrt{1-x^2}}dx=?$ **[BUTex'13-14]**

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

সমাধান: (a) ; ques এর limit missing Limit 0 হতে 1.

40. $\int_0^{2a}\frac{dx}{\sqrt{2ax-x^2}}$ এর মান কত? **[CUET'13-14]**

(a) π (b) $\pi/2$ (c) 2π (d) None of these

সমাধান: (a); $\int_0^{2a}\frac{dx}{\sqrt{2ax-x^2}}=\int_0^{2a}\frac{dx}{\sqrt{a^2-(a^2-2ax+x^2)}}=\int_0^{2a}\frac{dx}{\sqrt{a^2-(x-a)^2}}=\left[\sin^{-1}\frac{x-a}{a}\right]_0^{2a}$

$=\sin^{-1}(1)-\sin^{-1}(-1)=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi$

41. $\int x\ln x dx$ এর মান কত? **[CUET'13-14]**

(a) $\frac{x^2}{2}\ln(x)-\frac{x^2}{2}+c$ (b) $x^2\ln(x)-\frac{x^2}{4}+c$

(c) $\frac{x^2}{2}\ln(x)+\frac{x^2}{4}+c$ (d) None of these

সমাধান:(d); $\int x\ln x\,dx=\ln x\int x\,dx-\int\left\{\frac{d}{dx}\ln x\int xdx\right\}dx=\frac{x^2}{2}\ln x-\int\frac{x}{2}dx=\frac{1}{2}x^2\ln x-\frac{x^2}{4}+c$

42. $\int_0^{\pi/2}\frac{\cos x}{4-\sin x}dx$ এর মান– **[RUET'13-14]**

(a) $\frac{1}{2}\ln(2)$ (b) $\frac{1}{4}\ln\left(\frac{1}{3}\right)$ (c) $\frac{1}{4}\ln(3)$ (d) $\ln\left(\frac{1}{3}\right)$ (e) None

সমাধান: (e) ; $\int_0^{\pi/2}\frac{\cos x}{4-\sin x}dx=\int_4^3\frac{-dz}{z}=-[\ln z]_4^3=\ln\frac{4}{3}$.

ধরি, $4-\sin x=z$ $\therefore \cos xdx=-dz$ $\therefore \cos xdx=-dz$; $x=0$ হলে, $z=4$; $x=\frac{\pi}{2}$ হলে, $z=3$.

43. $\int_0^{\infty}e^{-2x}\cos 4xdx$ এর মান কোনটি? **[KUET'13-14]**

(a) e^{-2x} (b) 0 (c) $\frac{2}{5}$ (d) $\frac{1}{10}$ (e) $\frac{1}{20}$

সমাধান: (d); ধরি, $I=\int e^{-2x}\cos 4x dx$ $=\cos 4x\int e^{-2x}dx-\int\left[\frac{d}{dx}(\cos 4x)\int e^{-2x}dx\right]dx$

$=\frac{e^{-2x}\cos 4x}{-2}-\int\left[-4\sin 4x\times\frac{e^{-2x}}{-2}\right]dx$ $=-\frac{e^{-2x}\cos 4x}{2}-2\int e^{-2x}\sin 4x dx$

$=\frac{-e^{-2x}\cos 4x}{2}-2\left[\sin 4x\int e^{-2x}dx-\int\left\{\frac{d}{dx}(\sin 4x)\int e^{-2x}dx\right\}dx\right]$

$=-\frac{e^{-2x}\cos 4x}{2}-2\sin 4x\frac{e^{-2x}}{-z}+2\int 4^2\cos 4x\frac{e^{-2x}}{-2}dx$ $=5I=e^{-2x}\sin 4x-\frac{e^{-2x}\cos 4x}{2}$

$\therefore I=\frac{e^{-2x}\sin 4x}{5}-\frac{e^{-2x}\cos 4x}{10}$ $\therefore\int_0^{\infty}e^{-2x}\cos^{4x}=\left[\frac{e^{-2x}\cos 4x}{5}-\frac{e^{-2x}\cos 4x}{10}\right]_0^{\infty}$

$=0-0-0+\frac{1}{10}$ $[\therefore e^{-2\infty}=0]$ $=\frac{1}{10}$

44. $\int\frac{1+\tan^2 x}{(1+\tan x)^2}dx$ এর মান কোনটি? **[KUET'13-14]**

(a) $\frac{1}{1+\cot x}+c$ (b) $\frac{1}{1-\tan x}+c$ (c) $\frac{1}{1+\cos x}+c$ (d) $\frac{1}{1-\cot x}+c$ (e) $-\frac{1}{1+\tan x}+c$

সমাধান:(e); $\int\frac{1+\tan^2 x}{(1+\tan x)^2}dx$ $=\int\frac{dz}{z^2}=-\frac{1}{z}+c$ $=-\frac{1}{1+\tan x}+c$

ধরি, $1+\tan x=z$ $\therefore\sec^2 x dx=dz$ $\Rightarrow(1+\tan^2 x)dx=dz$

45. $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ এর মান হল- **[BUET'12-13]**

(a) $e^x\cos\left(\frac{x}{2}\right)+c$ (b) $e^x\sin\left(\frac{x}{2}\right)+c$ (c) $e^x\tan\left(\frac{x}{2}\right)+c$ (d) $e^x\cot\left(\frac{x}{2}\right)+c$

সমাধান: $I=\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ $=\int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx$ $=\int e^x\left(\frac{\sec^2\frac{x}{2}}{2}+\tan\frac{x}{2}\right)dx$

ধরি, $f(x)=\tan\frac{x}{2}$ $f(x)=\tan\frac{x}{2}$

$\therefore I=\int e^x[f'(x)+f(x)]dx$ $=e^x f(x)+c=e^x f(x)+c$ $=e^x\left(\tan\frac{x}{2}\right)+c$

46. $\int_0^{\frac{\pi}{2}}\frac{\cos x}{9-\sin^2 x}dx$ এর মান কোনটি? **[KUET'07-08, RUET'12-13]**

(a) $\frac{1}{6}\ln 2$ (b) $2\ln 6$ (c) $\frac{1}{2}\ln 6$ (d) $\frac{1}{3}\ln 2$ (e) $6\ln 3$

সমাধান: $\int_0^{\pi/2}\frac{\cos x}{9-\sin^2 x}dx$; ধরি, $\sin x = z \Rightarrow \cos x dx = dz$ যখন $x=0, z=0$; যখন $x=\pi/2; z=1$

$\therefore I=\int_0^1\frac{dz}{9-z^2}=\frac{1}{2\times3}\ln\left|\frac{3+z}{3-z}\right|_0^1=\frac{1}{6}\left[\ln\frac{3+1}{3-1}-\ln\frac{3+0}{3-0}\right]=\frac{1}{6}\left[\ln\frac{4}{2}-0\right]=\frac{1}{6}\ln 2$

47. $\int_0^1\frac{dx}{\sqrt{(2x-x^2)}}$ এর মান হলো— **[KUET'12-13]**

(a) $\frac{-\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{2}$ (e) $\frac{3\pi}{2}$

সমাধান: (d) ; $I=\int_0^1\frac{dx}{\sqrt{2x-x^2}}$

$=\int_0^1\frac{dx}{\sqrt{1-(1-2x+x^2)}}=\int_0^1\frac{dx}{\sqrt{1-(1-x)^2}}$

ধরি, $1-x=t$ $x=0\rightarrow t=1$

$\therefore -dx=dt$ $x=1\rightarrow t=0$

$\therefore I=\int_1^0\frac{-dt}{\sqrt{1-t^2}}=-[\sin^{-1}t]_1^0$

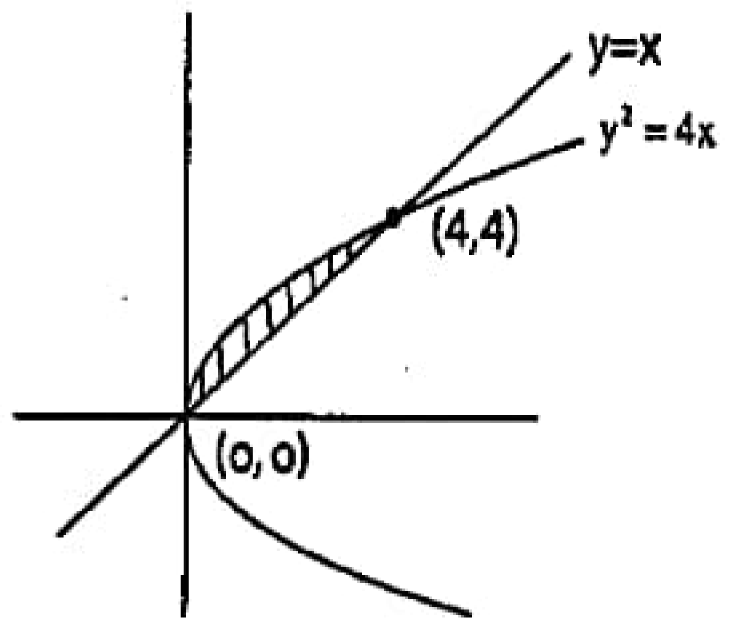
$=-[\sin^{-1}0-\sin^{-1}1]=-\left(0-\frac{\pi}{2}\right)=\frac{\pi}{2}$

$=2\pi+\frac{\pi}{2}=\frac{5\pi}{2}$

48. $y^2=4x$ এবং $y=x$ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল— **[BUTex'12-13]**

(a) $\frac{3}{8}$ sq. units (b) $\frac{8}{3}$ sq. units (c) 3 sq. units (d) 8 sq. units

সমাধান: (b) ; $y=x$..........(i); $y^2=4x$.......(ii)



$\Rightarrow (x)^2=4x$ [(i) হতে] $\Rightarrow x^2-4x=0 \Rightarrow x(x-4)=0$

$\therefore x=0, 4 \therefore y=0, 4$ [(i) হতে]

$Area=\int_0^4(\sqrt{4x}-x)dx=\int_0^4(2\sqrt{x}-x)dx$

$=2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4-\left[\frac{x^2}{2}\right]_0^4=\frac{4}{3}\left[4^{\frac{3}{2}}-0\right]-\frac{1}{2}[4^2-0]=\frac{8}{3}$ sq. unit

49. $\int_0^1\frac{dx}{e^x+e^{-x}}$ এর মান কত? **[KUET'09-10, BUTex'12-13]**

(a) $\tan^{-1}e-\frac{\pi}{4}$ (b) $\tan^{-1}+\frac{\pi}{4}$ (c) $\frac{\pi}{4}-\tan^{-1}e$ (d) $\frac{\pi}{2}+\tan^{-1}e$

সমাধান: (a); $I=\int_0^1\frac{dx}{e^x+e^{-x}}=\int_0^1\frac{dx}{e^x+\frac{1}{e^x}}$

$=\int_0^1\frac{e^xdx}{(e^x)^2+1}$

ধরি, $e^x=t \therefore e^xdx=dt$

যখন, $x=0$ তখন, $t=1$ যখন, $x=1$ তখন, $t=e$

$\therefore I=\int_1^e\frac{dt}{t^2+1}=[\tan^{-1}t]_1^e=\tan^{-1}e-\frac{\pi}{4}$

50. $\int\frac{(\tan x+\tan^3 x)dx}{e^{\sec^2 x}+e^{-\sec^2 x}}$ এর মান হলো— **[KUET'12-13]**

(a) $\frac{1}{2}\tan^{-1}(e^{\sec^2 x})+c$ (b) $\tan^{-1}\left(\frac{1}{2}e^{\sec^2 x}\right)+c$ (c) $2\tan^{-1}(e^{\sec^2 x})$

(d) $\tan^{-1}(2e^{\sec^2 x})+c$ (e) $\frac{1}{2}\tan^{-1}(e^{-\sec^2 x})+c$

সমাধান: (a) ; $I=\int\frac{(\tan x+\tan^3 x)dx}{e^{\sec^2 x}+e^{-\sec^2 x}}=\int\frac{\tan x(1+\tan^2 x)dx}{e^{\sec^2 x}+\frac{1}{e^{\sec^2 x}}}=\int\frac{e^{\sec^2 x}.\tan x.\sec^2 x}{(e^{\sec^2 x})^2+1}dx$

ধরি, $e^{\sec^2 x}=t \therefore e^{\sec^2 x}.2\sec x.\sec x\tan x\ dx=dt$

$\therefore I=\frac{1}{2}\int\frac{dt}{t^2+1}=\frac{1}{2}\tan^{-1}t+c=\frac{1}{2}\tan^{-1}(e^{\sec^2 x})+c$

51. $\int_0^{4/a} e^{\sqrt{ax}}d(\sqrt{x})$ এর মান কত? [SUST'12-13]

(a) $(e^2-1)/\sqrt{a}$ (b) $(e^2-1)/\sqrt{a}$ (c) (e^2-1) (d) $(1-e^2)/\sqrt{a}$ (e) $(1-e^2)/a$

সমাধান: $\int_0^{\frac{4}{a}} e^{\sqrt{ax}}d(\sqrt{x})=\int_0^2\frac{e^z dz}{\sqrt{a}}=\frac{1}{\sqrt{a}}[e^z]_0^2=\frac{1}{\sqrt{a}}(e^2-e^0)=\frac{e^2-1}{\sqrt{a}}$

ধরি, $\sqrt{ax}=z \therefore \sqrt{a}\ d(\sqrt{x})=dz \therefore d(\sqrt{x})=\frac{dz}{\sqrt{a}}$; $x=\frac{4}{a}$ হলে, $z=\sqrt{ax}=\sqrt{a\times\frac{4}{a}}=2$

$x=0$ হলে, $z=\sqrt{ax}=0$.

52. $\int_1^{\ln a} xe^x dx=3a$ হলে, a এর মান কত? [SUST'12-13]

(a) e (b) e^2 (c) e^3 (d) e^4 (e) e^5

সমাধান: $\int_1^{\ln a} xe^x dx=3a$

এখন, $\int xe^x dx=\int e^x(x-1+1)dx=\int[e^x\{(x-1)+1\}]dx$

এখন, $x-1=f(x)$ হলে, $f'(x)=1$.

আমরা জানি, $\int[e^x\{f(x)+f'(x)\}]dx=e^x f(x)$.

$\therefore \int xe^x dx=e^x(x-1)+c \quad \therefore \int_1^{\ln a} xe^x dx=[e^x(x-1)]_1^{\ln a}$

$\therefore 3a=e^{\ln a}(\ln a-1)+e^1(1-1)=a(\ln a-1)=a\ln a-a \quad \therefore a\ln a=4a$

$\therefore \ln a=4\ [\because a\neq 0]. \quad \therefore a=e^4$.

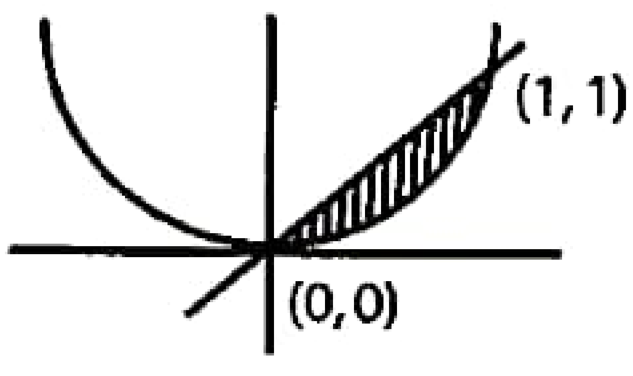
53. $y=x^2$ পরাবৃত্ত ও $y=x$ সরলরেখা দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল কত? [SUST'12-13]

(a) 1 (b) 3/2 (c) 2/3 (d) 1/6 (e) 1/3

সমাধান: $y=x^2, y=x$

$\therefore x^2=x \therefore x=0,1. \therefore y=0,1. \therefore$ ছেদবিন্দুদ্বয় $=(0,0)$; $(1,1)$.

$\therefore$ ক্ষেত্রফল $=\int_0^1(x-x^2)dx=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ বর্গএকক।



54. $\int_0^{\pi/4}\frac{\cos\theta}{\cos^2\theta}d\theta=$ কত? [BUTex'12-13]

(a) $1-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2}-1$ (d) $\frac{\pi}{2}-2$

সমাধান: $I=\int_0^{\frac{\pi}{4}}\frac{\cos\theta}{\cos^2\theta}d\theta=\int_0^{\frac{\pi}{4}}\frac{1}{\cos\theta}d\theta=\int_0^{\frac{\pi}{4}}\sec\theta\ d\theta=[\ell n|\tan\theta+\sec\theta|]_0^{\frac{\pi}{4}}$

$=\ell n\left|\tan\frac{\pi}{4}+\sec\frac{\pi}{4}\right|-\ell n|\tan 0+\sec 0|=\ell n|1+\sqrt{2}|-\ell n 1=\ell n|1+\sqrt{2}|-0$ [Ans: Blank]

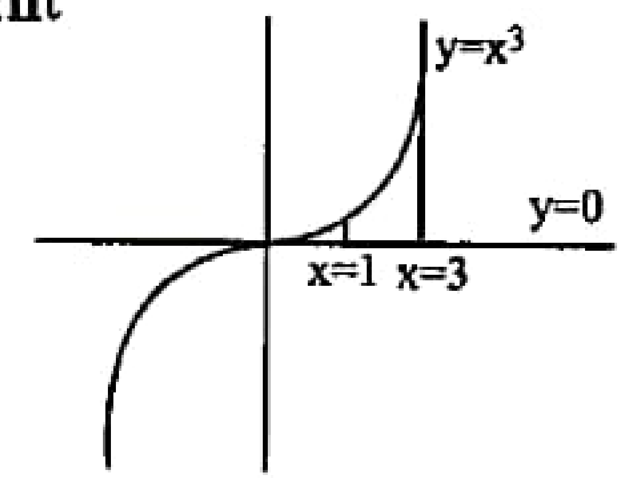
55. দেয়া আছে, $F(x)=\int_0^x \frac{t-3}{t^2+7}dt$ । x -এর মান কত হলে F(x) ন্যূনতম হবে? [BUET'11-12]

(a) 3 (b) 0 (c) $\sqrt{7}$ (d) $-\sqrt{7}$

সমাধান: $F'(x)=0$; $\frac{x-3}{x^2+7}=0 \therefore x=3$

56. $y=x^3$ বক্ররেখা এবং $y=0, x=1$ ও $x=3$ সরলরেখা তিনটি দিয়ে সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল হবে- [BUET'11-12]

(a) 5 sq. unit (b) 20 sq. unit (c) 10 sq. unit (d) 15 sq. unit



সমাধান: ক্ষেত্রফল $=\int_1^3 x^3dx$

$=\left[\frac{x^4}{4}\right]_1^3=\frac{3^4-1^4}{4}=20$ sq. unit

57. $\int\frac{e^x(1+x)}{\cos^2(xe^x)}dx$ এর মান হল- [BUET'11-12]

(a) $\sin(xe^x)+C$ (b) $\cos(xe^x)+C$ (c) $\tan(xe^x)+C$ (d) $\cos^2(xe^x)+C$

সমাধান: $\int\frac{e^x(1+x)}{\cos^2(xe^x)}dx=\int\frac{dz}{\cos^2 z}$ | ধরি, $z=xe^x$

$=\int\sec^2 z\ dz=\tan z+C \quad =\tan(xe^x)+C$ | $dz=e^x(1+x)dx$

58. $y=x$ এবং $y^2=16x$ রেখাদুটি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল কত? [CUET'11-12]

(a) 512/3 sq. Unit (b) 128 sq. Unit (c) 128/3 sq. Unit (d) None of these

সমাধান: $y=x$ এবং $y^2=16x \Rightarrow x^2=16x \Rightarrow x^2-16x=0 \quad \therefore x=16, 0$

$\therefore$ রেখাদুটি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $=\int_0^{16}(4\sqrt{x}-x)\,dx=\left[4\times\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^{16}=\frac{8}{3}\times16^{3/2}-\frac{16^2}{2}-0+0=\frac{128}{3}$

59. $\int_0^1\sqrt{\frac{1-x}{1+x}}\,dx$ এর মান কোনটি? [SUST'11-12,KUET'11-12,BUET'11-12]

(a) 1 (b) π (c) $\frac{\pi}{2}-1$ (d) $\frac{\pi}{2}+1$ (e) $1-\pi$

সমাধান: $\int_0^1\sqrt{\frac{1-x}{1+x}}\,dx=\int_0^1\frac{1-x}{\sqrt{1-x^2}}dx=\int_0^1\frac{1}{\sqrt{1-x^2}}dx-\int_0^1\frac{x\,dx}{\sqrt{1-x^2}}=\int_0^1\frac{1}{\sqrt{1-x^2}}dx+\int_0^1\frac{\frac{1}{2}(-2x)}{\sqrt{1-x^2}}dx$

$=[\sin^{-1}x]_0^1+[\sqrt{1-x^2}]_0^1 \quad =\sin^{-1}(1)-0+0-1=\frac{\pi}{2}-1$

60. $\int_{-\pi/2}^{\pi/2}(\sin x+\cos x)^2dx=?$ [RUET'11-12]

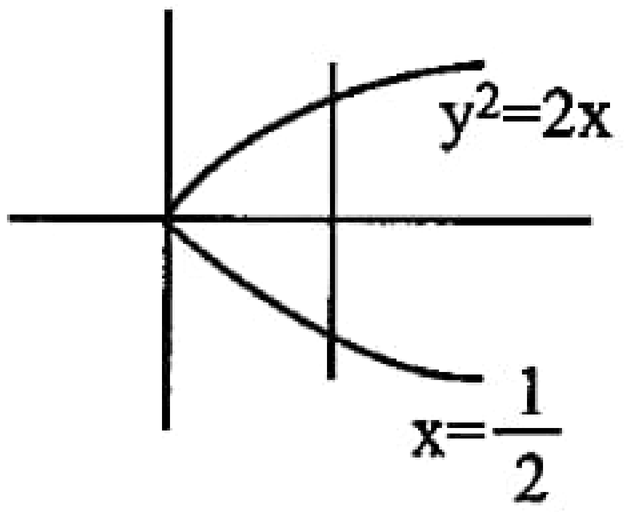
(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{2}$ (e) π

সমাধান: $\int_{-\pi/2}^{\pi/2}(\sin x+\cos x)^2dx=2\int_0^{\pi/2}(\sin^2x+\cos^2x+\sin 2x)dx=2\int_0^{\pi/2}dx+2\int_0^{\pi/2}\sin 2xdx$

$=2.\frac{\pi}{2}-2.\frac{1}{2}[\cos 2x]_0^{\pi/2}=\pi-\cos\pi+\cos 0=\pi+2 \therefore$ no correct answer.

61. $y^2 = 2x$ পরাবৃত্ত (Parabola) এবং এর উপকেন্দ্রিক লম্ব দ্বারা বেষ্টিত ক্ষেত্রের ক্ষেত্রফল কত বর্গ একক?

(a) $\frac{1}{3}$ (b) $\frac{8}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$ **[BUET'07-08, BUTex'11-12]**



সমাধান: ক্ষেত্রফল $= 2\int_0^{1/2} y dx = 2\sqrt{2}\int_0^{\frac{1}{2}} \sqrt{x}\ dx = 2\sqrt{2}.\frac{2}{3}[x^{3/2}]_0^{1/2}$

$= 2\sqrt{2} \times \frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} = 4\sqrt{2}.\frac{1}{3}.\left(\frac{1}{2}\right)^{3/2} = \frac{4\sqrt{2}}{3}.\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{4}{3}.\frac{1}{2} = \frac{2}{3}$

62. $\int e^x \sec x(1+\tan x)dx$ এর মান নির্ণয় কর : **[CUET'11-12]**

(a) $e^x \sec x + c$ (b) $e^x \operatorname{cosec} x + c$ (c) $e^x \tan x + c$ (d) None

সমাধান: $\int e^x \sec x(1+\tan x)dx = \int (e^x \sec x + e^x \sec x.\tan x)dx = e^x \sec x + c$

$[\because \int e^x\{f(x)+f'(x)\}dx = e^x f(x) + c]$

63. মান নির্ণয় কর ঃ $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ $\left\{\left[\tan^{-1}(x)\right]_1^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}1 = \frac{\pi}{12}\right\}$ **[Ans : d]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$ **[KUET'05-06, SUST'11-12]**

64. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ এর মান কোনটি? **[KUET'11-12]**

(a) $\tan x + c$ (b) $\cot x + c$ (c) $2\sqrt{\tan x} + c$ (d) $\frac{\sqrt{\tan x}}{2} + c$ (e) $\log(\sin 2x) + c$

সমাধান: $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx = \int \frac{\sec^2 x\sqrt{\tan x}}{\tan x}dx = \int \frac{\sec^2 x\ dx}{\sqrt{\tan x}} = 2\sqrt{\tan x} + c$

65. $\int \cos^{-1} x\ dx$ এর মান কোনটি? **[KUET'11-12]**

(a) $x\cos^{-1}x - \sqrt{1-x^2} + c$ (b) $x\cos^{-1}x + \sqrt{1-x^2} + c$

(c) $x[\cos^{-1}x - \sqrt{1-x^2}] + c$ (d) $x[\cos^{-1}x + \sqrt{1-x^2}] + c$

(e) $\cos^{-1}x - \sqrt{1-x^2} + c$

সমাধান: $\int \cos^{-1}x\ dx = \cos^{-1}x\int dx - \int\left\{\frac{d}{dx}(\cos^{-1}x)\int dx\right\}dx = x\cos^{-1}x - \int\left\{\frac{-1}{\sqrt{1-x^2}}x\right\}dx$

$= x\cos^{-1}x - \frac{1}{2}\int\frac{(-2x)}{\sqrt{1-x^2}}dx = x\cos^{-1}x - \frac{1}{2}\times 2\times\sqrt{1-x^2} + c = x\cos^{-1}x - \sqrt{1-x^2} + c$

66. $\int \frac{5e^{2x}}{1+e^{4x}} = ?$ **[RUET'11-12]**

(a) $\frac{5}{4}\ln(1+e^{4x}) + c$ (b) $\frac{5}{2}\ln(1+e^{4x}) + c$ (c) $\frac{5}{2}\tan^{-1}(e^{2x}) + c$

(d) $\frac{5}{4}\ln(1+e^{2x}) + c$ (e) None

সমাধান: $e^{2x} = z\ \therefore e^{2x}dx = \frac{dz}{2}$ $\int\frac{5e^{2x}}{1+(e^{2x})^2}dx = 5\int\frac{(dz)/2}{1+z^2} = \frac{5}{2}\int\frac{dz}{1+z^2} = \frac{5}{2}\tan^{-1}(e^{2x}) + c$

67. $\int x^{-1}dx$ এর মান? **[BUTex'11-12]**

(a) $\ln x$ (b) ∞ (c) 0 (d) $\frac{1}{x^2}$

সমাধান: (a) ; $\int x^{-1}dx = \ln x + c$

68. মান নির্ণয় করঃ $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}\, dx$ **[BUET'10-11]**

(a) -2 (b) $\frac{8}{21}$ (c) $\frac{4}{15}$ (d) None of the above

সমাধান: $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}\, dx$; $\sin x = z \Rightarrow \cos dx = dz$; $x=0 \Rightarrow z=0$; $x=\frac{\pi}{2} \Rightarrow z=1$

$$\int_0^1 (1-z^2)\sqrt{z}\, dz = \int_0^1 \left(\sqrt{z} - z^{\frac{5}{2}}\right) dz = \left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{z^{\frac{5}{2}+1}}{\frac{5}{2}+1}\right]_0^1 = \frac{8}{21} \text{ (Ans.)}$$

69. $\int \frac{\sec^2(\cot^{-1}x)}{1+x^2}dx$ এর মান হচ্ছে- **[BUET'10-11]**

(a) $-\frac{1}{x}+c$ (b) $\frac{1}{x}+c$ (c) $x+c$ (d) $-x+c$

সমাধান: $\int \frac{\sec^2(\cot^{-1}x)}{1+x^2}dx$; $\cot^{-1}x = z \Rightarrow -\frac{1}{1+x^2}dx = dz$

$$\therefore -\int \sec^2 z\, dz = -\tan z + c = -\tan(\cot^{-1}x) + c = -\tan\left(\tan^{-1}\frac{1}{x}\right) + c = -\frac{1}{x} + c$$

70. The result of $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ is – **[BUET'10-11]**

(a) $e^x\cos\left(\frac{x}{2}\right)+c$ (b) $e^x\sin\left(\frac{x}{2}\right)+c$ (c) $e^x\tan\left(\frac{x}{2}\right)+c$ (d) $e^x\cot\left(\frac{x}{2}\right)+c$

সমাধান:

$$I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

$$= \int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx$$

$$= \int e^x\left(\frac{\sec^2\frac{x}{2}}{2} + \tan\frac{x}{2}\right)dx$$

let, $f(x) = \tan\frac{x}{2}$

$\therefore f'(x) = \sec^2\frac{x}{2}.\frac{1}{2}$

$\therefore I = \int e^x[f'(x)+f(x)]dx = e^x f(x) + c$

$[\because \int e^x[f(x)+f'(x)]dx = e^x f(x) + c]$ $= e^x\left(\tan\frac{x}{2}\right) + c$

71. $\int_1^2 \log x \, dx$ এর মান- **[CUET'10-11]**

(a) $\log 2$ (b) $2\log 2$ (c) $2\log 2 - 1$ (d) None of these

সমাধান: $I = \int \log x \, dx = \log x \int dx - \int \left[\frac{d}{dx}(\log x) \int dx\right] dx = x\log x - \int \frac{1}{x}.x.dx = x\log x - x$

$\therefore \int_1^2 \log x \, dx = [x\log x - x]_1^2 = (2\log 2 - 2) - (1\log 1 - 1) = 2\log 2 - 1$

72. $\int_0^{\pi/2} \cos^5 x \, dx$ এর মান কত? **[KUET'10-11]**

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$ (c) $\frac{7}{15}$ (d) $\frac{11}{15}$ (e) $\frac{8}{15}$

সমাধান: (e) ; Use Calculator বা, $\int_0^{\pi/2} \cos^5 x \, dx = \frac{5-1}{5} \times \frac{5-3}{5-2} = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

73. যদি $\int \frac{dx}{a + b\cos x} = \frac{1}{\sqrt{a^2 - b^2}} \cos^{-1} \frac{b + a\cos x}{a + b\cos x}$ হয়, $\int_0^{\pi} \frac{dx}{a + b\cos x}$ এর মান হবে- **[RUET'10-11]**

(a) $\frac{2}{\sqrt{a^2 - b^2}}$ (b) $\frac{-2}{\sqrt{a^2 - b^2}}$ (c) $\frac{1}{\sqrt{a^2 - b^2}}$ (d) $\frac{-1}{\sqrt{a^2 - b^2}}$ (e) $\frac{\pi}{\sqrt{a^2 - b^2}}$

সমাধান: (e) ; $\int_0^{\pi} \frac{dx}{a + b\cos x} = \left[\frac{1}{\sqrt{a^2 - b^2}} \cos^{-1} \frac{b + a\cos x}{a + b\cos x}\right]_0^{\pi}$

$= \frac{1}{\sqrt{a^2 - b^2}} \left[\cos^{-1} \frac{b - a}{a - b} - \cos^{-1} \frac{b + a}{a + b}\right] = \frac{1}{\sqrt{a^2 - b^2}} [\cos^{-1}(-1) - \cos^{-1} 1] = \frac{\pi}{\sqrt{a^2 - b^2}}$

74. মান নির্ণয় কর : $\int \frac{dx}{\cos^2 x \sqrt{1 + \tan x}}$ **[CUET'10-11]**

(a) $2\sqrt{1 + \tan x} + C$ (b) $\sqrt{1 + \tan x} + C$ (c) $2\sqrt{1 + \tan x}$ (d) None of these

সমাধান: (a) ; $\int \frac{dx}{\cos^2 x \sqrt{1 + \tan x}} = \int \frac{\sec^2 x dx}{\sqrt{1 + \tan x}} = \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} + c = 2\sqrt{1 + \tan x} + c$

Shortcut: $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$

75. যোগজ নির্ণয় কর: $\int \frac{dx}{x\sqrt{(x^2 + 1)}}$ **[CUET'10-11]**

(a) $\sec^{-1} x + C$ (b) $\operatorname{cosec}^{-1} x + C$ (c) $-x^2\sqrt{x^2 - 1} + C$ (d) None of these

সমাধান: (d) ; ধরি, $x^2 + 1 = z^2 \Rightarrow 2xdx = 2zdz \quad \therefore xdx = zdz$

$\int \frac{dx}{x\sqrt{(x^2 + 1)}} = \int \frac{xdx}{x^2\sqrt{x^2 + 1}} = \int \frac{zdz}{(z^2 - 1) z} = \int \frac{dz}{z^2 - 1} = \frac{1}{2} \ln \frac{z - 1}{z + 1} + c = \frac{1}{2} \ln \frac{\sqrt{x^2 + 1} - 1}{\sqrt{x^2 + 1} + 1} + c$

76. $I=\int_0^{\pi/4}\frac{\sin 2\theta}{\sin^4\theta+\cos^4\theta}d\theta$ এর মান কোনটি? **[KUET'08-09, BUET'10-11]**

(a) $\frac{\pi}{3}$ (b) $\frac{1}{4}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{4}$ (e) $\frac{\pi}{6}$

সমাধান: (d) ; $1=\int_0^{\pi/4}\frac{\sin 2\theta}{\sin^4\theta+\cos^4\theta}d\theta$ [$\cos^4\theta$ দ্বারা ভাগ করে]

$$1=\int_0^{\pi/4}\frac{2\sin\theta\cos\theta\times\frac{1}{\cos^2\theta.\cos\theta.\cos\theta}}{1+\tan^4\theta}d\theta=\int_0^{\pi/4}\frac{2\tan\theta\sec^2\theta}{1+\tan^4\theta}d\theta$$

Let, $\tan^2\theta=p \Rightarrow 2\tan\theta\sec^2\theta d\theta=dp$

When, $\theta=0,\ \theta=\frac{\pi}{4}$; $p=0,\ p=1$ $\therefore I=\int_0^1\frac{dp}{1+p^2}=[\tan^{-1}p]_0^1=\frac{\pi}{4}$

77. $\int\frac{x^2}{e^{x^3}+e^{-x^3}}dx$ এর মান কত? **[KUET'10-11]**

(a) $\frac{1}{2}\tan^{-1}(e^{-x^3})+C$ (b) $\frac{1}{3}\tan^{-1}(e^{x^3})+C$ (c) $\tan^{-1}(e^{x^3})+C$

(d) $\tan^{-1}3x+C$ (e) $\tan^{-1}x+C$

সমাধান: $\int\frac{x^2}{e^{x^3}+e^{-x^3}}dx=\int\frac{\frac{1}{3}dz}{e^z+e^{-z}}=\frac{1}{3}\int\frac{e^z\ dz}{(e^z)^2+1}=\frac{1}{3}\int\frac{dp}{1+p^2}=\frac{1}{3}\tan^{-1}(p)+c=\frac{1}{3}\tan^{-1}(e^{x^3})+c$

Let, $x^3=z\Rightarrow 3x^2dx=dz$; $e^z=p\Rightarrow e^zdz=dp$

78. $\int_{-\pi/4}^{\pi/4}(3\tan x+2\sin x)=?$ **[Ans:c][SUST'10-11]**

(a) 4.4 (b) 3.0 (c) 0 (d) –4.4

79. $\int_0^5\frac{xdx}{x^2-5x+6}=?$ **[Ans: a][SUST'10-11]**

(a) ln(32/9) (b) ln(9/32) (c) ln(41) (d) ln(1/41)

80. $\int_0^5\frac{xdx}{1-x^2}=?$ **[SUST'10-11]**

(a) $1-2\sqrt{6}i$ (b) $1-2\sqrt{6}i$ (c) $1-2\sqrt{6}$ (d) $-1+2\sqrt{6}$

সমাধান: সঠিক উত্তর নেই। [Ans: $\ln\left(\frac{1}{2\sqrt{6}i}\right)$]