# অধ্যায়- ০৭ : সংযুক্ত ও যৌগিক কোণের ত্রিকোণমিতিক অনুপাত

**Written**

01. যদি $\sin x + \sin y = 1$ এবং $\cos x + \cos y = 0$ হয় তবে প্রমাণ কর যে, $x + y = \pi$। **[RUET'18-19]**

**সমাধান:** $\sin x + \sin y = 1 \Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 1$ .... (i)

$\cos x + \cos y = 0 \Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 0$ ..... (ii)

$(ii) \div (i) \Rightarrow \frac{\cos\left(\frac{x+y}{2}\right)}{\sin\left(\frac{x+y}{2}\right)} = 0 \Rightarrow \cot\left(\frac{x+y}{2}\right) = 0 \Rightarrow \frac{x+y}{2} = \frac{\pi}{2}\left[\because \cot\frac{\pi}{2} = 0\right] \Rightarrow x + y = \pi$ (Ans.)

02. যদি $\sin x + \sin y = a$ এবং $\cos x + \cos y = b$ হয় তবে দেখাও যে $\sin\frac{1}{2}(x-y) = \pm\frac{1}{2}\sqrt{4 - a^2 - b^2}$। **[BUET'16-17]**

**সমাধান:** $\sin x + \sin y = a$ ..... (i); $\cos x + \cos y = b$ ..... (ii)

$(i)^2 + (ii)^2 \Rightarrow 1 + 1 + 2\sin x \sin y + 2\cos x \cos y = a^2 + b^2 \Rightarrow 2 + 2(\cos x \cos y + \sin x \sin y) = a^2 + b^2$

$\Rightarrow 2 + 2\cos(x-y) = a^2 + b^2 \Rightarrow 2 + 2\left\{1 - 2\sin^2\frac{1}{2}(x-y)\right\} = a^2 + b^2 \Rightarrow 4 - 4\sin^2\frac{1}{2}(x-y) = a^2 + b^2$

$\Rightarrow \sin^2\frac{1}{2}(x-y) = \frac{4-a^2-b^2}{4} \therefore \sin\frac{1}{2}(x-y) = \pm\frac{1}{2}\sqrt{4 - a^2 - b^2}$ (Showed)

03. $\tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(\frac{2\pi}{3} + \theta\right)$ কে $\tan 3\theta$ এর মাধ্যমে প্রকাশ কর। **[BUET'16-17]**

**সমাধান:** $\tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(\frac{2\pi}{3} + \theta\right) = \tan\theta + \frac{\tan\frac{\pi}{3} + \tan\theta}{1 - \tan\frac{\pi}{3}\tan\theta} + \frac{\tan\frac{2\pi}{3} + \tan\theta}{1 - \tan\frac{2\pi}{3}\tan\theta}$

$= \tan\theta + \frac{\sqrt{3} + \tan\theta}{1 - \sqrt{3}\tan\theta} + \frac{-\sqrt{3} + \tan\theta}{1 + \sqrt{3}\tan\theta} = \tan\theta + \frac{\sqrt{3} + 3\tan\theta + \tan\theta + \sqrt{3}\tan^2\theta - \sqrt{3} + 3\tan\theta + \tan\theta - \sqrt{3}\tan^2\theta}{(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta)}$

$= \tan\theta + \frac{8\tan\theta}{1 - 3\tan^2\theta} = \frac{\tan\theta - 3\tan^3\theta + 8\tan\theta}{1 - 3\tan^2\theta} = \frac{3(3\tan\theta - \tan^3\theta)}{1 - 3\tan^2\theta} = 3\tan 3\theta$

04. যদি $\tan\frac{\theta}{2} = \sqrt{\frac{1-P}{1+p}}\tan\frac{\alpha}{2}$, show that $\cos\alpha = \frac{\cos\theta - P}{1 - P\cos\theta}$. **[BUET'14-15]**

**সমাধান:** Given, $\tan\frac{\theta}{2} = \sqrt{\frac{1-p}{1+p}}\tan\left(\frac{\alpha}{2}\right)$

এখন, $\cos\alpha = \frac{1 - \tan^2\left(\frac{\alpha}{2}\right)}{1 + \tan^2\left(\frac{\alpha}{2}\right)} = \frac{1 - \frac{1+p}{1-p}.\tan^2\frac{\theta}{2}}{1 + \frac{1+p}{1-p}.\tan^2\frac{\theta}{2}} = \frac{1 - p - (1+p)\tan^2\frac{\theta}{2}}{1 - p + (1+p)\tan^2\frac{\theta}{2}}$

আবার, $\frac{\cos\theta - p}{1 - p\cos\theta} = \frac{\frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} - p}{1 - p.\frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}} = \frac{1 - p - (1+p)\tan^2\frac{\theta}{2}}{1 - p + (1+p)\tan^2\frac{\theta}{2}} \quad \therefore \cos\alpha = \frac{\cos\theta - p}{1 - p\cos\theta}$ [Showed]

05. সমাধান কর: $\cot x + \cot 2x + \cot 3x = \cot x \cot 2x \cot 3x$. **[BUET'14-15]**

**সমাধান:** $\cot x + \cot 2x + \cot 3x = \cot x \cot 2x \cot 3x \Rightarrow \cot 2x + \cot x = \cot 3x(\cot x \cot 2x - 1)$

$\Rightarrow \frac{\cot 2x + \cot x}{\cot x \cot 2x - 1} = \cot 3x \Rightarrow \frac{1}{\cot(x+2x)} = \cot 3x \quad \left[\therefore (A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}\right]$

$\Rightarrow \tan 3x = \frac{1}{\tan 3x} \Rightarrow \tan^2 3x = 1 \Rightarrow \tan 3x = \pm 1 = \tan\left(\pm\frac{\pi}{4}\right)$

$\therefore 3x = n\pi \pm \frac{\pi}{4} \quad \therefore x = \frac{n\pi}{3} \pm \frac{\pi}{12}; n\epsilon\mathbb{Z}$

06. যদি $\theta = \frac{\pi}{36}$ হয়, তবে $\sin^2 3\theta + \sin^2 4\theta + \sin^2 5\theta + \ldots\ldots + \sin^2 15\theta$ এর মান নির্ণয় কর। **[BUET'13-14]**

সমাধান: প্রদত্ত রাশি $= \sin^2 15^\circ + \sin^2 20^\circ + \sin^2 25^\circ + \ldots\ldots + \sin^2 75^\circ$

$= (\sin^2 15^\circ + \cos^2 15^\circ) + (\sin^2 20^\circ + \cos^2 20^\circ) + (\sin^2 25^\circ + \cos^2 25^\circ) + (\sin^2 30^\circ + \cos^2 30^\circ)$

$+ (\sin^2 35^\circ + \cos^2 35^\circ) + (\sin^2 40^\circ + \cos^2 40^\circ) + \sin^2 45^\circ = 6 + \frac{1}{2} = \boxed{6.5}$

07. $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$, হলে $\cos 4\theta = ?$ **[RUET'12-13]**

সমাধান: $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right) \Rightarrow \cos 2\theta = 2\cos^2\theta - 1 = 2 \times \frac{1}{4}\left(x^2 + \frac{1}{x^2} + 2\right) - 1 = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$

$\therefore \cos 4\theta = 2 \times \left\{\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)\right\}^2 - 1 = 2 \times \frac{1}{4}\left(x^4 + \frac{1}{x^4} + 2\right) - 1 \quad \therefore \cos 4\theta = \frac{1}{2}\left(x^4 + \frac{1}{x^4}\right)$

08. যদি $\theta = \frac{\pi}{20}$ হয়, তবে $\cot\theta.\cot 3\theta.\cot 5\theta.\ldots\ldots\ldots\ldots\ldots\cot 19\theta$ এর মান নির্ণয় কর। **[BUET'11-12]**

সমাধান: $\cot\theta.\cot 3\theta.\cot 5\theta.\cot 7\theta.\cot 9\theta.\cot 11\theta.\cot 13\theta.\cot 15\theta.\cot 17\theta.\cot 19\theta$

$= \cot\theta.\cot 3\theta.1.\cot 7\theta.\cot 9\theta.\cot\left(\frac{\pi}{2} + \theta\right).\cot\left(\frac{\pi}{2} + 3\theta\right).(-1).\cot\left(\frac{\pi}{2} + 7\theta\right).\cot\left(\frac{\pi}{2} + 9\theta\right)$ $\left[\because \theta = \frac{\pi}{2\theta}\right]$

$= -\cot\theta.(-\tan\theta).\cot 3\theta.(-\tan 3\theta).\cot 7\theta.(-\tan 7\theta).\cot 9\theta.(-\tan 9\theta)$

$= -(\cot\theta.\tan\theta).(\cot 3\theta.\tan 3\theta).(\cot 7\theta.\tan 7\theta).(\cot 9\theta.\tan 9\theta)$

$= -1.1.1.1$ [$\because \cot x.\tan x = 1$] $= -1$

09. প্রমাণ কর যে, $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \sqrt{3}$ **[BUET'04-05, BUTex'11-12]**

সমাধান: $\text{L.H.S} = \frac{2\sin 20^\circ \sin 40^\circ \sin 80^\circ}{2\cos 20^\circ \cos 40^\circ \cos 80^\circ} = \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{(\cos 20^\circ + \cos 60^\circ)\cos 80^\circ} = \frac{\cos 20^\circ \sin 80^\circ - \frac{1}{2}\sin 80^\circ}{\cos 20^\circ \cos 80^\circ + \frac{1}{2}\cos 80^\circ}$

$= \frac{\frac{1}{2}(\sin 100^\circ + \sin 60^\circ) - \frac{1}{2}\sin 80^\circ}{\frac{1}{2}(\cos 100^\circ + \cos 60^\circ) + \frac{1}{2}\cos 80^\circ} = \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ}{\cos 100^\circ + \frac{1}{2} + \cos 80^\circ} = \frac{2\sin 10^\circ \cos 90^\circ + \sqrt{3}/2}{2\cos 90^\circ \cos 10^\circ + \frac{1}{2}}$

$= \frac{0 + \sqrt{3}/2}{0 + \frac{1}{2}} = \sqrt{3} = \text{R.H.S (Sowed)}$

10. $A + B = \pi/4$ হলে দেখাও যে, $(1 + \tan A)(1 + \tan B) = 2$ **[RUET'10-11]**

সমাধান: বামপক্ষ $= (1 + \tan A)(1 + \tan B) = (1 + \tan A)\left\{1 + \tan\left(\frac{\pi}{4} - A\right)\right\}$

$= (1 + \tan A)\left\{1 + \frac{\tan\frac{\pi}{4} - \tan A}{1 + \tan\frac{\pi}{4}.\tan A}\right\} = (1 + \tan A).\frac{(1 + \tan A + 1 - \tan A)}{(1 + \tan A)} = 2 =$ ডানপক্ষ

11. $\sin 3A = ?$ **[BUTex'10-11]**

**সমাধান:** $3\sin A - 4\sin^3 A$

12. $\sin 210^\circ + \cot 225^\circ = ?$ **[BUTex'10-11]**

**সমাধান:** $\sin(180^\circ + 30^\circ) + \cot(180^\circ + 45^\circ) = -\sin 30^\circ + \cot 45^\circ = \frac{1}{2}$ (Ans.)

13. যদি $A + B + C = \pi$ হয় তবে প্রমাণ কর যে, $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$ **[BUET'09-10]**

**সমাধান:** L.H.S. $= \sin 2A + \sin 2B + \sin 2C = 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin(\pi - C)\cos(A-B) + \sin 2C = 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C[\cos(A-B) + \cos[\pi - (A-B)] = 2\sin C\,[\cos(A-B) - \cos(A+B)]$

$= 2\sin C \times 2\sin A\sin B = 4\sin A\sin B\sin C = $ R.H.S. (Proved)

14. একটি সমকোণী ত্রিভুজের সূক্ষ্মকোণদ্বয়কে নিম্নলিখিত সমীকরণ দ্বারা প্রকাশ করা যায়। সূক্ষ্মকোণদ্বয় এর মান নির্ণয় কর।

সমীকরণটি ঃ $\sin\theta + 2\cos\theta = 1$ **[CUET'09-10]**

**সমাধান:** $\sin\theta + 2\cos\theta = 1 \Rightarrow (\sin\theta - 1)^2 = 4\cos^2\theta \Rightarrow \sin^2\theta - 2\sin\theta + 1 = 4 - 4\sin^2\theta$

$\Rightarrow 5\sin^2\theta - 2\sin\theta - 3 = 0 \Rightarrow 5\sin^2\theta - 5\sin\theta + 3\sin\theta - 3 = 0$

$(\sin\theta - 1)(5\sin\theta + 3) = 0 \therefore \theta = 90^\circ,\ \sin^{-1}\frac{-3}{5}$ $\therefore$ এরূপ কোন সমকোণী ত্রিভুজ সম্ভব নয়। (Ans.)

15. যদি $\alpha$ ও $\beta$ ধনাত্মক ও সূক্ষ্মকোণ হয় এবং $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$ হয়, তবে দেখাও যে, $\tan\alpha = \sqrt{2}\tan\beta$

**সমাধান:** $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta} \Rightarrow \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{4(1 - \cos 2\beta)}{2(1 + \cos 2\beta)}$ **[CUET'09-10]**

$\Rightarrow \frac{2\sin^2\alpha}{2\cos^2\alpha} = 2.\frac{\sin^2\beta}{\cos^2\beta} \Rightarrow \tan^2\alpha = 2\tan^2\beta$

$\therefore \tan\alpha = \sqrt{2}\tan\beta$ [$\because \alpha, \beta$ ধনাত্মক সূক্ষ্মকোণ, তাই $-\sqrt{2}\tan\beta$ গ্রহণযোগ্য নয়] (Ans.)

16. যদি $\tan\theta\tan\varphi = \sqrt{\frac{a-b}{a+b}}$ হয়, তবে প্রমাণ কর যে, $(a - b\cos 2\theta)(a - b\cos 2\varphi) = a^2 - b^2$

**সমাধান:** $\tan\theta\,\tan\varphi = \sqrt{\frac{a-b}{a+b}} \Rightarrow \tan^2\theta\tan^2\varphi = \frac{a-b}{a+b} \Rightarrow (a-b) = (a+b)\tan^2\theta\tan^2\varphi$

L.H.S. $= (a - b\cos 2\theta)(a - b\cos 2\varphi)$ **[BUTex'08-09]**

$= \left(a - b\times\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\left(a - b\times\frac{1-\tan^2\varphi}{1+\tan^2\varphi}\right) = \left(\frac{a + a\tan^2\theta - b + b\tan^2\theta}{1+\tan^2\theta}\right)\left(\frac{a + a\tan^2\varphi - b + b\tan^2\varphi}{1+\tan^2\varphi}\right)$

$= \left\{\frac{(a-b) + (a+b)\tan^2\theta}{1+\tan^2\theta}\right\}\left\{\frac{(a-b) + (a+b)\tan^2\varphi}{1+\tan^2\varphi}\right\}$

$= \left\{\frac{(a+b)\tan^2\theta\,(1+\tan^2\varphi)}{1+\tan^2\theta}\right\}\left\{\frac{(a+b)\tan^2\varphi\,(1+\tan^2\theta)}{1+\tan^2\varphi}\right\}$ [মান বসিয়ে]

$= (a+b)^2\tan^2\theta\tan^2\varphi = (a+b)^2 \times \frac{a-b}{a+b} = (a+b)(a-b) = a^2 - b^2 = $ R.H.S. (Proved)

17. ABC ত্রিভুজ থেকে প্রমাণ কর যে, $a\sin\left(\frac{A}{2}+B\right)=(b+c)\sin\left(\frac{A}{2}\right)$ **[RUET'07-08]**

**সমাধান:** আমরা জানি, $\frac{a}{b+c}=\frac{2R\sin A}{2R\sin B+2R\sin C}=\frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}$

$$=\frac{\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left[\frac{B-(\pi-A-B)}{2}\right]}=\frac{\sin\frac{A}{2}\cos\frac{A}{2}}{\cos\frac{A}{2}\cos\left[\frac{\pi}{2}-\left(\frac{A}{2}+B\right)\right]}$$

$$=\frac{\sin\frac{A}{2}}{\cos\left[\frac{\pi}{2}-\left(\frac{A}{2}+B\right)\right]}=\frac{\sin\frac{A}{2}}{\sin\left(\frac{A}{2}+B\right)} \quad \therefore a\sin\left(\frac{A}{2}+B\right)=(b+c)\sin\left(\frac{A}{2}\right) \text{ [Proved]}$$

18. প্রমাণ কর ঃ $2\sin\frac{\pi}{16}=2\sin 11^{o}15'=\sqrt{2-\sqrt{2+\sqrt{2}}}$ . **[BUTex'07-08,CUET'05-06]**

**সমাধান:** $2\sin\frac{\pi}{16}=\sqrt{2.2\sin^2\frac{\pi}{16}}=\sqrt{2.\left(1-\cos\frac{\pi}{8}\right)}=\sqrt{2-2\cos\frac{\pi}{8}}=\sqrt{2-\sqrt{2.2\cos^2\frac{\pi}{8}}}$

$$=\sqrt{2-\sqrt{2\left(1+\cos\frac{\pi}{4}\right)}}=\sqrt{2-\sqrt{2+2\cos\frac{\pi}{4}}}=\sqrt{2-\sqrt{2+\sqrt{2}}}$$

আবার, $\left(\frac{\pi}{16}\right)^{\circ}=\left(\frac{\pi}{16}\times\frac{180}{\pi}\right)^{0}=11.25^{\circ}=11^{\circ}\,15'$

$\therefore 2\sin\frac{\pi}{16}=2\sin 11^{\circ}15=\sqrt{2-\sqrt{2+\sqrt{2}}}$ (প্রমাণিত)

19. সমাধান কর $(Sol^{n})$ : $4\cos x\ \cos 2x\ \cos 3x = 1;\ 0 < x < \pi$ **[KUET'06-07]**

**সমাধান:** $4\cos x\ \cos 2x\ \cos 3x = 1 \Rightarrow 2\cos 2x\ (2\cos 3x\ \cos x) = 1$

$\Rightarrow 2\cos 2x\ (\cos 4x + \cos 2x) = 1 \Rightarrow 2\cos 2x\cos 4x + 2\cos^2 2x = 1$

$\Rightarrow 2\cos 2x\cos 4x + 2\cos^2 2x - 1 = 0$

$\Rightarrow 2\cos 2x\cos 4x+\cos 4x=0$

$\Rightarrow \cos 4x\ (2\cos 2x + 1) = 0;\ \cos 4x = 0 \quad x=(2n+1)\frac{\pi}{8}\longrightarrow$ (i) $n\in z$

Again, $\cos 2x = -\frac{1}{2}=\cos\frac{2\pi}{3} \Rightarrow 2x=2n\pi\pm\frac{2\pi}{3}$

$\therefore x=n\pi\pm\frac{\pi}{3}\longrightarrow$ (ii)

(i) ও (ii) হতে পাই

When $n=0,\ x=\frac{\pi}{8},\frac{\pi}{3}$

$n=1,\ x=\frac{3\pi}{8},\frac{2\pi}{3}$

$n=2,\ x=\frac{5\pi}{8}$

$n=3,\ x=\frac{7\pi}{8}$ (Ans.)

20. (a) সমাধান কর ঃ $\frac{(\sec x+\tan x)^2-(\sec 2x+\tan x)^2}{\sin 2x-\sin x}=2;\ [-\pi\le x\le\pi]$ **[RUET'06-07]**

(b) ABC ত্রিভুজ থেকে প্রমাণ কর যে, $a^2\ (\sin^2B-\sin^2C)+b^2\ (\sin^2C-\sin^2A)+c^2\ (\sin^2A-\sin^2B)=0$

**সমাধান: (a)** $(\sec x+\tan x)^2-(\sec 2x+\tan x)^2-2(\sin 2x-\sin x)=0$

$$\Rightarrow\left(\frac{1+\sin x}{\cos x}\right)^2-\left(\frac{1}{\cos 2x}+\frac{\sin x}{\cos x}\right)^2-2(\sin 2x-\sin x)=0$$

$$\Rightarrow\frac{1+2\sin x+\sin^2 x}{\cos x}-\frac{\cos^2 x+2\cos x.\cos 2x.\sin x}{\cos 2x.\cos x}-2\sin 2x+2\sin x=0$$

$$\Rightarrow\frac{\cos 2x+2\cos 2x.\sin x+\sin^2 x.\cos 2x-\cos^2 x-\cos 2x.\sin 2x-2\sin 2x.\cos 2x.\cos x+2\sin x.\cos 2x.\cos x}{\cos 2x.\cos x}=0$$

$$\Rightarrow\cos 2x+2\cos 2x.\sin x+\sin^2 x.\cos 2x-\cos^2 x-\cos 2x.\sin 2x-\sin 4x.\cos x+\sin 2x.\cos 2x=0$$

$$\Rightarrow\cos 2x+2\cos 2x.\sin x+\sin^2 x.\cos 2x-\cos^2 x-\sin 4x.\cos x=0$$

$$\Rightarrow\cos^2 x-\sin^2 x+2\sin x(\cos^2 x-\sin^2 x)+\sin^2 x(\cos^2 x-\sin^2 x)-\cos^2 x-2\sin 2x.\cos 2x.\cos x=0$$

$$\Rightarrow 1-2\sin^2 x+2\sin x-4\sin^3 x+\sin^2 x-2\sin^4 x-1+\sin^2 x-4\sin x(1-\sin^2 x)(1-2\sin^2 x)=0$$

$$\Rightarrow 1-2\sin^2 x+2\sin x-4\sin^3 x+\sin^2 x-2\sin^4 x-1+\sin^2 x-4\sin x+12\sin^3 x-8\sin^5 x=0$$

$$\Rightarrow -8\sin^5 x-2\sin^4 x+8\sin^3 x-6\sin x=0\ \Rightarrow -2\sin x(4\sin^4 x+\sin^3 x+2\sin^2 x+3)=0$$

$-2\sin x=0,\ \therefore x=n\pi$ Where, $n\in z$

or, $4\sin^4 x+\sin^3 x+2\sin^2 x+3=0$

But, $4\sin^4 x+\sin^3 x+2\sin^2 x+3=\left(4-\frac{1}{4}\right)\sin^4 x+\sin^2 x+\sin^2 x\left(\frac{1}{2}\sin x+1\right)^2+3\geq 3$

$\therefore x=n\pi;\ n\in z$

**(b)** $\text{L.H.S}=a^2\left(\frac{b^2}{4R^2}-\frac{c^2}{4R^2}\right)+b^2\left(\frac{c^2}{4R^2}-\frac{a^2}{4R^2}\right)+c^2\left(\frac{a^2}{4R^2}-\frac{b^2}{4R^2}\right)=0$

21. ABC একটি স্থূলকোণী ত্রিভুজ। প্রমাণ কর যে, cot A cotB + cotB cotC + cotC cotA = 1। **[BUET'05-06]**

**সমাধান:** যে কোন ত্রিভুজ ABC এর জন্য $A+B+C=\pi\ \Rightarrow A+B=\pi-C$

$$\cot(A+B)=\cot(\pi-c)\ \Rightarrow\ \frac{\cot A\cot B-1}{\cot B+\cot A}=-\cot C$$

$\Rightarrow$ cot A cot B + cot B cot C + cot A. cotC = 1. (Proved)

22. $\Delta ABC$ ত্রিভুজে cos A = sinB – cosC হলে দেখাও যে, ত্রিভুজটি সমকোণী। **[BUET'04-05,05-06]**

**সমাধান:** cosA - sinB – cosC $\Rightarrow$ cosA + cosC = sinB $\Rightarrow$ cosA – cos (A + B) = sinB

$$\Rightarrow 2\sin\left(\frac{2A+B}{2}\right)\sin\frac{B}{2}=2\sin\frac{B}{2}\cos\frac{B}{2}\ \Rightarrow\sin\left(\frac{2A+B}{2}\right)=\cos\frac{B}{2}$$

$$\Rightarrow\sin\left(A+\frac{B}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{B}{2}\right)\ \Rightarrow A+\frac{B}{2}=\frac{\pi}{2}-\frac{B}{2}\ \Rightarrow A+B=\frac{\pi}{2}$$

$\therefore\ \Delta ABC$ সমকোণী (Showed)

**Alternate:** $\cos A=\sin B-\cos C\Rightarrow\cos A+\cos C=\sin B$

$$\Rightarrow 2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=2\sin\frac{B}{2}\cos\frac{B}{2}\Rightarrow\cos\left(\frac{r}{2}-\frac{B}{2}\right)\cos\frac{A-C}{2}=\sin\frac{B}{2}\cos\frac{B}{2}$$

$$\Rightarrow\sin\frac{B}{2}\cos\frac{A-C}{2}=\sin\frac{B}{2}\cos\frac{B}{2}\Rightarrow\cos\frac{A-C}{2}=\cos\frac{B}{2}$$

$\therefore A-C=B\Rightarrow A=B+C$

$\therefore A+B+C=\pi\Rightarrow 2A=\pi\Rightarrow A=\frac{\pi}{2}$

23. দেখাও যে, $\frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1} = \cot 3A$ **[BUET'04-05]**

সমাধান: $R.H.S = \cot 3A = \cot(2A + A) = \frac{\cot A \cot 2A - 1}{\cot A + \cot 2A}$

$$= \frac{\cot A\left(\frac{\cot^2 A - 1}{2\cot A}\right) - 1}{\cot A + \frac{\cot^2 A - 1}{2\cot A}} = \frac{\frac{\cot^3 A - \cot A - 2\cot A}{2\cot A}}{\frac{2\cot^2 A + \cot^2 A - 1}{2\cot A}} = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1} = \text{L.H.S. (Showed)}$$

24. $\tan 36° + \tan 9° + \tan 36° \tan 9°$ এর মান কত? **[KUET'04-05]**

সমাধান: $\tan 45° = 1 \Rightarrow \tan(36° + 9°) = 1 \Rightarrow \frac{\tan 36° + \tan 9°}{1 - \tan 36° \tan 9°} = 1$

$\Rightarrow \tan 36° + \tan 9° + \tan 36° \tan 9° = 1$

25. $\cot B \cot C + \cot C \cot A + \cot A \cot B = 1$ হলে, A, B এবং C এর মধ্যে সম্পর্ক স্থাপন কর। **[BUTex'04-05]**

সমাধান: $\cot B \cot C + \cot C \cot A + \cot A \cot B = 1$

$\Rightarrow \cot B \cot C + \cot C \cot A + \cot A \cot B - 1 = 0 \Rightarrow \cot C(\cot B + \cot A) + \cot A \cot B - 1 = 0$

$\Rightarrow \cot C + \frac{\cot A \cot B - 1}{\cot A + \cot B} = 0 \Rightarrow \cot C + \cot(A + B) = 0 \Rightarrow \cot(A + B) = -\cot C = \cot(\pi - C)$

$\Rightarrow A + B = \pi - C \quad \therefore A + B + C = \pi$ (Ans.)

26. যদি $a = 2b$ এবং $A = 3B$ হয়, তবে ত্রিভুজের কোণগুলো নির্ণয় কর। **[BUET'03-04]**

সমাধান: আমরা জানি, $\Delta ABC$-এ $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2b}{\sin 3B} = \frac{b}{\sin B}$

$\Rightarrow 2\sin B = \sin 3B \Rightarrow 2\sin B = 3\sin B - 4\sin^3 B \Rightarrow 4\sin^2 B = 1 \Rightarrow \sin B = \frac{1}{2} \quad \therefore B = 30°$ (Ans.)

$\therefore A = 3B = 90°;\ C = 180° - (A + B) = 60°$ (Ans.)

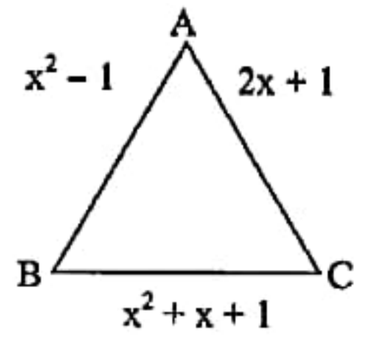
27. দেখাও যে, কোন ত্রিভুজের বাহু $x^2 + x + 1$, $2x + 1$ এবং $x^2 - 1$ হলে, সর্বোচ্চ কোণটির মান $120°$। **[CUET'03-04]**

সমাধান: আমরা জানি, বৃহত্তর বাহুর বিপরীত কোণ বৃহত্তর|

এখানে বৃহত্তর বাহু হচ্ছে $x^2 + x + 1$ এর বিপরীত কোণ A

$$\therefore \cos A = \frac{(x^2 - 1)^2 + (2x + 1)^2 - (x^2 + x + 1)^2}{2(x^2 - 1)(2x + 1)}$$

$$= \frac{x^4 - 2x^2 + 1 + 4x^2 + 4x + 1 - x^4 - x^2 - 1 - 2x^3 - 2x - 2x^2}{2(x^2 - 1)(2x + 1)}$$




$$= \frac{2x - x^2 - 2x^3 + 1}{2(x^2 - 1)(2x + 1)} = \frac{(2x + 1)(1 - x^2)}{2(x^2 - 1)(2x + 1)} = -\frac{1}{2} \quad \therefore A = 120° \quad \text{(Showed)}$$

**Alternate:** Putting $x = 2$ [Any valid value for which $a + b > c$; $b + c > a$; $c + a > b$

$a = x^2 + x + 1 = 4 + 2 + 1 = 7$

$b = 2x + 1 = 5$

$c = x^2 - 1 = 4 - 1 = 3$

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 9 - 49}{2 \times 5 \times 3} = -\frac{15}{30} = -\frac{1}{2} \quad \therefore \Delta = 120°$

28. θ কোণকে α ও β অংশে এমনভাবে বিভক্ত করা হলো যেন $\tan\alpha = k\tan\beta$ হয়, প্রমাণ কর যে, $\sin(\alpha-\beta) = \frac{k-1}{k+1}\sin\theta$ ।

সমাধান: Given, $\tan\alpha = k\tan\beta \Rightarrow \frac{\sin\alpha}{\cos\alpha} = k\frac{\sin\beta}{\cos\beta} \Rightarrow \frac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta} = \frac{k}{1}$ **[KUET'03-04]**

$\Rightarrow \frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta} = \frac{k-1}{k+1}$ [বিয়োজন - যোজন]

$\Rightarrow \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} = \frac{k-1}{k+1} \Rightarrow \sin(\alpha-\beta) = \frac{k-1}{k+1}\sin\theta \quad [\alpha+\beta=\theta]$ (Proved)

29. যদি $\sin x+\sin y=1$ এবং $\cos x+\cos y=0$ তবে প্রমাণ কর যে, $x+y=\pi$ **[BUET'02-03]**

সমাধান: $\sin x+\sin y=1;\ 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}=1$.....(i)

$\cos x+\cos y=0;\ 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}=0$.....(ii)

(ii) হতে, $\cos\frac{x-y}{2} \neq 0$ কেননা তা না হলে (i) সিদ্ধ হবে না।

$\therefore \cos\frac{x+y}{2} = 0 = \cos\frac{\pi}{2}; \frac{x+y}{2} = \frac{\pi}{2} \therefore x+y=\pi$ (Proved)

30. যদি $a=2, b=1+\sqrt{3}, C=60°$ হয়, তবে ত্রিভুজটি সমাধান কর। **[BUET'02-03]**

সমাধান: $\cos C = \frac{a^2+b^2-c^2}{2ab} \Rightarrow \cos60° = \frac{1}{2} = \frac{4+1+3+2\sqrt{3}-c^2}{2\times2\times(1+\sqrt{3})}$

$\Rightarrow 4+4\sqrt{3} = 16+4\sqrt{3}-2c^2 \Rightarrow 2c^2=12 \Rightarrow c^2=6 \Rightarrow c=\sqrt{6}$

$\therefore \cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{6+4-1-2\sqrt{3}-3}{2\times\sqrt{6}\times2} \Rightarrow \cos B = \frac{6-2\sqrt{3}}{4\sqrt{6}} \quad \therefore B = 75°$

$\therefore A = 180 - (B+C) \therefore A = 180 - (75+60) = 45°$ (Ans.)

31. সমাধান কর ঃ $\cos\theta - \cos7\theta = \sin4\theta$. **[BUTex'02-03]**

সমাধান: $\cos\theta - \cos7\theta = \sin4\theta$ বা, $2\sin4\theta\sin3\theta - \sin4\theta = 0$ বা, $\sin4\theta(2\sin3\theta-1)=0$

বা, $\sin4\theta=0$ বা, $4\theta=n\pi \Rightarrow \theta=\frac{n\pi}{4}$ (Ans.)

অথবা, $2\sin3\theta-1=0$ বা, $\sin3\theta=\frac{1}{2}=\sin\frac{\pi}{6}$ বা, $3\theta = n\pi+(-1)^n\frac{\pi}{6} \quad \therefore \theta=\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$. (Ans.)

32. α ও β দুইটি ধনাত্মক সুক্ষকোণ এবং $\cos2\alpha=\frac{3\cos2\beta-1}{3-\cos2\beta}$ হলে, প্রমাণ কর যে, $\tan\alpha=\pm\sqrt{2}\tan\beta$ **[BUET'01-02]**

সমাধান: $\cos2\alpha=\frac{3\cos2\beta-1}{3-\cos2\beta} \Rightarrow \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{3\frac{1-\tan^2\beta}{1+\tan^2\beta}-1}{3-\frac{1-\tan^2\beta}{1+\tan^2\beta}}$

$= \frac{3-3\tan^2\beta-1-\tan^2\beta}{3+3\tan^2\beta-1+\tan^2\beta} = \frac{2-4\tan^2\beta}{2+4\tan^2\beta} = \frac{1-2\tan^2\beta}{1+2\tan^2\beta} \Rightarrow \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{1-2\tan^2\beta}{1+2\tan^2\beta}$

$\Rightarrow \frac{2}{2\tan^2\alpha} = \frac{2}{4\tan^2\beta}$ [যোজন-বিযোজন] $\Rightarrow \tan^2\alpha = 2\tan^2\beta \Rightarrow \tan\alpha = \pm\sqrt{2}\tan\beta$ (Proved)

33. সমাধান কর ঃ $\cos x+\sqrt{3}\sin x=\sqrt{2}$ **[BUTex'01-02]**

**সমাধান:** $\cos x+\sqrt{3}\sin x=\sqrt{2} \Rightarrow \cos x.\cos\frac{\pi}{3}+\sin x.\sin\frac{\pi}{3}=\cos \pi/4 \Rightarrow \cos\left(x-\frac{\pi}{3}\right)=\cos\frac{\pi}{4}$

$\Rightarrow x-\pi/3=2n\pi\pm\pi/4 \Rightarrow x=2n\pi\pm\frac{\pi}{4}+\frac{\pi}{3}$

$\therefore x=2n\pi+\frac{7\pi}{12}$ এবং $\Rightarrow x=2n\pi-\frac{\pi}{4}+\frac{\pi}{3}=2n\pi+\frac{\pi}{12}$

34. যে কোন ত্রিভুজ ABC এর জন্য দেখাও যে, $\frac{1}{a}\cos^2\frac{A}{2}+\frac{1}{b}\cos^2\frac{B}{2}+\frac{1}{c}\cos^2\frac{C}{2}=\frac{s^2}{abc}$ **[BUET'00-01**

**সমাধান:** $\text{L.H.S.}=\frac{1}{a}\cos^2\frac{A}{2}+\frac{1}{b}\cos^2\frac{B}{2}+\frac{1}{c}\cos^2\frac{C}{2}=\frac{1}{a}\times\frac{s(s-a)}{bc}+\frac{1}{b}\times\frac{s(s-b)}{ca}+\frac{1}{c}\times\frac{s(s-c)}{ab}$

$=\frac{s(3s-a-b-c)}{abc}=\frac{s(3s-2s)}{abc}=\frac{s^2}{abc}=\text{R.H.S (Proved)}$

35. প্রমাণ কর যে, $16\ \cos\frac{2\pi}{15}\ \cos\frac{4\pi}{15}\ \cos\frac{8\pi}{15}\ \cos\frac{14\pi}{15}=1$ **[BUET'00-01]**

**সমাধান:** Let, $\frac{2\pi}{15}=\theta$

$\text{L.H.S}=16\ \cos\theta\ \cos2\theta\cos4\theta\ \cos7\theta\ =\frac{8}{\sin\theta}.(2\sin\theta\cos\theta)\cos2\theta\cos4\theta\cos7\theta$

$=\frac{4}{\sin\theta}(2\sin2\theta\cos2\theta)\cos4\theta\cos7\theta\ =\frac{2}{\sin\theta}(2\sin4\theta\cos4\theta)\cos7\theta=\frac{1}{\sin\theta}(2\sin8\theta\cos7\theta)$

$=\frac{1}{\sin\theta}(\sin15\theta+\sin\theta)\ =\frac{1}{\sin\theta}(\sin2\pi+\sin\theta)=1=\text{R.H.S. (Proved)}$

## MCQ

01. $\text{cosec}(x-y)$ এর মান কোনটি? **[Ans: e] [KUET'18-19]**

(a) $\cos x-\cos y$ (b) $\sin x-\sin y$ (c) $\tan x-\tan y$ (d) $\frac{\sin x-\sin y}{\cos x+\cos y}$ (e) $\frac{\sec x.\sec y}{\tan x-\tan y}$

02. $k, l$ এর কোন মানের জন্য $5\sin(k\theta)=(10l+9)\sin\theta+(15l+6)\cos\theta$ **[SUST'18-19]**

(a) $-1,-\frac{2}{5}$ (b) $1,-\frac{2}{5}$ (c) $1,-\frac{5}{2}$ (d) $-1,\frac{2}{5}$ (e) $2,\frac{5}{2}$

**সমাধান:** (b); প্রদত্ত সমীকরণটি অভেদ হলে, $\theta$ এর যেকোনো মানের জন্য এটি সিদ্ধ হব।

$\theta=0$ হলে, $5\sin(k.0)=0+(15l+6)\cos0\ \therefore l=-\frac{2}{5}$

$\theta=\frac{\pi}{2}$ হলে, $5\sin\left(k\frac{\pi}{2}\right)=10l+9+0=10\left(-\frac{2}{5}\right)+9=5$

বা, $\sin\left(k\frac{\pi}{2}\right)=1$; $k=1$ এর জন্য সমীকরণটি সিদ্ধ হয়।

03. $\sqrt{3}\tan6\theta-\sqrt{3}\tan4\theta+\tan6\theta\tan4\theta+1=0$ এর মুখ্য সমাধান হলো- **[KUET'17-18]**

(a) $60°$ (b) $165°$ (c) $75°$ (d) $30°$ (e) $135°$

**সমাধান:** (c); $\sqrt{3}(\tan6\theta-\tan4\theta)=-(1+\tan6\theta\tan4\theta)$

$\Rightarrow\frac{\tan6\theta-\tan4\theta}{1+\tan6\theta\tan4\theta}=\frac{-1}{\sqrt{3}}\Rightarrow\tan(6\theta-4\theta)=\tan\left(\pi-\frac{\pi}{6}\right)\Rightarrow2\theta=\frac{5\pi}{6}\therefore\theta=\frac{5\pi}{12}=75°$

04. $\cos^2A+\cos^2(A+\pi/3)+\cos^2(A-\pi/3)$ এর মান কোনটি? **[KUET'17-18]**

(a) $\frac{3}{2}$ (b) $\frac{3}{2}+1$ (c) $\frac{1}{2}+5$ (d) $\frac{5}{7}$ (e) $\frac{7}{9}$

**সমাধান:** (a); $\cos^2A+\cos^2\left(A+\frac{\pi}{3}\right)+\cos^2\left(A-\frac{\pi}{3}\right)$, রাশিটিতে $A=\frac{\pi}{6}$ বসিয়ে পাই, $=\frac{3}{2}$

05. ABC ত্রিভূজে যদি $a = 3, b = 3\sqrt{3}$ এবং $A = 30^\circ$ হয় তবে B ও C এর মান কোনটি? **[KUET'16-17]**

(a) $45^\circ, 30^\circ$ (b) $30^\circ, 40^\circ$ (c) $20^\circ, 45^\circ$ (d) $10^\circ, 70^\circ$ (e) $30^\circ, 90^\circ$

**সমাধান:** (No correct answer); $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow B = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}\left(\frac{3\sqrt{3} \times \sin 30^\circ}{3}\right) = 60^\circ$

$\therefore C = 180^\circ - (A + B) = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$

06. $\frac{\sin x}{\sin y} = \sqrt{2}$ এবং $\frac{\tan x}{\tan y} = \sqrt{3}$ হলে, x এবং y এর মান কোনটি? **[KUET'16-17]**

(a) $\frac{\pi}{4}, \frac{\pi}{6}$ (b) $\frac{\pi}{3}, \frac{\pi}{5}$ (c) $-\frac{\pi}{3}, -\frac{\pi}{8}$ (d) $\frac{\pi}{7}, \frac{\pi}{5}$ (e) $-\pi, \pi$

**সমাধান:** (a); $\frac{\tan x}{\tan y} = \sqrt{3} \Rightarrow \frac{\sin x}{\sin y} = \sqrt{3}\left(\frac{\cos x}{\cos y}\right) \Rightarrow \frac{\cos x}{\cos y} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \frac{\cos^2 x}{\cos^2 y} = \frac{2}{3} \Rightarrow 3\cos^2 x = 2\cos^2 y$

$\Rightarrow 3 - 3\sin^2 x = 2 - 2\sin^2 y \Rightarrow 3\sin^2 x - 2\sin^2 y = 1 \ldots\ldots\ldots (i)$

আবার, $\frac{\sin x}{\sin y} = \sqrt{2} \Rightarrow \sin^2 x - 2\sin^2 y = 0 \ldots\ldots\ldots (ii)$

(i) ও (ii) সমাধান করে পাই, $x = \frac{\pi}{4}$ , $y = \frac{\pi}{6}$

07. $\theta = \sin^{-1}\frac{3}{5}$ হলে $\frac{1-\tan^2\theta}{1+\tan^2\theta}$ এর মান কত? **[BUTex'16-17]**

(a) $\frac{9}{25}$ (b) $\frac{7}{25}$ (c) $\frac{16}{25}$ (d) $\frac{6}{25}$

**সমাধান:** (b); $\sin\theta = \frac{3}{5} \therefore \tan\theta = \frac{3}{4} \therefore \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{7}{25}$

08. $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = ?$ **[BUTex'16-17]**

(a) 4 (b) $\frac{1}{4}$ (c) 0 (d) $\sqrt{3}$

**সমাধান:** (a); $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ} = \frac{4(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{2\sin 10^\circ \cos 10^\circ} = \frac{4\sin 20^\circ}{\sin 20^\circ} = 4$

09. ABC ত্রিভুজে $a = 6cm, b = 8cm$ এবং $c = 10cm$ হলে cosB এর মান কত? **[BUTex'16-17]**

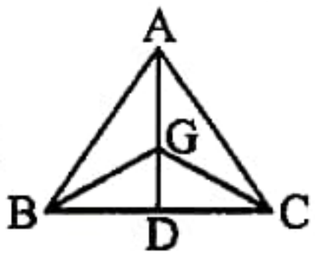
(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{2}{3}$

**সমাধান:** (b); $\cos B = \frac{a^2+c^2-b^2}{2ac} \Rightarrow \cos B = \frac{3}{5}$

10. একটি সমবাহু ত্রিভুজের ভরকেন্দ্রের সাথে যে কোন দুটি কৌণিক বিন্দু সংযুক্ত করে নতুন ত্রিভুজ তৈরি করা হল। নতুন ত্রিভুজটির ক্ষেত্রফল সমবাহু ত্রিভুজের ক্ষেত্রফলের কত অংশ হবে? **[Ans: d] [SUST'16-17]**

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

**সমাধান:** (d);




$\Delta$ ক্ষেত্র $GBC = \frac{1}{2} \times GD \times BC = \frac{1}{2} \times \frac{AD}{3} \times BC = \frac{1}{3} \times \Delta$ ক্ষেত্র ABC

11. $\sin 54^\circ = \frac{1}{4}(1 + \sqrt{5})$ হলে $\cos 36^\circ$ এর মান কত? **[Ans: a] [SUST'16-17]**

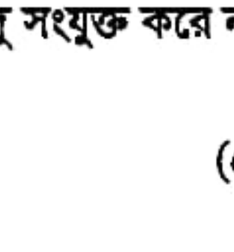
(a) $\frac{1}{4}(1 + \sqrt{5})$ (b) $\frac{1}{4}(1 - \sqrt{5})$ (c) $\frac{1}{4}(\sqrt{5} - 1)$ (d) $\frac{1}{4}(-1 - \sqrt{5})$ (e) $\frac{1}{4}(\pm\sqrt{5} \pm 1)$

**সমাধান:** (a); $\sin 54^\circ = \sin(90^\circ - 36^\circ) = \cos 36^\circ = \frac{1}{4}(1 + \sqrt{5})$

12. একটি সমদ্বিবাহু ত্রিভুজের অসম বাহুর দৈর্ঘ্য ও পরিব্যাসার্ধ যথাক্রমে 8 ও 5 একক। পরিকেন্দ্রটি ত্রিভুজের অভ্যন্তরে হলে সমবাহুদ্বয়ের প্রত্যেকটির দৈর্ঘ্য কত একক? **[SUST'16-17]**

(a) $4\sqrt{2}$ (b) $2\sqrt{5}$ (c) $4\sqrt{5}$ (d) $2\sqrt{3}$ (e) $3\sqrt{5}$

**সমাধান:** (b); পরিকেন্দ্র ত্রিভুজের অভ্যন্তরে, তাই ত্রিভুজটি সূক্ষ্মকোণী




এখন, $\frac{a}{\sin A} = \frac{b}{\sin(180^\circ - 2A)} = 2R$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin 2A} = 2R \Rightarrow \sin 2A = \frac{b}{2R} \therefore A = \frac{1}{2}\sin^{-1}\frac{b}{3R} \therefore a = 2R\sin\left(\frac{1}{2}\sin^{-1}\frac{b}{2R}\right) = 2\sqrt{5}$

13. $\sin\cot^{-1}\tan\cos^{-1}x = ?$ **[BUTex'15-16]**

(a) x (b) $\pi$ (c) $\frac{1}{x}$ (d) $\sqrt{1-x}$

**সমাধান:** (a); $\sin\cot^{-1}\tan\cos^{-1}x = \sin\cot^{-1}\cot\left(\frac{\pi}{2} - \cos^{-1}x\right) = \sin\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos\cos^{-1}x = x$

14. ABC ত্রিভুজে $\angle A = 75^\circ$ এবং $\angle B = 45^\circ$ হলে c: b =? [BUTex'15-16]

(a) $\sqrt{5}:\sqrt{7}$ (b) $\sqrt{2}:\sqrt{3}$ (c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{3}:\sqrt{5}$

**সমাধান:** (c); $\angle C = 180^\circ - 45^\circ - 75^\circ = 60^\circ$ $\therefore \frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \frac{c}{b} = \frac{\sin C}{\sin B} = \frac{\sin 60^\circ}{\sin 45^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{3}}{\sqrt{2}}$ $\therefore$ c: b = $\sqrt{3}:\sqrt{2}$

15. যদি $\tan\alpha - \tan\beta = p, \cot\beta - \cot\alpha = q, \alpha - \beta = \theta$ হয়, তবে $\cot\theta$ এর মান কত? [BUTex'15-16]

(a) $\frac{1}{p} - \frac{1}{q}$ (b) $\frac{1}{q} - \frac{1}{p}$ (c) $\frac{1}{p} + \frac{1}{q}$ (d) $1 - \frac{p}{q}$

**সমাধান:** (c); $p = \tan\alpha - \tan\beta = \frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta} = \frac{\sin\alpha.\cos\beta - \sin\beta.\cos\alpha}{\cos\alpha.\cos\beta} = \frac{\sin(\alpha-\beta)}{\cos\alpha.\cos\beta} \Rightarrow \frac{1}{p} = \frac{\cos\alpha.\cos\beta}{\sin(\alpha-\beta)}$

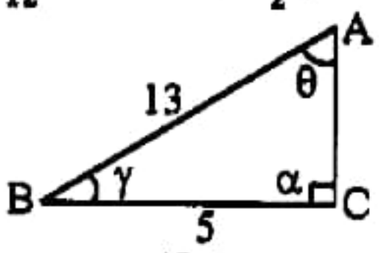
$q = \cot\beta - \cot\alpha = \frac{\cos\beta}{\sin\beta} - \frac{\cos\alpha}{\sin\alpha} = \frac{\sin\alpha.\cos\beta - \cos\alpha.\sin\beta}{\sin\alpha.\sin\beta} = \frac{\sin(\alpha-\beta)}{\sin\alpha.\sin\beta}$

$\therefore \frac{1}{p} + \frac{1}{q} = \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)} = \cot(\alpha - \beta)$ ; $\cot\theta = \frac{1}{p} + \frac{1}{q}$

16. যদি একটি ত্রিভুজের দুইটি বাহুর দৈর্ঘ্য 13 ও 5 একক হয় এবং 13 একক বাহুর পাশের একটি কোণের পরিমাণ $\text{cosec}^{-1}\frac{13}{5}$ হলে, অপর কোণ দুইটির পরিমাণ ও অপর বাহুর দৈর্ঘ্য কত হবে? [KUET'15-16]

(a) $\frac{\pi}{4}, \sec^{-1}\frac{13}{12}, 12$ (b) $\frac{\pi}{2}, \sec^{-1}\frac{13}{12}, 12$ (c) $\frac{\pi}{2}, \cos^{-1}\frac{5}{13}, 12$ (d) $\frac{\pi}{2}, \sin^{-1}\frac{5}{13}, 12$ (e) কোনটিই নয়

**সমাধান:** (c);




$AC = \sqrt{13^2 - 5^2} = 12$

এখানে, $\theta = \text{cosec}^{-1}\frac{13}{5}$ $\therefore \alpha = \frac{\pi}{2}; \gamma = \cos^{-1}\frac{5}{13}$

17. যদি $\cos(A + B)\sin(C + D) = \cos(A - B)\sin(C - D)$ হয়, তাহলে tan D এর মান কোনটি? [KUET'15-16]

(a) tan A tan B tan C (b) cot A cot B cot C (c) sin A sin B sin C (d) cos A cos B cos C (e) sec A sec B sec C

**সমাধান:** (a); $\cos(A + B)\sin(C + D) = \cos(A - B)\sin(C - D) \Rightarrow \frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C-D)}{\sin(C+D)}$

$\Rightarrow \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)} = \frac{\sin(C-D)+\sin(C+D)}{\sin(C-D)-\sin(C+D)} \Rightarrow \frac{\cos A\cos B}{-\sin A\sin B} = \frac{\sin C\cos D}{-\cos C\sin D} \Rightarrow \tan D = \tan A\tan B\tan C$

18. $A + B + C = (2n + 1)\frac{\pi}{2}$ হলে $\tan B\tan C + \tan C\tan A + \tan A\tan B$ এর মান বের কর। [CUET'15-16]

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{1}{2}$

**সমাধান:** (c); $A + B + C = (2n + 1)\frac{\pi}{2} \Rightarrow \tan(A + B + C) = \tan\left\{(2n + 1)\frac{\pi}{2}\right\}$

$\Rightarrow \frac{\tan A+\tan B+\tan C-\tan A\times\tan B\times\tan C}{1-(\tan A\tan B+\tan B\tan C+\tan C\tan A)} = \frac{1}{0} \Rightarrow 1 - (\tan A\tan B + \tan B\tan C + \tan C\tan A) = 0$

$\Rightarrow \tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

19. যদি $\sin\alpha + \sin\beta = a$ এবং $\cos\alpha + \cos\beta = b$ হয়, তাহলে $\cos(\alpha - \beta)$ এর মান কত? [CUET'14-15]

(a) $\frac{a^2+b^2+2}{2}$ (b) $\frac{a^2+b^2-2}{2}$ (c) $\frac{a^2-b^2-2}{2}$ (d) $\frac{a^2-b^2+2}{2}$

**সমাধান:** (b); $\sin\alpha + \sin\beta = a \therefore \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = a^2$ ..... (i)

$\cos\alpha + \cos\beta = b \therefore \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = b^2$ ..... (ii)

(i) + (ii) $\Rightarrow (\sin^2\alpha + \cos^2\beta) + \sin^2\beta + \cos^2\beta + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta = a^2 + b^2$

$\Rightarrow 2 + 2\cos(\alpha - \beta) = a^2 + b^2 \therefore \cos(\alpha - \beta) = \frac{a^2+b^2-2}{2}$

20. A এর কোন মানের জন্য $\cos A\sin\left(A - \frac{\pi}{6}\right)$ এর মান বৃহত্তম হবে? [CUET'14-15]

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) None of them

**সমাধান:**(b); $\sin\left(A - \frac{\pi}{6}\right)\cos A = \frac{1}{2}\left\{2\cos A\sin\left(A - \frac{\pi}{6}\right)\right\} = \frac{1}{2}\left\{\sin\left(2A - \frac{\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right\} = \frac{1}{2}\sin\left(2A - \frac{\pi}{6}\right) - \frac{1}{4}$

For maximum value, $\sin\left(2A - \frac{\pi}{6}\right) = 1 \therefore 2A - \frac{\pi}{6} = \frac{\pi}{2} \therefore 2A = \frac{2\pi}{3} \therefore A = \frac{\pi}{3}$

21. ABC ত্রিভুজের ক্ষেত্রে $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$ হলে cosC এর মান হবে- [CUET'14-15]

(a) $\frac{1}{\sqrt{2}}$ (b) $\pm\frac{1}{\sqrt{2}}$ (c) $\pm\frac{\sqrt{3}}{2}$ (d) None of them

**সমাধান:** (b); $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$

$\Rightarrow (a^2)^2 + (b^2)^2 + (-c^2)^2 + 2a^2(-c^2) + 2b^2(-c^2) + 2a^2b^2 = 2a^2b^2$

$\Rightarrow (a^2 + b^2 - c^2)^2 = 2a^2b^2 \Rightarrow a^2 + b^2 - c^2 = \pm\sqrt{2}ab \Rightarrow \frac{a^2+b^2-c^2}{2ab} = \pm\frac{1}{\sqrt{2}} \therefore \cos C = \pm\frac{1}{\sqrt{2}}$

22. $\tan 2\theta \tan\theta = 1$ সমীকরণে $\theta$ এর মান হবে- **[BUET'13-14]**

(a) $n\pi + \frac{\pi}{6}$ (b) $n\pi - \frac{\pi}{6}$ (c) $2n\pi + \frac{\pi}{6}$ (d) $2n\pi - \frac{\pi}{6}$

**সমাধান:** (a) ; $\tan\theta = y \Rightarrow \frac{2y}{1-y^2}.y = 1$ ; $2y^2 = 1 - y^2$ $\therefore 3y^2 = 1$ $y = \pm\frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \pm\frac{1}{\sqrt{3}}$ $\theta = n\pi \pm \frac{\pi}{6}$ But, first option is the first choice.

23. একটি ত্রিভুজের তিনটি শীর্ষবিন্দু A(5,12), B(−12, 5) এবং C(−7, 17) হলে, $\angle ACB$ কোণের মান হবে- **[BUET'13-14]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

**সমাধান:** (c); $AB = c = 13\sqrt{2}$; $BC = a = 13$; $AC = b = 13$ $\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ba} = 0 \therefore \angle C = \angle ACB = \frac{\pi}{2}$

24. (a+b+c) (b+c–a)=3bc হলে sin A-এর মান হবে: **[CUET'13-14]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) None of these

**সমাধান:**(c); $(b+c+a)(b+c-a) = 3bc \Rightarrow (b+c)^2 - a^2 = 3bc \Rightarrow b^2 + c^2 - a^2 = bc$

$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2} \Rightarrow \cos A = \frac{1}{2} \Rightarrow \sin A = \frac{\sqrt{3}}{2}$

25. $\cos\theta = \frac{a\cos\varphi - b}{a - b\cos\varphi}$ হলে $\frac{\tan\frac{\theta}{2}}{\tan\frac{\varphi}{2}}$ এর মান কোনটি? **[KUET'13-14]**

(a) $\frac{a + b\sin\varphi}{b - a\sin\varphi}$ (b) $\sqrt{\frac{a + b\cos\varphi}{a - b\sin\varphi}}$ (c) $\sqrt{\frac{a+b}{a-b}}$ (d) $\frac{\sqrt{a}+b}{b}$ (e) $\frac{(a+b)^2}{a-b}$

**সমাধান:** (c); $\cos\theta = \frac{a\cos\varphi - b}{a - b\cos\varphi}$

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{a\frac{1-\tan^2\frac{\varphi}{2}}{1+\tan^2\frac{\varphi}{2}} - b}{a - b\frac{1-\tan^2\frac{\varphi}{2}}{1+\tan^2\frac{\varphi}{2}}} = \frac{a - a\tan^2\frac{\varphi}{2} - b - b\tan^2\frac{\varphi}{2}}{a + a\tan^2\frac{\varphi}{2} - b + b\tan^2\frac{\varphi}{2}} = \frac{(a-b) - \tan^2\frac{\varphi}{2}(a+b)}{(a-b) + \tan^2\frac{\varphi}{2}(a+b)}$$

$$\Rightarrow \frac{1+\tan^2\frac{\theta}{2} + 1 - \tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2} - 1 + \tan^2\frac{\theta}{2}} = \frac{(a-b) + \tan^2\frac{\varphi}{2}(a+b) + (a-b) - \tan^2\frac{\varphi}{2}(a+b)}{(a-b) + \tan^2\frac{\varphi}{2}(a+b) - (a-b) + \tan^2\frac{\varphi}{2}(a+b)}$$

$$\Rightarrow \frac{2}{2\tan^2\frac{\theta}{2}} = \frac{2(a-b)}{2\tan^2\frac{\varphi}{2}(a+b)} \Rightarrow \frac{\tan^2\frac{\theta}{2}}{\tan^2\frac{\varphi}{2}} = \frac{a+b}{a-b} \quad \therefore \frac{\tan\frac{\theta}{2}}{\tan\frac{\varphi}{2}} = \sqrt{\frac{a+b}{a-b}}$$

26. যদি $A+B+C=\pi$ ও $\cos A+\cos B=\sin C$ হয় তবে B কোনের মান হলো- **[KUET'13-14]**

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/2$ (e) $2\pi/3$

**সমাধান:** (d); $A+B+C=\pi$, $\cos A+\cos B=\sin C \Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2\sin\frac{C}{2}\cos\frac{C}{2}$

$\Rightarrow 2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)=2\sin\frac{C}{2}\cos\frac{C}{2} \Rightarrow 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)=2\sin\frac{C}{2}\cos\frac{C}{2}$

$\Rightarrow \cos\left(\frac{B-A}{2}\right)=\cos\frac{C}{2} \quad \therefore B=A+C=\frac{\pi}{2}$

27. যদি $3\sec^4\theta+8=10\sec^2\theta$ হয়, তবে $\tan\theta$ এর মান হবে- **[BUET'12-13]**

(a) $\pm\frac{1}{\sqrt{3}}$ (b) $\pm1$ (c) $\pm\frac{1}{\sqrt{2}},\pm1$ (d) $\pm\frac{1}{\sqrt{3}},\pm1$

**সমাধান:** $3\sec^4\theta+8=10\sec^2\theta$

$\Rightarrow 3x^2+8-10x=0$ [$x=\sec^2\theta$ ধরে]

$\Rightarrow x=\frac{10\pm\sqrt{100-96}}{6}=\frac{10\pm2}{6}=2,\frac{4}{3}$

$\sec^2\theta=2 \Rightarrow 1+\tan^2\theta=2 \Rightarrow \tan\theta=\pm1$

$\sec^2\theta=\frac{4}{3} \Rightarrow 1+\tan^2\theta=\frac{4}{3} \Rightarrow \tan\theta=\pm\frac{1}{\sqrt{3}}$

28. $\tan x+\tan 2x+\tan 3x=\tan x\tan 2x\tan 3x$ সমীকরণে x এর মান হবে- **[BUET'12-13]**

(a) $\frac{n\pi}{12}$ (b) $\frac{n\pi}{4}$ (c) $\frac{n\pi}{3}$ (d) $\frac{n\pi}{5}$

**সমাধান:** $\tan x+\tan 2x+\tan 3x=\tan x\tan 2x\tan 3x \Rightarrow \tan x+\tan 2x=-\tan 3x(1-\tan x\tan 2x)$

$\Rightarrow \frac{\tan x+\tan 2x}{1-\tan x\tan 2x}=-\tan 3x \Rightarrow \tan 3x=-\tan 3x \Rightarrow 2\tan 3x=0 \Rightarrow \tan 3x=0 \Rightarrow 3x=n\pi \Rightarrow x=\frac{n\pi}{3}$

**Shortcut:** Use Calculator

29. যদি $\sin\theta+\operatorname{cosec}\theta=2$ হয়, তবে $\sin^n\theta+\operatorname{cosec}^n\theta$ এর মান হলো- **[Ans: c] [KUET'12-13]**

(a) 1 (b) $-1$ (c) 2 (d) $-2$ (e) 3

**সমাধান:** $\sin\theta+\operatorname{cosec}\theta=2 \Rightarrow \sin\theta+\frac{1}{\sin\theta}=2 \Rightarrow \sin^2\theta+1=2\sin\theta \Rightarrow \sin^2\theta+1-2\sin\theta=0$

$\Rightarrow (\sin\theta-1)^2=0 \Rightarrow \sin\theta-1=0 \quad \therefore \sin\theta=1$

Now, $\sin^n\theta+\operatorname{cosec}^n\theta=\sin^n\theta+\frac{1}{\sin^n\theta}=(1)^n+\frac{1}{(1)^n}=1+1=2$

30. যদি $\sin\left(\frac{\pi}{2}\cos\alpha\right)=\cos\left(\frac{\pi}{2}\sin\alpha\right)$ হয়, তবে $\alpha$ এর মান হলো- **[Ans: d] [KUET'12-13]**

(a) $0,\frac{\pi}{4}$ (b) $\frac{\pi}{4},\frac{\pi}{2}$ (c) $\frac{\pi}{2},\frac{3\pi}{4}$ (d) $0,\frac{\pi}{2}$ (e) $-\frac{\pi}{4},\frac{\pi}{4}$

**সমাধান:** $\sin\left(\frac{\pi}{2}\cos\alpha\right)=\cos\left(\frac{\pi}{2}\sin\alpha\right) \Rightarrow \sin\left(\frac{\pi}{2}\cos\alpha\right)=\sin\left(\frac{\pi}{2}\pm\frac{\pi}{2}\sin\alpha\right) \Rightarrow \frac{\pi}{2}\cos\alpha=\frac{\pi}{2}\pm\frac{\pi}{2}\sin\alpha$

$\Rightarrow \cos\alpha=1\pm\sin\alpha \Rightarrow \cos\alpha\pm\sin\alpha=1 \Rightarrow \cos^2\alpha+\sin^2\alpha\pm2\sin\alpha\cos\alpha=1 \Rightarrow 1\pm\sin 2\alpha=1$

$\Rightarrow \sin 2\alpha=0 \Rightarrow 2\alpha=n\pi \Rightarrow \alpha=n\frac{\pi}{2}$

$n=0$ হলে, $\alpha=0$; $n=1$ হলে, $\alpha=\frac{\pi}{2}$ $\therefore \alpha=0,\frac{\pi}{2}$ [Shortcut: Use calculator.]

31. যদি $\tan\alpha-\tan\beta=p, \cot\beta-\cot\alpha=q$ ও $\theta=\alpha-\beta$ হয়, তবে $\cot\theta$ এর মান হলো- **[KUET'12-13]**

(a) $\frac{1}{p}-\frac{1}{q}$ (b) $\frac{1}{q}-\frac{1}{p}$ (c) $\frac{1}{p}+\frac{1}{q}$ (d) $1-\frac{p}{q}$ (e) $1+\frac{p}{q}$

**সমাধান:** $\tan\alpha-\tan\beta=p \Rightarrow \frac{1}{\cot\alpha}-\frac{1}{\cot\beta}=p \Rightarrow \frac{\cot\beta-\cot\alpha}{\cot\alpha\cot\beta}=p \Rightarrow \frac{q}{\cot\alpha\cot\beta}=p \Rightarrow \cot\alpha\cot\beta=\frac{q}{p}$

$$\cot\theta=\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}=\frac{\frac{q}{p}+1}{q}=\frac{q+p}{pq}=\frac{1}{p}+\frac{1}{q}$$

32. যদি $A+B+C=\pi$ হয় তবে $\cot B\cot C+\cot C\cot A+\cot A\cot B$ এর মান কোনটি? **[Ans: e][RUET'12-13]**

(a) $\pi/2$ (b) $\pi/4$ (c) $1/4$ (d) $1/2$ (e) 1

**সমাধান:** $\cot C=\cot\{\pi-(A+B)\}$ or, $\cot C=-\cot(A+B)$ or, $\cot C=-\frac{\cot A\cot B-1}{\cot A+\cot B}$

or, $\cot A\cot c+\cot c\cot B=-\cot A\cot B+1$ or, $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$

33. $\sin^2 18^\circ+\sin^2 36^\circ+\sin^2 54^\circ+\sin^2 72^\circ =$ কত? **[Ans: d] [BUTex'12-13]**

(a) –2 (b) ±2 (c) 0 (d) 2

**সমাধান:** $\sin^2 18^\circ+\sin^2 36^\circ+\sin^2 54^\circ+\sin^2 72^\circ$

$=\sin^2 18^\circ+\sin^2 36^\circ+[\sin(90^\circ-36^\circ)]^2+[\sin(90^\circ-18^\circ)]^2$

$=\sin^2 18^\circ+\sin^2 36^\circ+\cos^2 36^\circ+\cos^2 18^\circ=(\sin^2 18^\circ+\cos^2 18^\circ)+(\sin^2 36^0+\cos^2 36^\circ)=1+1=2$

34. $\operatorname{cosec}\theta$ এর পূর্ণরূপ কি? **[Ans: c] [BUTex'12-13]**

(a) $\cos ec\theta$ (b) $\sec ant\theta$ (c) $cosecant\theta$ (d) $coversed\ cosec\theta$

35. $\sec\theta=\frac{13}{12}$ হলে $\cot\theta$ এর মান কত? **[Ans: b] [BUTex'12-13]**

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{13}{25}$ (d) $\frac{25}{144}$

**সমাধান:** $\sec\theta=\frac{13}{12} \Rightarrow \sec^2\theta=\frac{169}{144} \Rightarrow 1+\tan^2\theta=\frac{169}{144} \Rightarrow \tan^2\theta=\frac{25}{144} \Rightarrow \cot^2\theta=\frac{144}{25} \therefore \cot\theta=\pm\frac{12}{5}$

36. ABC ত্রিভুজে $\angle B=55^\circ, \angle C=80^\circ, a=20\sqrt{2}$ cm হলে ত্রিভুজটির পরিলিখিত বৃত্তের ব্যাসার্ধ কত cm হবে? **[SUST'08-09,12-13]**

(a) 10 (b) $10\sqrt{2}$ (c) 20 (d) $20\sqrt{2}$ (e) $15\sqrt{2}$

**সমাধান:** $\angle B=55^\circ, \angle C=80^\circ \therefore \angle A=45^\circ$ এখন, $\frac{a}{\sin A}=2R \therefore R=\frac{a}{2\sin A}=20$ cm

37. কোন ত্রিভুজের বাহুগুলি $2x+3, x^2+3x+3$ এবং $x^2+2x$ হলে, বৃহত্তম কোণটি হবে- **[BUET'11-12]**

(a) $90^\circ$ (b) $120^\circ$ (c) $60^\circ$ (d) $180^\circ$

**সমাধান:** $a=2x+3, b=x^2+3x+3; c=x^2+2x$ $\therefore$ বৃহত্তম কোণটি হলো B

$$b^2=a^2+c^2-2ac\cos B \Rightarrow \cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{(2x+3)^2+(x^2+2x)^2-(x^2+3x+3)^2}{2(2x+3)(x^2+2x)}=-\frac{1}{2}$$

[$x=1$ ধরলে $a=5, b=7, c=3$ হয় তখন $\cos B=-\frac{1}{2}$ হয় এভাবে Shortcut এ করা যায়]

$\therefore \cos B=-\frac{1}{2} \Rightarrow B=120^\circ$

38. যদি $\cot\theta = 2$ হয়, তবে $10\sin 2\theta - 6\tan 2\theta$ এর মান হবে- **[BUET'11-12]**

(a) 1 (b) 3 (c) 2 (d) 0

**সমাধান:** $\cot\theta = 2 \Rightarrow \tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\frac{1}{2} = 26.565$

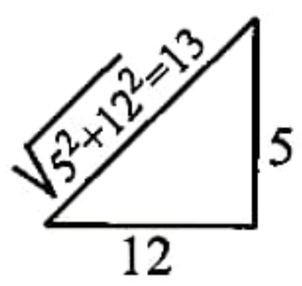
Calculator এ $\theta$ এর মান বসিয়ে পাই, $10\sin 2\theta - 6\tan 2\theta \approx 0$

39. যদি $\tan\theta = \frac{5}{12}$ এবং $\cos\theta$ ধনাত্মক হয়, তবে $\frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}$ এর মান হবে- **[BUET'11-12]**

(a) $\frac{34}{39}$ (b) $\frac{34}{40}$ (c) $\frac{30}{39}$ (d) $\frac{35}{50}$

**সমাধান:** $\tan\theta = \frac{5}{12}; \cos\theta = \frac{12}{13}; \sin\theta = \frac{5}{13}$ [$\theta$ কোনটি ১ম চতুর্ভাগে অবস্থিত]

$$\therefore \frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta} = \frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} = \frac{\frac{5}{13} + \frac{12}{13}}{\frac{13}{12} + \frac{5}{12}} = \frac{17}{13} \times \frac{12}{18} = \frac{34}{39}$$




40. যদি $A + B + C = \frac{\pi}{2}$ এবং $\sin B.\sin C = -\sin A$ হয়, তবে $\cot A + \cot B + \cot C$ এর মান কোনটি?

(a) 1 (b) $-1$ (c) 0 (d) 2 (e) $-2$

**সমাধান:** $\cot A + \cot B + \cot C = \cot A + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C} = \cot A + \frac{\sin C.\cos B + \sin B.\cos C}{\sin B.\sin C}$

$$= \cot A + \frac{\sin(B+C)}{-\sin A} = \cot A + \frac{\sin\left(\frac{\pi}{2} - A\right)}{-\sin A} = \cot A - \cot A = 0$$ **[KUET'11-12]**

41. যদি $\cos x + \cos y = a$ এবং $\sin x + \sin y = b$ হয়, তবে $\cos(x+y)$ এর মান কোনটি?

(a) $\frac{a-b}{a+b}$ (b) $\frac{a^2-b^2}{a^2+b^2}$ (c) $\frac{2a^2-b^2}{a^2+b^2}$ (d) $\frac{a^3+2b^2}{a^3-b^2}$ (e) $\frac{a^2+3b^2}{a^2-2b^2}$

**সমাধান:** $a^2 = \cos^2 x + \cos^2 y + 2\cos x.\cos y$; $b^2 = \sin^2 x + \sin^2 y + 2\sin x.\sin y$

$\therefore a^2 - b^2 = \cos 2x + \cos 2y + 2\cos(x+y)$; $a^2 + b^2 = 2 + 2\cos(x-y)$ **[KUET'11-12,10-11]**

এখন, $a^2 - b^2 = 2\cos(x+y).\cos(x-y) + 2\cos(x+y)$

$= \cos(x+y)\{2\cos(x-y)+2\} = \cos(x+y) \times (a^2+b^2)$ $\therefore \cos(x+y) = \frac{a^2-b^2}{a^2+b^2}$

42. যদি $A + B = \frac{\pi}{2}$ হয়, তবে $\cos^2 A - \cos^2 B$ এর মান কত? **[KUET'11-12]**

(a) $\sin(A-B)$ (b) $\sin(B-A)$ (c) $\cos(B-A)$ (d) $-\cos(B-A)$ (e) 1

**সমাধান:** $\cos^2 A - \cos^2 B = \sin^2 B - \sin^2 A = \sin(B+A).(\sin B - A)$

$= 1.\sin(B-A)\left[\because (A+B) = \frac{\pi}{2}\right] = \sin(B-A)$

43. $\sin^3 x + \sin^3(120^\circ + x) + \sin^3(240^\circ + x) = ?$ **[RUET'11-12]**

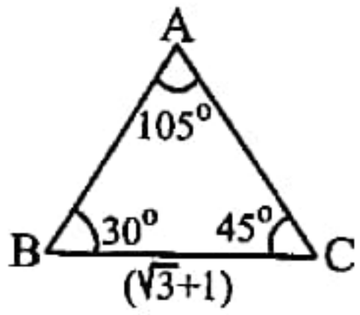
(a) $-3\sin 3x$ (b) $-\frac{1}{4}\sin 3x$ (c) $\frac{3}{4}\sin 3x$ (d) $-\frac{3}{4}\sin 3x$ (e) $-\frac{1}{3}\sin 3x$

**সমাধান:** (d); $x = 30^\circ$ বসাই, $\sin^3 30^\circ + \sin^3 150^\circ + \sin^3 270^\circ = -\frac{3}{4}\sin(3\times 30^\circ)$

44. একটি ত্রিভুজের $(\sqrt{3}+1)$cm দৈর্ঘ্য বিশিষ্ট বাহু সংলগ্ন দুটি কোন $30^0$ ও $45^0$। ত্রিভুজটির ক্ষেত্রফল কত? **[CUET'11-12]**

(a) $1/2\sqrt{2}$ (b) 2 (c) $\frac{1}{2}(\sqrt{3}+1)$ (d) None of these

**সমাধান:** (c); ত্রিভুজের sine সূত্র মতে, $\frac{AB}{\sin 45^\circ}=\frac{(\sqrt{3}+1)}{\sin 105^\circ} \Rightarrow AB=\frac{(\sqrt{3}+1)\sin 45^\circ}{\sin 105^\circ}$




বা, $AB=2$ (ক্যালকুলেটর ব্যবহার করে)

$\therefore$ ত্রিভুজ এর ক্ষেত্রফল $=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}|=\frac{1}{2}\times AB.BC\sin 30^\circ$

$=\frac{1}{2}\times 2\times(\sqrt{3}+1)\times\frac{1}{2}=\frac{1}{2}(\sqrt{3}+1)$

45. $\sin^2(3/100)+\cos^2(3/100)=?$ **[Ans: c] [SUST'11-12]**

(a) 0.06 (b) 0.018 (c) 1.00 (d) 0 (e) 0.3

46. $(a+b+c)(b+c-a)=3bc$ হলে A কোণের মান নির্ণয় কর। **[BUET'10-11]**

(a) $30^\circ$ (b) $0^\circ$ (c) $60^\circ$ (d) $45^\circ$

**সমাধান:** $(a+b+c)(b+c-a)=3bc \Rightarrow (b+c)^2-a^2=3bc \Rightarrow b^2+c^2-a^2=bc$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc}=\frac{1}{2} \quad \therefore A=60^\circ$

47. যদি $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$ হয়, তবে $A+B+C$ এর মান কত? **[Ans:b] [KUET'10-11]**

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{3\pi}{2}$ (d) $2\pi$ (e) $\frac{7\pi}{2}$

48. $\frac{\cos 27^\circ-\cos 63^\circ}{\cos 27^\circ+\cos 63^\circ}=?$ **[RUET'10-11]**

(a) $\sin 18^\circ$ (b) $\tan 18^\circ$ (c) $\cos 18^\circ$ (d) $\tan 15^\circ$ (e) $\cot 15^\circ$

**সমাধান:** (b); Use Calculator.

49. $A-B=\pi/4$ হলে, $(1+\tan A)(1-\tan B)=?$ **[Ans: a] [SUST'10-11]**

(a) 2 (b) –2 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

**সমাধান:** (a); $A$ ও $B$ এর দুইটি মান ধরে ক্যালকুলেটরে বসিয়ে পরীক্ষা কর।

50. $\sin\theta+\frac{1}{2}\sin 2\theta=m\cos\theta$ এবং $\sin\theta-\frac{1}{2}\sin 2\theta=n\cos\theta$ হলে $m^2-n^2=?$ **[SUST'10-11]**

(a) 4mn (b) $4\sqrt{(mn)}$ (c) $1/mn$ (d) $1/\sqrt{mn}$

**সমাধান:** (b); $\sin\theta+\frac{1}{2}\sin 2\theta=m\cos\theta$

$\Rightarrow \sin\theta+\sin\theta\cos\theta=m\cos\theta \quad \therefore \tan\theta+\sin\theta=m \ldots\ldots\ldots (i)$

$\sin\theta-\frac{1}{2}\sin 2\theta=n\cos\theta \quad \therefore \tan\theta-\sin\theta=n \ldots\ldots\ldots (ii)$

$m^2-n^2=4\tan\theta\sin\theta=4\sqrt{\tan^2\theta\sin^2\theta}=4\sqrt{\sin^2\theta(\sec^2\theta-1)}$

$=4\sqrt{\tan^2\theta-\sin^2\theta}=4\sqrt{mn}$