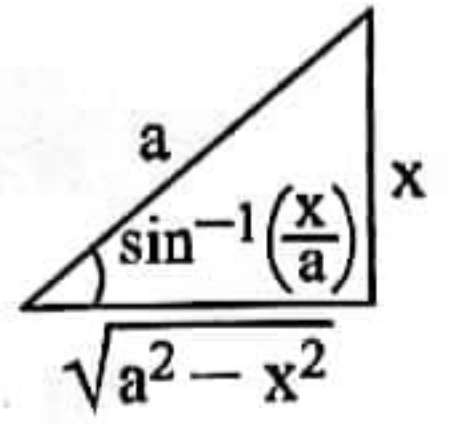




**Question Type-01: মান সংক্রান্ত**

**Formula & Concept:**

এক্ষেত্রে সঠিকভাবে চিত্র ঐকে একটি অনুপাত থেকে অপর অনুপাতে রূপান্তর করে মান বের করতে হবে।  
এছাড়াও সকল বিপরীত ত্রিকোণমিতিক সূত্রাবলি মনে রাখতে হবে এবং যথাযথভাবে প্রয়োগ করতে হবে।  
সবশেষে কোণের মান বের করার ক্ষেত্রে অবশ্যই মানের সীমা ও শর্তাবলি খেয়াল রাখতে হবে।



যেমন:  $\sin^{-1} \frac{x}{a} = \cos^{-1} \frac{\sqrt{a^2 - x^2}}{a} = \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$

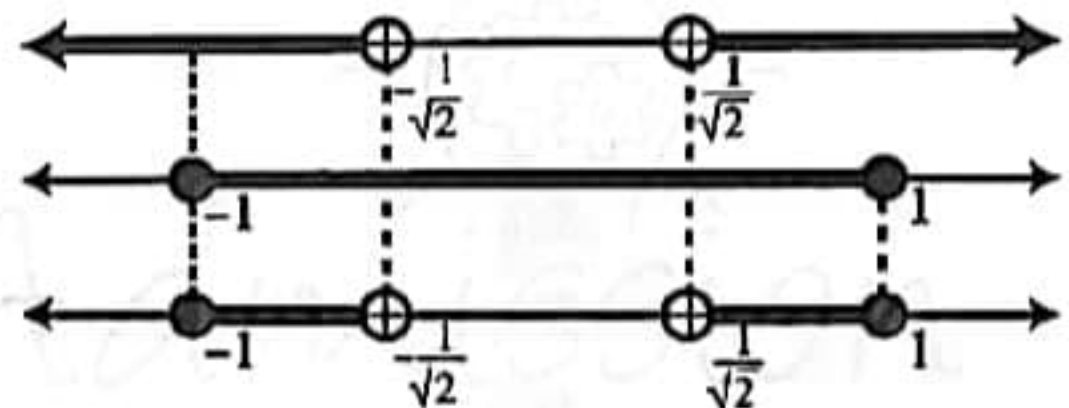
**MCQ**

01.  $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$  হলে,  $x^2 + y^2$  এর মান বের কর। [CKRUET'21-22]  
 (a) 1 (b) 0 (c) tan 1 (d) cos 1 (e) sin 1

সমাধান: (a);  $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} \Rightarrow \cos^{-1} \{xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2}\} = \frac{\pi}{2}$   
 $\Rightarrow xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2} = \cos \frac{\pi}{2} = 0 \Rightarrow \sqrt{1-x^2} \cdot \sqrt{1-y^2} = xy \Rightarrow (1-x^2) \cdot (1-y^2) = x^2 y^2$   
 $\Rightarrow 1 - x^2 - y^2 + x^2 y^2 = x^2 y^2 \Rightarrow x^2 + y^2 = 1$

02. Given,  $\frac{\cos^{-1} x}{\sin^{-1} x} < 1$ , find the most appropriate value of x. [IUT'21-22]  
 (a)  $1 \geq x > \frac{1}{\sqrt{2}}$  (b)  $x > \frac{1}{\sqrt{2}}$  (c)  $x < \frac{1}{\sqrt{2}}$  (d)  $-1 \leq x < \frac{1}{\sqrt{2}}$

Solution: (a);  $\cos^{-1} x < \sin^{-1} x \Rightarrow \sin^{-1} \sqrt{1-x^2} < \sin^{-1} x \Rightarrow \sqrt{1-x^2} < x$   
 $\Rightarrow 1 - x^2 < x^2 \Rightarrow 1 - 2x^2 < 0 \Rightarrow 1 < 2x^2 \Rightarrow x^2 > \frac{1}{2}$



$\therefore x > \frac{1}{\sqrt{2}}$  or,  $x < -\frac{1}{\sqrt{2}}$   
 $-1 \leq x \leq 1$  Intersection  
 $-1 \leq x < -\frac{1}{\sqrt{2}}$  or,  $\frac{1}{\sqrt{2}} < x \leq 1$

But for  $\sin^{-1} x$  and  $\cos^{-1} x - 1 \leq x \leq 1 \therefore -1 \leq x < -\frac{1}{\sqrt{2}}$  or,  $\frac{1}{\sqrt{2}} < x \leq 1$  (Ans.)

03.  $\cos^{-1} \cos \frac{4\pi}{3}$  is equal to- [IUT'21-22]  
 (a)  $\frac{4\pi}{3}$  (b)  $\pi$  (c)  $\frac{2\pi}{3}$  (d)  $\frac{\pi}{3}$

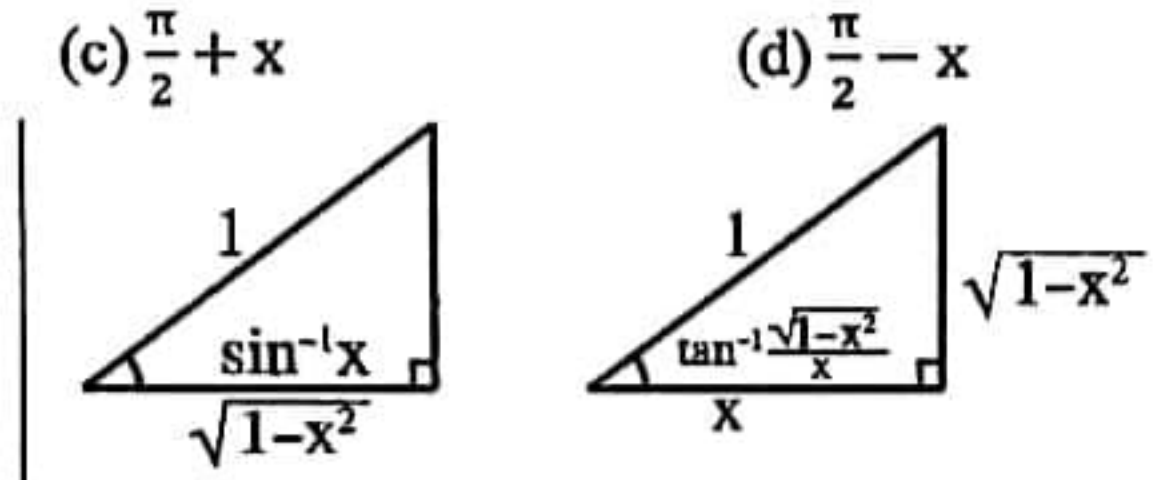
Solution: (c);  $\cos^{-1} \left( \cos \frac{4\pi}{3} \right) = \cos^{-1} \left( -\frac{1}{2} \right) = \frac{2\pi}{3}$

04. যদি  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$  হয়, তবে  $x\sqrt{1-y^2} + y\sqrt{1-x^2}$  এর মান নির্ণয় কর। [CKRUET'20-21]  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d) 1 (e) None of them

সমাধান: (d);  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2} \Rightarrow \sin^{-1} (x\sqrt{1-y^2} + y\sqrt{1-x^2}) = \frac{\pi}{2}$   
 $\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin \frac{\pi}{2} = 1$

05. What is the value of  $\cos \tan^{-1} \cot \sin^{-1} x$ ? [IUT'20-21]  
 (a) x (b) -x (c)  $\frac{\pi}{2} + x$  (d)  $\frac{\pi}{2} - x$

Solution: (a);  $\cos \tan^{-1} \cot \cot^{-1} \frac{\sqrt{1-x^2}}{x}$   
 $= \cos \tan^{-1} \frac{\sqrt{1-x^2}}{x} = \cos \cos^{-1} \frac{x}{1} = x$





06. যদি  $\tan^{-1} a + \frac{1}{2} \sec^{-1} \frac{1+b^2}{1-b^2} + \frac{1}{2} \operatorname{cosec}^{-1} \frac{1+c^2}{2c} = \pi$  হয়, তাহলে  $a + b + c$  এর মান কত? [KUET'17-18]  
 (a)  $5abc$  (b)  $7abc$  (c)  $11abc$  (d)  $2abc$  (e)  $abc$

সমাধান: (e);  $\tan^{-1} a + \frac{1}{2} \sec^{-1} \frac{1+b^2}{1-b^2} + \frac{1}{2} \operatorname{cosec}^{-1} \frac{1+c^2}{2c} = \pi$   
 $\Rightarrow \tan^{-1} a + \frac{1}{2} \cos^{-1} \frac{1-b^2}{1+b^2} + \frac{1}{2} \sin^{-1} \frac{2c}{1+c^2} = \pi \Rightarrow \tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$   
 $\Rightarrow \tan(\tan^{-1} b + \tan^{-1} c) = \tan(\pi - \tan^{-1} a) \Rightarrow \frac{b+c}{1-bc} = -a \Rightarrow b + c = abc - a \therefore abc = a + b + c$

07.  $\sin(\pi \cos \theta) = \cos(\pi \sin \theta)$  হলে  $\theta$  এর মান কোনটি? [KUET'16-17]  
 (a)  $\pm \frac{\pi}{4} + \cos^{-1} \left( \frac{1}{2\sqrt{2}} \right)$  (b)  $\pm \frac{\pi}{2} + \cos^{-1} \left( \frac{1}{2\sqrt{2}} \right)$  (c)  $\pm \frac{\pi}{4} + \sin^{-1} \left( \frac{1}{2\sqrt{2}} \right)$   
 (d)  $\pm \frac{\pi}{2} + \sin^{-1} \left( \frac{1}{2\sqrt{2}} \right)$  (e)  $\pm \frac{\pi}{4} + \cos^{-1} \left( \frac{1}{1\sqrt{2}} \right)$

সমাধান: (a);  $\sin(\pi \cos \theta) = \cos(\pi \sin \theta) \Rightarrow \sin(\pi \cos \theta) = \sin \left( \frac{\pi}{2} \pm \pi \sin \theta \right) \Rightarrow \pi \cos \theta = \frac{\pi}{2} \pm \pi \sin \theta$   
 $\Rightarrow \cos \theta \pm \sin \theta = \frac{1}{2} \Rightarrow \frac{1}{\sqrt{2}} \cos \theta \pm \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2\sqrt{2}} \Rightarrow \cos \left( \theta \pm \frac{\pi}{4} \right) = \frac{1}{2\sqrt{2}} \Rightarrow \theta = \pm \frac{\pi}{4} + \cos^{-1} \left( \frac{1}{2\sqrt{2}} \right)$

08.  $\tan \frac{1}{2} \left( \tan^{-1} x + \tan^{-1} \frac{1}{x} \right) = ?$  [IUT'16-17]  
 (a)  $\frac{x-1}{2x}$  (b)  $\tan \frac{x-1}{2x}$  (c) 1 (d)  $\sqrt{2}$

Solution: (c);  $\tan \left( \frac{1}{2} \tan^{-1} \frac{x+\frac{1}{x}}{1-\frac{x}{x}} \right) = \tan \left( \frac{1}{2} \cdot \tan \infty \right) = \tan \left( \frac{1}{2} \times \frac{\pi}{2} \right) = \tan \frac{\pi}{4} = 1$

09.  $\cot^{-1}(\tan 2x) + \cot^{-1}(-\tan 3x) = ?$  [IUT'16-17]  
 (a)  $\frac{2}{3}x$  (b)  $x$  (c)  $\frac{3}{2}x$  (d)  $2x$

Solution: (b);  $\tan^{-1} \frac{1}{\tan 2x} - \tan^{-1} \frac{1}{\tan 3x} \Rightarrow \tan^{-1} \frac{\tan 3x - \tan 2x}{\tan 2x \tan 3x + 1} = x$

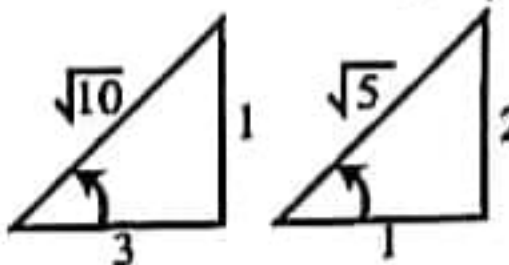
10.  $\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$  এর মান কোনটি? [KUET'15-16]  
 (a)  $\frac{\pi}{2}$  (b)  $\frac{-\pi}{2}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{4}$  (e)  $\frac{\pi}{6}$

সমাধান: (a);  $\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} = \frac{\pi}{2}$  [ক্যালকুলেটর ব্যবহার করে]

11.  $\sin \cot^{-1} \tan \cos^{-1} x = ?$  [BUTEX'15-16]  
 (a)  $x$  (b)  $\pi$  (c)  $\frac{1}{x}$  (d)  $\sqrt{1-x}$

সমাধান: (a);  $\sin \cot^{-1} \tan \cos^{-1} x = \sin \cot^{-1} \tan \tan^{-1} \frac{\sqrt{1-x^2}}{x} = \sin \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \sin \sin^{-1} \frac{x}{1} = x$

12.  $\sec^2(\cot^{-1} 3) + \operatorname{cosec}^2(\tan^{-1} 2)$  এর মান কত? [KUET'14-15]  
 (a)  $2\frac{13}{36}$  (b)  $3\frac{11}{13}$  (c)  $5\frac{7}{9}$  (d)  $4\frac{3}{11}$  (e)  $5\frac{12}{13}$

সমাধান: (a);   $\sec^2(\cot^{-1} 3) + \operatorname{cosec}^2(\tan^{-1} 2) = \frac{10}{9} + \frac{5}{4} = 2\frac{13}{36}$

13.  $\cot \cos^{-1} \sin \tan^{-1} \frac{3}{4} = ?$  [BUTEX'14-15]  
 (a)  $\frac{3}{2}$  (b) 1 (c)  $\frac{3}{4}$  (d)  $\frac{1}{4}$

সমাধান: (c);  $\cot \cos^{-1} \sin \tan^{-1} x = x$

14.  $\tan^{-1} x + \tan^{-1} \frac{1-x}{1+x} = ?$  [IUT'14-15]  
 (a)  $\frac{\pi}{2}$  (b)  $\frac{\pi}{3}$  (c)  $\frac{\pi}{4}$  (d) Both a and c

Solution: (c);  $\tan^{-1} x + \tan^{-1} \frac{1-x}{1+x} = \tan^{-1} x + \tan^{-1} 1 - \tan^{-1} x = \frac{\pi}{4}$



15.  $\sec^2(\tan^{-1} 2) + \sin \cot^{-1} \tan \cos^{-1} x$  এর মান হবে- [BUET'13-14]  
 (a)  $x$  (b)  $x+5$  (c)  $x^2$  (d)  $x^2 + 5$

সমাধান: (b);  $y = 5 + x$  [Using calculator for first part].

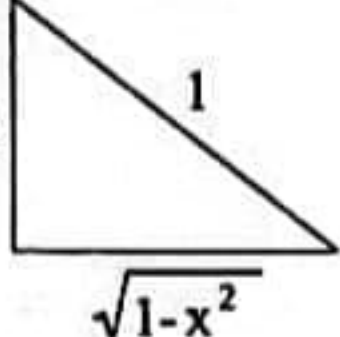
16.  $\cos \tan^{-1} \cot \sin^{-1} x$  এর মান কত? [KUET'13-14]  
 (a) 1 (b)  $x$  (c)  $\frac{1}{x}$  (d)  $\frac{\pi}{4}$  (e)  $\frac{1}{4}$

সমাধান: (b);  $\cos \tan^{-1} \cot \sin^{-1} x = \cos \tan^{-1} \cot \left[ \cot^{-1} \left( \frac{\sqrt{1-x^2}}{x} \right) \right]$

$$= \cos \tan^{-1} \left( \frac{\sqrt{1-x^2}}{x} \right) = \cos \cos^{-1}(x) = x$$

বিকল্প:  $\cos \tan^{-1} \cot \sin^{-1} x = \cos \tan^{-1} \tan \left( \frac{\pi}{2} - \sin^{-1} x \right) = \cos \left( \frac{\pi}{2} - \sin^{-1} x \right) = \sin(\sin^{-1} x) = x$

17.  $\sin^{-1} x$  এর মান হবে- [BUTEX'13-14]  
 (a)  $\cot^{-1} \frac{\sqrt{1-x^2}}{x}$  (b)  $\cot^{-1} \frac{x}{\sqrt{1-x^2}}$  (c)  $\cot^{-1} \sqrt{1-x^2}$  (d)  $\cot^{-1} \frac{1}{\sqrt{1-x^2}}$

সমাধান: (a);  $\sin^{-1} x = \cot^{-1} \frac{\sqrt{1-x^2}}{x}$  

18. নিচের কোন সম্পর্কটি সত্য নয়? [Ans: d] [BUTEX'13-14]  
 (a)  $\sin x = \sin(2n\pi + x)$  (b)  $\sin(\sin^{-1} x) = x$   
 (c)  $\sin^2 x = (-\sin x)^2$  (d)  $\sin^{-1} x = (\sin x)^{-1}$

19.  $\sin \left[ \cos^{-1} \left( -\frac{1}{2} \right) + \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) \right]$  এর মান হবে- [BUET'12-13]  
 (a) 1 (b)  $\frac{1}{2}$  (c)  $\frac{1}{\sqrt{2}}$  (d)  $\frac{\sqrt{3}}{2}$

সমাধান: (b);  $\sin \left[ \cos^{-1} \left( -\frac{1}{2} \right) + \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) \right] = \sin[120^\circ + 30^\circ] = \sin 150^\circ = \frac{1}{2}$

20.  $\sin^{-1}(1)$  এর মান ( $n = 1, 2, 3, \dots$ ) [RUET'12-13]  
 (a)  $\frac{n\pi}{2}$  (b)  $\frac{(n+1)\pi}{2}$  (c)  $\frac{(2n+1)\pi}{2}$  (d)  $2n\pi + \frac{\pi}{2}$  (e) None

সমাধান: (d);  $\sin \theta = 1; \theta = 2n\pi + \frac{\pi}{2} [n \in \mathbb{Z}]$

21. যদি  $x = \sin \cos^{-1} y$  হয়, তবে  $x^2 + y^2$  এর মান হবে- [BUET'11-12]  
 (a) 2 (b) 1 (c) -1 (d) 0

সমাধান: (b);  $x = \sin \cos^{-1} y \Rightarrow \cos^{-1} y = \sin^{-1} x \Rightarrow \sin^{-1} x = \sin^{-1} \sqrt{1-y^2}$

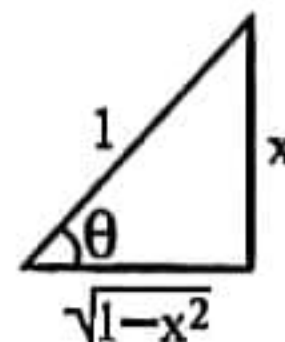
$$\Rightarrow x = \sqrt{1-y^2} \Rightarrow x^2 = 1-y^2 \Rightarrow x^2 + y^2 = 1$$

22.  $\tan^{-1} 2 + \cot^{-1} \frac{1}{3}$  এর মান কোনটি? [KUET'11-12]  
 (a)  $\frac{\pi}{4}$  (b)  $\frac{5\pi}{4}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{2\pi}{4}$  (e)  $\frac{5\pi}{6}$

সমাধান: (c) ক্যালকুলেটর ব্যবহার করে।

23.  $\cot(\sin^{-1} x)$  এর মান কত? [BUTEX'11-12]  
 (a)  $\sqrt{1-x^2}$  (b)  $\frac{1}{x}$  (c)  $\frac{x}{\sqrt{1-x^2}}$  (d) কোনটিই নয়

সমাধান: (d);  $\cot(\sin^{-1} x) = \cot \left( \cot^{-1} \frac{\sqrt{1-x^2}}{x} \right) = \frac{\sqrt{1-x^2}}{x}$





24. যদি  $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$  হয়, তাহলে  $(x^2 + y^2)$  এর মান হলো-

[BUET'10-11]

- (a) 4 (b) 3 (c) 2 (d) 1

সমাধান: (d);  $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2} \Rightarrow x = \sin\left(\frac{\pi}{2} - \sin^{-1}y\right) \Rightarrow x = \cos(\sin^{-1}y)$   
 $\Rightarrow x = \cos(\cos^{-1}\sqrt{1-y^2}) \Rightarrow x^2 + y^2 = 1$

25.  $\sin \cos^{-1} \tan \sec^{-1} \frac{x}{y}$  এর সরলীকৃত মান কত?

[KUET'10-11]

- (a)  $\frac{\sqrt{x^2-2y^2}}{x}$  (b)  $\frac{2y^2-x^2}{x}$  (c)  $\frac{\sqrt{y^2-2x^2}}{y}$  (d)  $\frac{\sqrt{y^2-2x^2}}{x}$  (e)  $\frac{\sqrt{2y^2-x^2}}{y}$

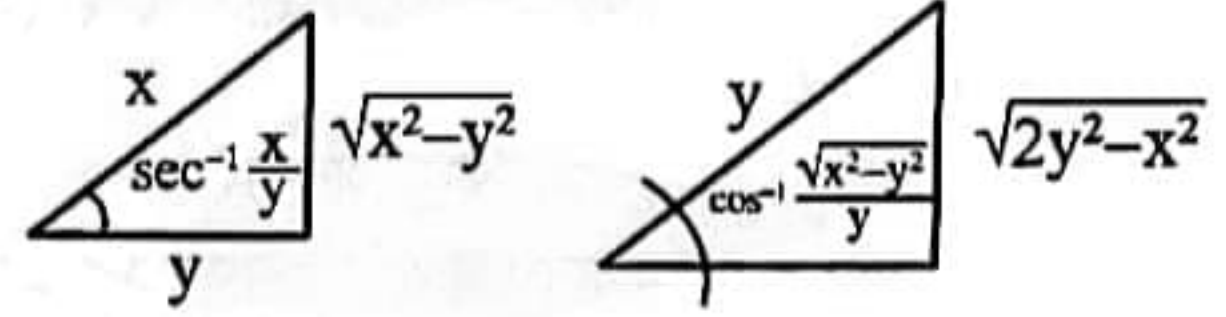
সমাধান: (e);  $\sec^{-1} \frac{x}{y} = \tan^{-1} \frac{\sqrt{x^2-y^2}}{y}$ ;  $\sin \cos^{-1} \frac{\sqrt{x^2-y^2}}{y} = \sin \sin^{-1} \frac{\sqrt{2y^2-x^2}}{y} = \frac{\sqrt{2y^2-x^2}}{y}$

বিকল্প:  $\sin \cos^{-1} \tan^{-1} \sec^{-1} \frac{x}{y}$

$$= \sin \cos^{-1} \tan \tan^{-1} \frac{\sqrt{x^2-y^2}}{y}$$

$$= \sin \cos^{-1} \frac{\sqrt{x^2-y^2}}{y} = \sin \sin^{-1} \frac{\sqrt{2y^2-x^2}}{y}$$

$$= \frac{\sqrt{2y^2-x^2}}{y}$$



26.  $\sec^2(\tan^{-1} 4) + \tan^2(\sec^{-1} 3)$  এর মান কত?

[CUET'10-11]

- (a) 5 (b) 25 (c) 7 (d) None of these

সমাধান: (b);  $\sec^2(\tan^{-1} 4) + \tan^2(\sec^{-1} 3) = 1 + \tan^2(\tan^{-1} 4) + \sec^2(\sec^{-1} 3) - 1 = 4^2 + 3^2 = 16 + 9 = 25$

27. Evaluate:  $\tan^{-1} \frac{m}{n} - \tan^{-1} \frac{m-n}{m+n}$ .

[IUT'10-11]

- (a)  $\frac{\pi}{3}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{9}$  (d)  $\frac{\pi}{4}$

Solution: (d);  $\tan^{-1} \frac{m}{n} - \tan^{-1} \frac{m-n}{m+n} = \tan^{-1} \left[ \frac{\frac{m}{n} - \frac{m-n}{m+n}}{1 + \frac{m}{n} \cdot \frac{m-n}{m+n}} \right]$

$$= \tan^{-1} \left[ \frac{m^2 + mn - mn + n^2}{n(m+n) + m(m-n)} \right] = \tan^{-1} \left( \frac{m^2 + n^2}{m^2 + n^2} \right) = \tan^{-1} 1 = \frac{\pi}{4}$$

### Written

28. মান নির্ণয় কর:  $\sin^2(\cos^{-1} \frac{1}{3}) - \cos^2(\sin^{-1} \frac{1}{\sqrt{3}})$

[BUTEX'09-10]

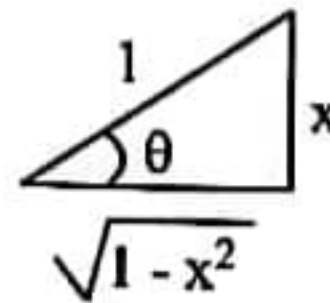
সমাধান:  $\sin^2(\cos^{-1} \frac{1}{3}) - \cos^2(\sin^{-1} \frac{1}{\sqrt{3}}) = 1 - \cos^2(\cos^{-1} \frac{1}{3}) - 1 + \sin^2(\sin^{-1} \frac{1}{\sqrt{3}})$

$$= 1 - \left(\frac{1}{3}\right)^2 - 1 + \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} - \frac{1}{9} = \frac{3-1}{9} = \frac{2}{9} \text{ (Ans.)}$$

29.  $\cot(\sin^{-1}x)$  এর মান কত?

[BUTEX'09-10]

সমাধান:  $\cot(\sin^{-1}x) = \cot \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \frac{\sqrt{1-x^2}}{x}$



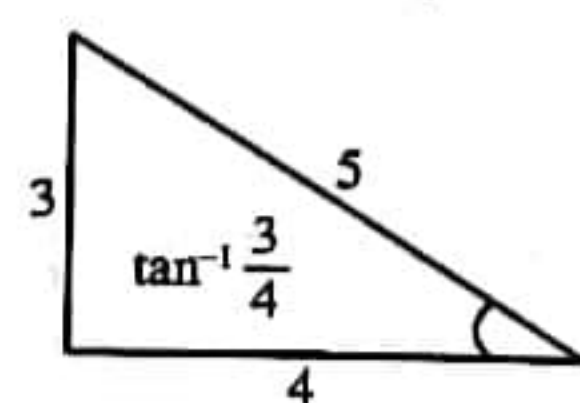
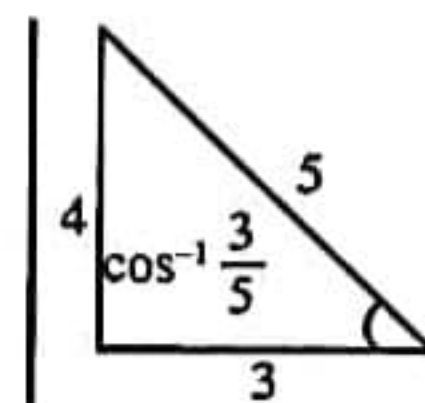
30.  $\cot \cos^{-1} \sin \tan^{-1} \frac{3}{4}$  এর মান কত?

[KUET'04-05]

সমাধান:  $\cot \cos^{-1} \sin \tan^{-1} \frac{3}{4}$

$$= \cot \cos^{-1} \sin \sin^{-1} \frac{3}{5}$$

$$\Rightarrow \cot \cos^{-1} \frac{3}{5} = \cot \cot^{-1} \frac{3}{4} = \frac{3}{4} \text{ (Ans.)}$$





31. প্রমাণ কর:  $\cos^{-1} x = 2 \sin^{-1} \sqrt{\frac{1-x}{2}} = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$

[KUET'03-04]

সমাধান: ধরি,  $\cos^{-1} x = \theta$ 

$$\therefore \cos \theta = x$$

$$\text{এখন, } \sin \frac{\theta}{2} = \sqrt{\frac{1-\cos \theta}{2}} = \sqrt{\frac{1-x}{2}}$$

$$\therefore \frac{\theta}{2} = \sin^{-1} \sqrt{\frac{1-x}{2}} \therefore \theta = 2 \sin^{-1} \sqrt{\frac{1-x}{2}}$$

$$\therefore \cos^{-1} x = 2 \sin^{-1} \sqrt{\frac{1-x}{2}}$$

$$\text{আবার, } \cos \frac{\theta}{2} = \sqrt{\frac{1+\cos \theta}{2}} = \sqrt{\frac{1+x}{2}}$$

$$\Rightarrow \frac{\theta}{2} = \cos^{-1} \sqrt{\frac{1+x}{2}} \Rightarrow \theta = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$$

$$\therefore \cos^{-1} x = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$$

$$\therefore \cos^{-1} x = 2 \sin^{-1} \sqrt{\frac{1-x}{2}} = 2 \cos^{-1} \sqrt{\frac{1+x}{2}} \text{ (Proved)}$$

### Question Type-02: বিপরীত ত্রিকোণমিতিক সমীকরণ সংক্রান্ত

#### Formula & Concept:

সমীকরণ সমাধানের ক্ষেত্রে সীমা অবশ্যই খেয়াল রাখতে হবে। সীমা উল্লেখ না থাকলে সাধারণ আকারে সমাধান বের করবে। সমাধানের পর এক্ষেত্রে অবশ্যই শুদ্ধি পরীক্ষা করতে হবে।

#### MCQ

01.  $\tan^{-1} \frac{2x}{1-x^2} = \sin^{-1} \frac{2a}{1+a^2} + \cos^{-1} \frac{1-b^2}{1+b^2}$  সমীকরণে  $x$  এর মান হল-

[CUET'11-12, IUT'16-17]

(a)  $\frac{a-b}{1+ab}$

(b)  $\frac{a+b}{1-ab}$

(c)  $\frac{2ab}{a^2+b^2}$

(d) None of these

$$\text{সমাধান: (b); } \tan^{-1} \frac{2x}{1-x^2} = \sin^{-1} \frac{2a}{1+a^2} + \cos^{-1} \frac{1-b^2}{1+b^2}$$

$$\Rightarrow 2 \tan^{-1} x = 2 \tan^{-1} a + 2 \tan^{-1} b = 2(\tan^{-1} a + \tan^{-1} b) \Rightarrow \tan^{-1} x = \tan^{-1} \frac{a+b}{1-ab} \therefore x = \frac{a+b}{1-ab}$$

02.  $\sin^{-1}(1)$  এর মান ( $n = 1, 2, 3, \dots$ ) -

[RUET'12-13]

(a)  $\frac{n\pi}{2}$

(b)  $\frac{(n+1)\pi}{2}$

(c)  $\frac{(2x+1)\pi}{2}$

(d)  $2n\pi + \frac{\pi}{2}$

(e) None

$$\text{সমাধান: (d); } \sin \theta = 1; \theta = 2n\pi + \frac{\pi}{2} [n \in \mathbb{Z}]$$

03. সমাধান কর:  $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

[BUET'10-11]

(a)  $n\pi \pm (-1)^n \frac{\pi}{4}$

(b)  $2n\pi \pm \frac{\pi}{3}$

(c)  $n\pi \pm \frac{\pi}{3}$

(d)  $n\pi + \frac{\pi}{4}$

$$\text{সমাধান: (d); } 2 \tan^{-1}(\cos x) = \tan^{-1} \frac{2 \cos x}{1-\cos^2 x} \therefore \frac{2 \cos x}{\sin^2 x} = \frac{2}{\sin x} \Rightarrow \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}$$

#### Written

04.  $\frac{1}{2} \tan^{-1} x = \tan^{-1} \left( \frac{1-x}{1+x} \right)$  হলে, প্রমাণ কর  $x = \pm \frac{1}{\sqrt{3}}$

[BUTEX'21-22]

$$\text{সমাধান: } \frac{1}{2} \tan^{-1} x = \tan^{-1} \left( \frac{1-x}{1+x} \right) \Rightarrow \frac{1}{2} \tan^{-1} x = \tan^{-1} \left( \frac{1-x}{1+x} \right) \Rightarrow \frac{1}{2} \tan^{-1} x = \tan^{-1}(1) - \tan^{-1}(x)$$

$$\Rightarrow \frac{3}{2} \tan^{-1} x = \tan^{-1}(1) \Rightarrow \tan^{-1} x = \frac{2}{3} \left( \frac{\pi}{4} \right) \Rightarrow \tan^{-1} x = \frac{\pi}{6} \therefore x = \frac{1}{\sqrt{3}}$$

$$\text{বিকল্প: } \tan^{-1} x = 2 \tan^{-1} \left( \frac{1-x}{1+x} \right) \Rightarrow \tan^{-1} x = \tan^{-1} \left\{ \frac{2 \left( \frac{1-x}{1+x} \right)}{1 - \left( \frac{1-x}{1+x} \right)^2} \right\}$$

$$\Rightarrow \tan^{-1} x = \tan^{-1} \left\{ \frac{2(1-x)(1+x)}{(1+x)^2 - (1-x)^2} \right\}$$

$$\Rightarrow \tan^{-1} x = \tan^{-1} \frac{2(1-x^2)}{4x} \Rightarrow x = \frac{1-x^2}{2x} \Rightarrow 3x^2 = 1 \therefore x = \pm \frac{1}{\sqrt{3}} \text{ [শুদ্ধি পরীক্ষা করলে পাওয়া যায় } x = \frac{1}{\sqrt{3}} \text{ হলো একমাত্র}$$

সঠিক সমাধান। যদিও প্রশ্নে  $x = \pm \frac{1}{\sqrt{3}}$  প্রমাণ করতে বলা হয়েছে তবে তা  $x = \frac{1}{\sqrt{3}}$  হবে]



05. যদি  $\tan^{-1}(x + \alpha) - \tan^{-1}x = \frac{\pi}{4}$  হয়, তবে  $x$  ও  $\alpha$ -এর মধ্যে সম্পর্ক নির্ণয় কর এবং এদের সম্ভাব্য মান বের কর।  
যেখানে,  $(x > 0, \alpha > 0)$  [RUET'19-20]

সমাধান:  $\tan^{-1}(x + \alpha) - \tan^{-1}x = \frac{\pi}{4} \Rightarrow \tan^{-1} \frac{x + \alpha - x}{1 + x(x + \alpha)} = \tan^{-1}(1)$

$\Rightarrow \frac{\alpha}{1 + x^2 + x\alpha} = 1 \Rightarrow \alpha = x^2 + x\alpha + 1 \Rightarrow x^2 + x\alpha + 1 - \alpha = 0 \therefore x = \frac{-\alpha \pm \sqrt{\alpha^2 - 4 \cdot 1 \cdot (1 - \alpha)}}{2 \cdot 1}$

$x$  ও  $\alpha$  এর সম্পর্ক,  $x = \frac{-\alpha \pm \sqrt{\alpha^2 + 4\alpha - 4}}{2}$ ;  $x > 0, \alpha > 0$  হলে  $x$  এর সম্ভাব্য মান =  $\frac{-\alpha + \sqrt{\alpha^2 + 4\alpha - 4}}{2}$

06. সমাধান কর:  $\sin^{-1} 2x + \sin^{-1} x = \frac{\pi}{3}$  [BUET'18-19]

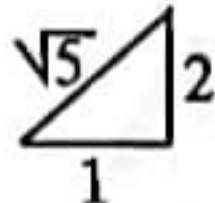
সমাধান:  $\sin^{-1} 2x = \frac{\pi}{3} - \sin^{-1} x \Rightarrow 2x = \sin\left(\frac{\pi}{3} - \sin^{-1} x\right) = \frac{\sqrt{3}}{2} \cos(\sin^{-1} x) - \frac{1}{2} \sin(\sin^{-1} x)$

$= \frac{\sqrt{3}}{2} \sqrt{1 - x^2} - \frac{x}{2} \Rightarrow \frac{5}{2}x = \frac{\sqrt{3}}{2} \sqrt{1 - x^2} \Rightarrow \frac{25}{3}x^2 = 1 - x^2 \Rightarrow x^2 = \frac{3}{28}$

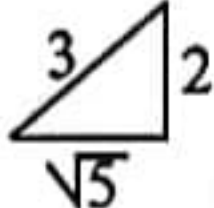
$\therefore x = \pm \sqrt{\frac{3}{28}}$  শুদ্ধি পরীক্ষা করে পাই, সঠিক উত্তর  $x = \sqrt{\frac{3}{28}} = \frac{\sqrt{21}}{14}$  (Ans.)

07.  $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2)$  সমীকরণটির সমাধান বের কর। [BUET'12-13, BUTEX'18-19]

সমাধান:  $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2)$

$\tan(\cos^{-1} x) = \sin\left(\sin^{-1} \frac{2}{\sqrt{5}}\right)$  

$\Rightarrow \tan(\cos^{-1} x) = \frac{2}{\sqrt{5}} \Rightarrow \cos^{-1} x = \tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\Rightarrow \cos^{-1} x = \cos^{-1} \frac{\sqrt{5}}{3}$  

$\Rightarrow x = \frac{\sqrt{5}}{3}$  [Ans.]

08. সমাধান কর:  $\cos^{-1} x - \sin^{-1} x = \sin^{-1}(1 - x)$  [BUET'17-18]

সমাধান: দেওয়া আছে,  $\cos^{-1}(x) - \sin^{-1} x = \sin^{-1}(1 - x)$

$\Rightarrow \sin^{-1}(\sqrt{1 - x^2}) - \sin^{-1}(x) = \sin^{-1}(1 - x)$

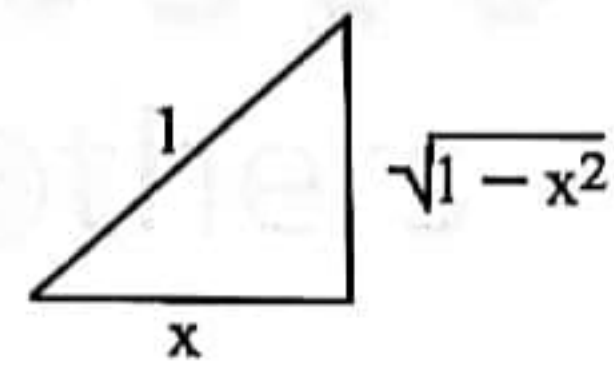
$\Rightarrow \sin^{-1}\left\{\sqrt{1 - x^2} \cdot \sqrt{1 - x^2} - x \cdot \sqrt{1 - (\sqrt{1 - x^2})^2}\right\} = \sin^{-1}(1 - x)$

$\Rightarrow \sin^{-1}\{1 - x^2 - x \cdot \sqrt{1 - 1 + x^2}\} = \sin^{-1}(1 - x)$

$\Rightarrow \sin^{-1}\{1 - x^2 - x \cdot x\} = \sin^{-1}(1 - x)$

$\Rightarrow 1 - x^2 - x^2 = 1 - x \Rightarrow 1 - 2x^2 = 1 - x \Rightarrow 2x^2 = x$

$\Rightarrow 2x^2 - x = 0 \Rightarrow x(2x - 1) = 0 \Rightarrow x = 0$  অথবা  $x = \frac{1}{2}$ ; সুতরাং নির্ণেয় সমাধান  $x = 0, \frac{1}{2}$  (Ans.)



09. সমাধান কর:  $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$  [BUET'10-11]

সমাধান:  $\tan^{-1} x + \cot^{-1} x + \cot^{-1} x = \frac{2}{3}\pi \Rightarrow \frac{\pi}{2} + \cot^{-1} x = \frac{2}{3}\pi \Rightarrow \cot^{-1} x = \frac{\pi}{6} \therefore x = \sqrt{3}$  (Ans.)

10. সমাধান কর:  $\tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x$  [BUET'06-07]

সমাধান:  $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} + \tan^{-1} x \Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2} \tan^{-1} x$

$\Rightarrow \frac{3}{2} \tan^{-1} x = \frac{\pi}{4} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \therefore x = \frac{1}{\sqrt{3}}$  (Ans.)



### Question Type-03: বিপরীত ত্রিকোণমিতিক সমীকরণের প্রমাণ সংক্রান্ত সমস্যা

#### Formula & Concept:

এক্ষেত্রে যা চাওয়া হয়েছে তার সাথে মিল রেখে সমাধান করে এগিয়ে যেতে হবে। কোনো সমস্যা সমাধানের একাধিক পদ্ধতি থাকতে পারে। এর মধ্যে সহজ পদ্ধতিতে সমাধান বের করাই বুদ্ধিমানের কাজ।

#### MCQ

01.  $\sin \cot^{-1} \tan \cos^{-1} x = ?$

[BUTEX'15-16]

- (a)  $x$  (b)  $\pi$  (c)  $\frac{1}{x}$  (d)  $\sqrt{1-x}$

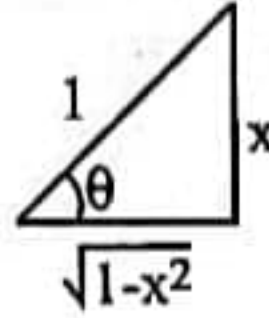
সমাধান: (a);  $\sin \cot^{-1} \tan \cos^{-1} x = \sin \cot^{-1} \cot \left( \frac{\pi}{2} - \cos^{-1} x \right) = \sin \left( \frac{\pi}{2} - \cos^{-1} x \right) = \cos \cos^{-1} x = x$

#### Written

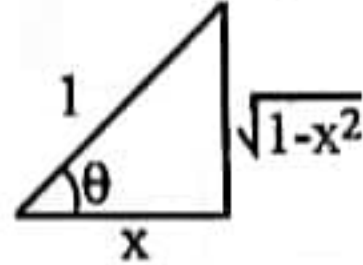
02. প্রমাণ কর যে,  $\cos \tan^{-1} \cot \sin^{-1} x = x$  (চিত্র আবশ্যিক)

[KUET'19-20]

সমাধান: L. H. S =  $\cos \tan^{-1} \cot \sin^{-1} x$



=  $\cos \tan^{-1} \cot \cot^{-1} \frac{\sqrt{1-x^2}}{x}$



=  $\cos \tan^{-1} \frac{\sqrt{1-x^2}}{x} = \cos \cos^{-1} \frac{x}{1} = x = \text{R. H. S}$

03. প্রমাণ করা যে,  $\sin^{-1} \left( \frac{4}{5} \right) + \sin^{-1} \left( \frac{5}{13} \right) + \sin^{-1} \left( \frac{16}{65} \right) = \frac{\pi}{2}$

[RUET'07-08, BUTEX'19-20]

সমাধান:  $\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} = \tan^{-1} \frac{4}{\sqrt{5^2-4^2}} + \tan^{-1} \frac{5}{\sqrt{13^2-5^2}} + \tan^{-1} \frac{16}{\sqrt{65^2-16^2}}$

=  $\tan^{-1} \frac{4}{3} + \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{16}{63} = \tan^{-1} \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} + \tan^{-1} \frac{16}{63} = \tan^{-1} \frac{63}{16} + \tan^{-1} \frac{16}{63} = \frac{\pi}{2}$  (Proved)

04. প্রমাণ কর:  $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \frac{1-x}{1+x}$

[BUTEX'08-09]

সমাধান: L. H. S =  $\tan^{-1} \sqrt{x} = \frac{1}{2} \times 2 \tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2} = \frac{1}{2} \cos^{-1} \frac{1-x}{1+x} = \text{R. H. S}$  (Proved)

05. প্রমাণ কর:  $\cos^{-1} x = 2 \sin^{-1} \sqrt{\frac{1-x}{2}} = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$

[KUET'03-04]

সমাধান: ধরি,  $\cos^{-1} x = \theta \therefore \cos \theta = x$

এখন,  $\sin \frac{\theta}{2} = \sqrt{\frac{1-\cos \theta}{2}} = \sqrt{\frac{1-x}{2}} \therefore \frac{\theta}{2} = \sin^{-1} \sqrt{\frac{1-x}{2}} \therefore \theta = 2 \sin^{-1} \sqrt{\frac{1-x}{2}} \therefore \cos^{-1} x = 2 \sin^{-1} \sqrt{\frac{1-x}{2}}$

আবার,  $\cos \frac{\theta}{2} = \sqrt{\frac{1+\cos \theta}{2}} = \sqrt{\frac{1+x}{2}}$  বা,  $\frac{\theta}{2} = \cos^{-1} \sqrt{\frac{1+x}{2}}$  বা,  $\theta = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$

$\therefore \cos^{-1} x = 2 \cos^{-1} \sqrt{\frac{1+x}{2}} \therefore \cos^{-1} x = 2 \sin^{-1} \sqrt{\frac{1-x}{2}} = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$

06. প্রমাণ কর:  $\cot^{-1} \frac{5}{3} + \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{27}{11}$

[RUET'03-04]

সমাধান:  $\cot^{-1} \frac{5}{3} + \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{27}{11}$

L.H.S =  $\cot^{-1} \frac{5}{3} + \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{3}{5} + \tan^{-1} \frac{3}{4} = \tan^{-1} \frac{\frac{3}{5} + \frac{3}{4}}{1 - \frac{3}{5} \times \frac{3}{4}} = \tan^{-1} \frac{12+15}{20} \times \frac{20}{11} = \tan^{-1} \frac{27}{11} = \text{R.H.S}$  (Proved)



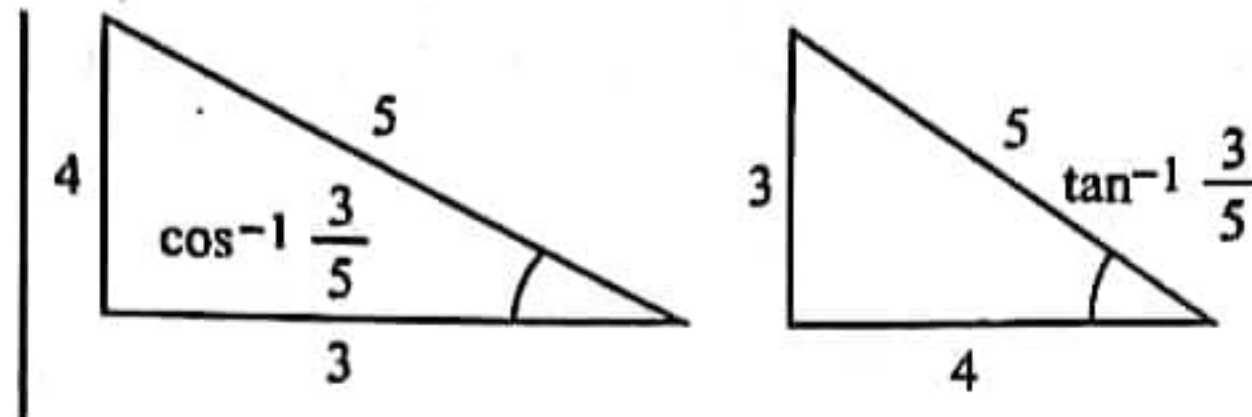
07.  $\cot \cos^{-1} \sin \tan^{-1} \frac{3}{4}$  এর মান কত?

[KUET'04-05]

সমাধান:  $\cot \cos^{-1} \sin \tan^{-1} \frac{3}{4}$

$$= \cot \cos^{-1} \sin \sin^{-1} \frac{3}{5} = \cot \cos^{-1} \frac{3}{5}$$

$$= \cot \cot^{-1} \frac{3}{4} = \frac{3}{4} \text{ (Ans.)}$$



08. প্রমাণ কর:  $2 \tan^{-1} \left\{ \sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right\} = \cos^{-1} \frac{b+a \cos \theta}{a+b \cos \theta}$

[BUTEX'00-01]

সমাধান: L.H.S =  $2 \tan^{-1} \left\{ \sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right\} = \cos^{-1} \left( \frac{1 - \left( \frac{\sqrt{a-b}}{\sqrt{a+b}} \right)^2 \sin^2 \frac{\theta}{2}}{1 + \left( \frac{\sqrt{a-b}}{\sqrt{a+b}} \right)^2 \sin^2 \frac{\theta}{2}} \right) = \cos^{-1} \left( \frac{a \cos^2 \frac{\theta}{2} + b \cos^2 \frac{\theta}{2} - a \sin^2 \frac{\theta}{2} + b \sin^2 \frac{\theta}{2}}{a \cos^2 \frac{\theta}{2} + b \cos^2 \frac{\theta}{2} + a \sin^2 \frac{\theta}{2} - b \sin^2 \frac{\theta}{2}} \right)$

$$= \cos^{-1} \left( \frac{a \cos 2 \times \frac{\theta}{2} + b}{a + b \cos 2 \times \frac{\theta}{2}} \right) = \cos^{-1} \left( \frac{b + a \cos \theta}{a + b \cos \theta} \right) = \text{R.H.S (Proved)}$$

**Question Type-04:**  $a \cos \theta + b \sin \theta = c$  [যেখানে  $|c| \leq \sqrt{a^2 + b^2}$ ] আকৃতির ত্রিকোণমিতিক সমীকরণ সংক্রান্ত সমস্যা

⇒ **Formula & Concept:**

এ ধরনের অংকে উভয়পক্ষকে  $\sqrt{a^2 + b^2}$  দ্বারা ভাগ করে  $\cos(A \pm B)$  এর সূত্র প্রয়োগ করাই উত্তম।

### MCQ

01.  $\sin \theta + \cos \theta = \sqrt{2}$  হলে  $\theta$  এর মান-  $[0^\circ < \theta < 90^\circ]$

[CUET'10-11, IUT'11-12]

- (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d) None of these

সমাধান: (b);  $\sin \theta + \cos \theta = \sqrt{2} \Rightarrow \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = 1 \Rightarrow \sin 45^\circ \sin \theta = \cos 45^\circ \cos \theta = 1$

$$\Rightarrow \cos(\theta - 45^\circ) = \cos 0 \Rightarrow \theta - 45^\circ = 0 \therefore \theta = 45^\circ$$

02.  $\cos \theta + \sqrt{3} \sin \theta = 2$ ,  $(0^\circ < \theta < 360^\circ)$  এর মান নির্ণয় কর।

[BUET'10-11]

- (a)  $45^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  (d)  $120^\circ$

সমাধান: (b);  $\cos \theta + \sqrt{3} \sin \theta = 2 \Rightarrow \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta = 1$

$$\Rightarrow \cos \left( \theta - \frac{\pi}{3} \right) = 1 \Rightarrow \theta - \frac{\pi}{3} = 2n\pi \Rightarrow \theta = 2n\pi + \frac{\pi}{3}$$

$$n = 0, n \in \mathbb{Z} \therefore \theta = \frac{\pi}{3} \text{ বা } 60^\circ$$

### Written

03. সমাধান কর:  $\sqrt{3} \sin x - \cos x = 2, 0 < x < 2\pi$

[BUET'07-08]

সমাধান:  $\sqrt{3} \sin x - \cos x = 2; 0 < x < 2\pi$  উভয় পক্ষকে  $\sqrt{(\sqrt{3})^2 + (-1)^2}$  বা 2 দ্বারা ভাগ করে পাই,

$$\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x = 1 \Rightarrow \sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6} = 1 \Rightarrow \sin \left( x - \frac{\pi}{6} \right) = 1$$

$$\Rightarrow x - \frac{\pi}{6} = (4n + 1) \frac{\pi}{2} \Rightarrow x = (4n + 1) \frac{\pi}{2} + \frac{\pi}{6}$$

$$n = 0 \text{ হলে, } x = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3} < 2\pi \text{ এবং } n = 1 \text{ হলে, } x = \frac{5\pi}{2} + \frac{\pi}{6} = \frac{8\pi}{3} > 2\pi$$

$$\therefore \text{নির্ণয় সমাধান: } x = \frac{2\pi}{3} \text{ (Ans.)}$$





04. সমাধান কর:  $\sqrt{3} \sin \theta - \cos \theta = 2$  যখন  $-2\pi < \theta < 2\pi$

[BUTEX'03-04, CUET'07-08]

সমাধান:  $\sqrt{3} \sin \theta - \cos \theta = 2$  বা,  $\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta = 1$  বা,  $\cos \frac{\pi}{3} \sin \frac{\pi}{3} \sin \theta = -1$

[উভয় পক্ষে  $\sqrt{(\sqrt{3})^2 + (-1)^2} = 2$  দ্বারা ভাগ করে]  $\Rightarrow \cos \frac{\pi}{3} \cos \theta - \sin \frac{\pi}{3} \sin \theta = -1$

বা,  $\cos\left(\theta + \frac{\pi}{3}\right) = -1$  বা,  $\theta + \frac{\pi}{3} = (2n+1)\pi = 2n\pi + \pi$  বা,  $\theta = 2n\pi + \frac{2\pi}{3}$   $\therefore$  সীমার মধ্যে  $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

05. সমাধান কর:  $\frac{\sqrt{3}}{\sin 2x} - \frac{1}{\cos 2x} = 4$

[BUET'06-07]

সমাধান:  $\frac{\sqrt{3}}{\sin 2x} - \frac{1}{\cos 2x} = 4 \Rightarrow \frac{\sqrt{3} \cos 2x - \sin 2x}{\sin 2x \cos 2x} = 4 \Rightarrow \frac{\sqrt{3}}{2} \cos 2x - \frac{1}{2} \sin 2x = 2 \sin 2x \cdot \cos 2x$

$\sin\left(\frac{\pi}{3} - 2x\right) = \sin 4x \Rightarrow \sin 4x + \sin\left(2x - \frac{\pi}{3}\right) = 0 \Rightarrow 2 \sin\left(3x - \frac{\pi}{6}\right) \cdot \cos\left(x + \frac{\pi}{6}\right) = 0$

$\Rightarrow \sin\left(3x - \frac{\pi}{6}\right) = 0 \Rightarrow 3x - \frac{\pi}{6} = n\pi \Rightarrow x = (6n+1)\frac{\pi}{18}$ ; [Where,  $n \in \mathbb{Z}$ ]

$= \cos\left(x + \frac{\pi}{6}\right) = 0 \Rightarrow x + \frac{\pi}{6} = (2n+1)\frac{\pi}{2} \Rightarrow x = (3n+1)\frac{\pi}{3}$

06. সমাধান কর:  $1 - 2 \sin \theta = \cos \theta$

[BUET'05-06]

সমাধান:  $\cos \theta + 2 \sin \theta = 1 \Rightarrow \frac{1}{\sqrt{5}} \cos \theta + \frac{2}{\sqrt{5}} \sin \theta = \frac{1}{\sqrt{5}}$  | ধরি,  $\frac{1}{\sqrt{5}} = \cos \alpha \therefore \sin \alpha = \frac{2}{\sqrt{5}}$

$\Rightarrow \cos(\theta - \alpha) = \cos \alpha \therefore \theta = 2n\pi + 2\alpha$ ;  $2n\pi$  যেখানে,  $\alpha = \cos^{-1} \frac{1}{\sqrt{5}}$  [ $n \in \mathbb{Z}$ ]

07. সমাধান কর:  $\cos \theta + \sin \theta = \sqrt{2}$

[BUET'02-03]

সমাধান:  $\cos \theta + \sin \theta = \sqrt{2} \Rightarrow \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta = \frac{\sqrt{2}}{\sqrt{2}}$  [উভয়পক্ষে,  $\sqrt{1^2 + 1^2} = \sqrt{2}$  দ্বারা ভাগ করে]

$\Rightarrow \sin \frac{\pi}{4} \cos \theta + \cos \frac{\pi}{4} \sin \theta = 1 \Rightarrow \sin\left(\frac{\pi}{4} + \theta\right) = 1 \Rightarrow \frac{\pi}{4} + \theta = (4n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (4n+1)\frac{\pi}{2} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{4}$  (Ans.)

08. সমাধান কর:  $\cos x + \sqrt{3} \sin x = \sqrt{2}$

[BUTEX'01-02]

সমাধান:  $\cos x + \sqrt{3} \sin x = \sqrt{2} \Rightarrow \cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3} = \cos \frac{\pi}{4} \Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos \frac{\pi}{4}$

$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{4} \Rightarrow x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{3}$

$\therefore x = 2n\pi + \frac{7\pi}{12}$  এবং  $\Rightarrow x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{3} = 2n\pi + \frac{\pi}{12}$

### Question Type-05: $\cot \theta$ , $\tan \theta$ , $\sec \theta$ , $\operatorname{cosec} \theta$ বিশিষ্ট ত্রিকোণমিতিক সমীকরণ সংক্রান্ত সমস্যা

#### Formula & Concept:

এই টাইপের সমস্যাগুলোতে  $\tan$  বা  $\cot$  এর সূত্র আনার চেষ্টা করতে হবে। যদি সূত্র না আনা যায়, তাহলে,

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ ,  $\cot \theta = \frac{\cos \theta}{\sin \theta}$ ,  $\sec \theta = \frac{1}{\cos \theta}$ ,  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$  আকারে ভেঙ্গে সমাধান করতে হবে।

#### MCQ

01. সমাধান নির্ণয় কর:  $\sqrt{2} \sec x + \tan x = 1$

[CKRUET'21-22]

(a)  $n\pi + \frac{\pi}{4}$

(b)  $n\pi - \frac{\pi}{4}$

(c)  $2n\pi - \pi$

(d)  $\frac{n\pi}{2} + \pi$

(e)  $2n\pi - \frac{\pi}{4}$

সমাধান: (e);  $\sqrt{2} \sec x + \tan x = 1 \Rightarrow \frac{\sqrt{2}}{\cos x} + \frac{\sin x}{\cos x} = 1 \Rightarrow \sqrt{2} + \sin x = \cos x \Rightarrow \cos x - \sin x = \sqrt{2}$

$\Rightarrow \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 1 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) = 1 \Rightarrow x + \frac{\pi}{4} = 2n\pi \Rightarrow x = 2n\pi - \frac{\pi}{4}$ ;  $n \in \mathbb{Z}$





02. General solution of the equation  $2(\cos x + \sec x) = 5$  is- [IUT'21-22]  
 (a)  $n\pi \pm \frac{\pi}{3}$  (b)  $2n\pi \pm \frac{\pi}{3}$  (c)  $2n\pi \pm \frac{\pi}{6}$  (d)  $n\pi \pm \frac{\pi}{6}$

**Solution: (b);**  $2(\cos x + \sec x) = 5 \Rightarrow 2\left(\cos x + \frac{1}{\cos x}\right) = 5 \Rightarrow 2\left(\frac{\cos^2 x + 1}{\cos x}\right) = 5 \Rightarrow 2\cos^2 x + 2 = 5\cos x$   
 $\Rightarrow 2\cos^2 x - 5\cos x + 2 = 0 \Rightarrow \cos x = \frac{1}{2}, 2$ ; But  $\cos x \neq 2 \therefore \cos x = \frac{1}{2} = \cos \frac{\pi}{3} \Rightarrow x = 2n\pi \pm \frac{\pi}{3} [n \in \mathbb{Z}]$

03.  $\sqrt{3} \tan 6\theta - \sqrt{3} \tan 4\theta + \tan 6\theta \tan 4\theta + 1 = 0$  এর মুখ্য সমাধান হলো- [KUET'17-18]  
 (a)  $60^\circ$  (b)  $165^\circ$  (c)  $75^\circ$  (d)  $30^\circ$  (e)  $135^\circ$

সমাধান: (c);  $\sqrt{3}(\tan 6\theta - \tan 4\theta) = -(1 + \tan 6\theta \tan 4\theta)$

$$\Rightarrow \frac{\tan 6\theta - \tan 4\theta}{1 + \tan 6\theta \tan 4\theta} = \frac{-1}{\sqrt{3}} \Rightarrow \tan(6\theta - 4\theta) = \tan\left(\pi - \frac{\pi}{6}\right) \Rightarrow 2\theta = \frac{5\pi}{6} \therefore \theta = \frac{5\pi}{12} = 75^\circ$$

04.  $\tan 2x \tan x = 1$  হলে  $x$  এর মান কত? [BUET'06-07, RUET'13-14, CUET'13-14, BUTEX'16-17]  
 (a)  $n\pi + \frac{\pi}{6}$  (b)  $n\pi \pm \frac{\pi}{6}$  (c)  $2n\pi \pm \frac{\pi}{6}$  (d)  $2n\pi - \frac{\pi}{6}$

সমাধান: (b);  $\tan 2x \tan x = 1 \Rightarrow \frac{2 \tan^2 x}{1 - \tan^2 x} = 1 \Rightarrow \tan x = \pm \frac{1}{\sqrt{3}} \Rightarrow x = n\pi \pm \frac{\pi}{6}$

05.  $\tan 2\theta \tan \theta = 1$  সমীকরণে  $\theta$  এর মান হবে- [BUET'13-14]  
 (a)  $n\pi + \frac{\pi}{6}$  (b)  $n\pi - \frac{\pi}{6}$  (c)  $2n\pi + \frac{\pi}{6}$  (d)  $2n\pi - \frac{\pi}{6}$

সমাধান: (a);  $\tan \theta = y \Rightarrow \frac{2y}{1-y^2} \cdot y = 1$ ;  $2y^2 = 1 - y^2 \therefore 3y^2 = 1 \Rightarrow y = \pm \frac{1}{\sqrt{3}}$

$\therefore \tan \theta = \pm \frac{1}{\sqrt{3}} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$  But, first option is the first choice.

06.  $\tan 2x \tan x = 1$  হলে  $x$  এর দুইটি মানই কোন কোন ক্ষেত্রে সঠিক নয়? [BUTEX'13-14]  
 (a)  $\frac{7\pi}{6}, \frac{5\pi}{6}$  (b)  $\frac{13\pi}{6}, \frac{15\pi}{6}$  (c)  $\frac{8\pi}{6}, \frac{9\pi}{6}$  (d)  $\frac{13\pi}{6}, \frac{11\pi}{6}$

সমাধান: (c); ক্যালকুলেটর ব্যবহার করে।

07.  $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$  সমীকরণে  $x$  এর মান হবে- [BUET'12-13]  
 (a)  $\frac{n\pi}{12}$  (b)  $\frac{n\pi}{4}$  (c)  $\frac{n\pi}{3}$  (d)  $\frac{n\pi}{5}$

সমাধান: (c);  $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x \Rightarrow \tan x + \tan 2x = -\tan 3x (1 - \tan x \tan 2x)$   
 $\Rightarrow \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = -\tan 3x \Rightarrow \tan 3x = -\tan 3x \Rightarrow 2 \tan 3x = 0 \Rightarrow \tan 3x = 0 \Rightarrow 3x = n\pi \Rightarrow x = \frac{n\pi}{3}$

**Shortcut:** Use Calculator

08.  $f(\theta) = \cos \theta - \sin \theta$  হলে,  $\theta$  এর কোন মানের জন্য  $f(\theta) = 0$  হবে? [RUET'12-13]  
 (a) 1 (b)  $\frac{\pi}{4}$  (c)  $\frac{\pi}{2}$  (d) 0 (e) -1

সমাধান: (b);  $f(\theta) = \cos \theta - \sin \theta$ ;  $f(\theta) = 0 \therefore \cos \theta - \sin \theta = 0$  or,  $\tan \theta = 1$  or  $\theta = \frac{\pi}{4}$

09.  $\tan 2\theta + \sec \theta = -1, 0 < \theta < 2\pi$  হলে  $\theta$  এর মান কত হবে? [BUTEX'11-12]  
 (a)  $\pi$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\frac{3\pi}{2}$

সমাধান: (a);  $\tan 2\theta + \sec \theta = -1, \theta = \pi$  হলে,  $\tan 2\pi + \sec \pi = -1$

10. How many solutions are there for  $\sec 4\theta - \sec 2\theta = 2; [0^\circ < \theta < 180^\circ]$  [IUT'10-11]  
 (a) 3 (b) 5 (c) 6 (d) 9

**Solution: (b);**  $\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2 \Rightarrow \cos 2\theta - \cos 4\theta = 2 \cos 4\theta \cos 2\theta$

$$\Rightarrow \cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta \Rightarrow \cos 6\theta + \cos 4\theta = 0 \Rightarrow 2 \cos 5\theta \cos \theta = 0$$

$$\Rightarrow \cos 5\theta = 0 \Rightarrow 5\theta = (2n+1)\frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{10} = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10} \Rightarrow \cos \theta = 0 \therefore \theta = \frac{\pi}{2}$$



## Written

11. সমাধান কর:  $\cot x + \cot 2x + \cot 3x = \cot x \cot 2x \cot 3x$ . [BUET'14-15]

সমাধান:  $\cot x + \cot 2x + \cot 3x = \cot x \cot 2x \cot 3x \Rightarrow \cot 2x + \cot x = \cot 3x (\cot x \cot 2x - 1)$

$$\Rightarrow \frac{\cot 2x + \cot x}{\cot x \cot 2x - 1} = \cot 3x \Rightarrow \frac{1}{\cot(x+2x)} = \cot 3x \quad \left[ \because (A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A} \right]$$

$$\Rightarrow \tan 3x = \frac{1}{\tan 3x} \Rightarrow \tan^2 3x = 1 \Rightarrow \tan 3x = \pm 1 = \tan\left(\pm \frac{\pi}{4}\right) \therefore 3x = n\pi \pm \frac{\pi}{4} \therefore x = \frac{n\pi}{3} \pm \frac{\pi}{12}; n \in \mathbb{Z}$$

12. সমাধান কর:  $\frac{(\sec x + \tan x)^2 - (\sec 2x + \tan x)^2}{\sin 2x - \sin x} = 2; [-\pi \leq x \leq \pi]$  [RUET'06-07]

সমাধান:  $(\sec x + \tan x)^2 - (\sec 2x + \tan x)^2 - 2(\sin 2x - \sin x) = 0$

$$\Rightarrow \left(\frac{1+\sin x}{\cos x}\right)^2 - \left(\frac{1}{\cos 2x} + \frac{\sin x}{\cos x}\right)^2 - 2(\sin 2x - \sin x) = 0$$

$$\Rightarrow \frac{1+2\sin x+\sin^2 x}{\cos x} - \frac{\cos^2 x+2\cos x \cdot \cos 2x \cdot \sin x}{\cos 2x \cdot \cos x} - 2\sin 2x + 2\sin x = 0$$

$$\Rightarrow \frac{\cos 2x+2\cos 2x \cdot \sin x+\sin^2 x \cdot \cos 2x-\cos^2 x-\cos 2x \cdot \sin 2x-2\sin 2x \cdot \cos 2x \cdot \cos x+2\sin x \cdot \cos 2x \cdot \cos x}{\cos 2x \cdot \cos x} = 0$$

$$\Rightarrow \cos 2x + 2\cos 2x \cdot \sin x + \sin^2 x \cdot \cos 2x - \cos^2 x - \cos 2x \cdot \sin 2x - \sin 4x \cdot \cos x + \sin 2x \cdot \cos 2x = 0$$

$$\Rightarrow \cos 2x + 2\cos 2x \cdot \sin x + \sin^2 x \cdot \cos 2x - \cos^2 x - \sin 4x \cdot \cos x = 0$$

$$\Rightarrow \cos^2 x - \sin^2 x + 2\sin x(\cos^2 x - \sin^2 x) + \sin^2 x(\cos^2 x - \sin^2 x) - \cos^2 x - 2\sin 2x \cdot \cos 2x \cdot \cos x = 0$$

$$\Rightarrow 1 - 2\sin^2 x + 2\sin x - 4\sin^3 x + \sin^2 x - 2\sin^4 x - 1 + \sin^2 x - 4\sin x(1 - \sin^2 x)(1 - 2\sin^2 x) = 0$$

$$\Rightarrow 1 - 2\sin^2 x + 2\sin x - 4\sin^3 x + \sin^2 x - 2\sin^4 x - 1 + \sin^2 x - 4\sin x + 12\sin^3 x - 8\sin^5 x = 0$$

$$\Rightarrow -8\sin^5 x - 2\sin^4 x + 8\sin^3 x - 6\sin x = 0 \Rightarrow -2\sin x(4\sin^4 x + \sin^3 x + 2\sin^2 x + 3) = 0$$

$$-2\sin x = 0, \therefore x = n\pi; \text{যেখানে, } n \in \mathbb{Z}$$

অথবা,  $4\sin^4 x + \sin^3 x + 2\sin^2 x + 3 = 0$

কিন্তু,  $4\sin^4 x + \sin^3 x + 2\sin^2 x + 3 = \left(4 - \frac{1}{4}\right)\sin^4 x + \sin^2 x + \sin^2 x \left(\frac{1}{2}\sin x + 1\right)^2 + 3 \geq 3 \therefore x = n\pi; n \in \mathbb{Z}$

13. সমাধান কর:  $2\sin \theta \tan \theta + 1 = \tan \theta + 2\sin \theta$  [CUET'05-06]

সমাধান:  $2\sin \theta \frac{\sin \theta}{\cos \theta} + 1 = \frac{\sin \theta}{\cos \theta} + 2\sin \theta \Rightarrow \frac{2\sin^2 \theta + \cos \theta}{\cos \theta} = \frac{\sin \theta + 2\sin \theta \cos \theta}{\cos \theta}$

$$\Rightarrow 2\sin^2 \theta + \cos \theta - 2\sin \theta \cos \theta - \sin \theta = 0 \Rightarrow 2\sin \theta (\sin \theta - \cos \theta) - 1(\sin \theta - \cos \theta) = 0$$

$$\Rightarrow (2\sin \theta - 1)(\sin \theta - \cos \theta) = 0$$

$$2\sin \theta - 1 = 0 \Rightarrow \frac{1}{2} \Rightarrow \sin \theta = \sin \frac{\pi}{6}$$

$$\therefore \theta = n\pi + (-1)^n \frac{\pi}{6} [n \in \mathbb{Z}] \text{ (Ans.)}$$

$$\left. \begin{array}{l} \text{বা, } \sin \theta - \cos \theta = 0 \\ \Rightarrow \tan \theta = \tan \frac{\pi}{4} \therefore \theta = n\pi + \frac{\pi}{4}, n \in \mathbb{Z} \end{array} \right\}$$

14. সমাধান কর:  $\sec 4\theta - \sec 2\theta = 2; 0^\circ < \theta < 180^\circ$  [CUET'03-04]

সমাধান:  $\sec 4\theta - \sec 2\theta = 2 \Rightarrow \frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2 \Rightarrow \cos 2\theta - \cos 4\theta = 2\cos 4\theta \cos 2\theta$

$$\cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta \Rightarrow \cos 6\theta + \cos 4\theta = 0 \Rightarrow \cos 5\theta \cos \theta = 0$$

$$\therefore \cos 5\theta = 0 \Rightarrow 5\theta = (2n+1)\frac{\pi}{2} \Rightarrow \theta = \frac{1}{10}(2n+1)\pi, n \in \mathbb{Z} \text{ and } \cos \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$$

$$n = 0, 1, 2, 3, 4 \text{ ইত্যাদি ধরে প্রদত্ত ব্যবধির মধ্যে পাই, } \theta = \frac{\pi}{10}, \frac{\pi}{2}, \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$$

### Question Type-06: sine, cosine কোণের বিজোড় সংখ্যক পদ সম্বলিত সমীকরণ সংক্রান্ত সমস্যা

#### Formula & Concept:

যখন সমীকরণে  $\sin \theta$  এবং  $\cos \theta$  এর বিজোড় সংখ্যক পদ থাকে তখন  $\sin C \pm \sin D$ ,  $\cos C \pm \cos D$  এর সূত্র ব্যবহার করলে সহজ হবে। প্রয়োজনে,  $1 \pm \cos 2\theta$  এর সূত্র ব্যবহার করা যেতে পারে।

#### MCQ

01.  $\cos \theta = -1$  হলে, সাধারণ সমাধান কত?

- (a) 0 (b)  $2n\pi$  (c)  $(2n+1)\pi$  (d)  $n\pi$

[BUTEX'13-14]

সমাধান: (c):  $\cos \theta = -1 \therefore \theta = \pi, 3\pi, \dots, (2n+1)\pi$

02. যদি  $\sin\left(\frac{\pi}{2} \cos \alpha\right) = \cos\left(\frac{\pi}{2} \sin \alpha\right)$  হয়, তবে  $\alpha$  এর মান হলো-

- (a)  $0, \frac{\pi}{4}$  (b)  $\frac{\pi}{4}, \frac{\pi}{2}$  (c)  $\frac{\pi}{2}, \frac{3\pi}{4}$  (d)  $0, \frac{\pi}{2}$  (e)  $-\frac{\pi}{4}, \frac{\pi}{4}$

[KUET'12-13]

সমাধান: (d):  $\sin\left(\frac{\pi}{2} \cos \alpha\right) = \cos\left(\frac{\pi}{2} \sin \alpha\right) \Rightarrow \sin\left(\frac{\pi}{2} \cos \alpha\right) = \sin\left(\frac{\pi}{2} \pm \frac{\pi}{2} \sin \alpha\right) \Rightarrow \frac{\pi}{2} \cos \alpha = \frac{\pi}{2} \pm \frac{\pi}{2} \sin \alpha$

$\Rightarrow \cos \alpha = 1 \pm \sin \alpha \Rightarrow \cos \alpha \pm \sin \alpha = 1 \Rightarrow \cos^2 \alpha + \sin^2 \alpha \pm 2 \sin \alpha \cos \alpha = 1 \Rightarrow 1 \pm \sin 2\alpha = 1$

$\Rightarrow \sin 2\alpha = 0 \Rightarrow 2\alpha = n\pi \Rightarrow \alpha = n\frac{\pi}{2}$

$n = 0$  হলে,  $\alpha = 0$ ;  $n = 1$  হলে,  $\alpha = \frac{\pi}{2} \therefore \alpha = 0, \frac{\pi}{2}$  [Shortcut: Use calculator.]

#### Written

03. সমাধান কর:  $\sin(x) + \sin\left(\frac{x}{2}\right) = 0$ , যখন  $0 \leq x \leq 2\pi$

[BUET'20-21]

সমাধান:  $\sin x + \sin\left(\frac{x}{2}\right) = 0 \Rightarrow 2 \sin \frac{x}{2} \cos \frac{x}{2} + \sin \frac{x}{2} = 0 \Rightarrow \sin \frac{x}{2} (2 \cos \frac{x}{2} + 1) = 0$

সুতরাং হয়,  $\sin \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = n\pi$

$\therefore x = 2n\pi$

$n = 0$  হলে,  $x = 0$ ;  $n = 1$  হলে,  $x = 2\pi$

$\therefore$  নির্ণেয় সমাধান,  $x = 0, \frac{4\pi}{3}, 2\pi$

অথবা,  $\cos \frac{x}{2} = -\frac{1}{2} = \cos \frac{2\pi}{3} \Rightarrow \frac{x}{2} = 2n\pi \pm \frac{2\pi}{3} \therefore x = 4n\pi \pm \frac{4\pi}{3}$

$x = 0$  হলে,  $x = \pm \frac{4\pi}{3} \therefore x = \frac{4\pi}{3}$  (গ্রহণযোগ্য মান)

বিকল্প:  $\sin x + \sin\left(\frac{x}{2}\right) = 0 \Rightarrow 2 \sin \frac{x+\frac{x}{2}}{2} \cos \frac{x-\frac{x}{2}}{2} = 0 \Rightarrow 2 \sin \frac{3x}{4} \cos \frac{x}{4} = 0$

সুতরাং হয়,  $\sin \frac{3x}{4} = 0 \Rightarrow \frac{3x}{4} = n\pi \therefore x = \frac{4n\pi}{3}$

$n = 0, 1$  হলে,  $x = 0, \frac{4\pi}{3}$

$\therefore$  নির্ণেয় সমাধান,  $x = 0, \frac{4\pi}{3}, 2\pi$  (Ans.)

অথবা,  $\cos \frac{x}{4} = 0 \Rightarrow \frac{x}{4} = (2n+1)\frac{\pi}{2}$

$\therefore x = (2n+1)2\pi$ ;  $n = 0$  হলে,  $x = 2\pi$

04. সমাধান কর:  $\sin \theta + \sin 2\theta + \sin 3\theta = 1 + \cos \theta + \cos 2\theta$ ,  $0 < \theta < \pi$

[BUET'08-09, RUET'15-16]

সমাধান:  $\sin \theta + \sin 2\theta + \sin 3\theta = 1 + \cos \theta + \cos 2\theta$

$\Rightarrow 2 \sin 2\theta \cos \theta + \sin 2\theta = \cos \theta + 2 \cos^2 \theta \Rightarrow \sin 2\theta (2 \cos \theta + 1) - \cos \theta (2 \cos \theta + 1) = 0$

$\Rightarrow (2 \cos \theta + 1)(2 \sin \theta \cos \theta - \cos \theta) = 0 \Rightarrow \cos \theta (2 \cos \theta + 1)(2 \sin \theta - 1) = 0$

$\therefore \cos \theta = 0 \Rightarrow \cos \theta = \cos \frac{\pi}{2} \therefore \theta = \frac{\pi}{2}$ ;  $\cos \theta = -\frac{1}{2} \Rightarrow \cos \theta = \cos \frac{2\pi}{3} \therefore \theta = \frac{2\pi}{3}$

$\sin \theta = \frac{1}{2} \Rightarrow \sin \theta = \sin \frac{\pi}{6} = \sin \frac{5\pi}{6} \therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6} \therefore \theta = \frac{\pi}{2}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$  (Ans.)



05. একটি সমকোণী ত্রিভুজের সুক্ষ্মকোণদ্বয়কে নিম্নলিখিত সমীকরণ দ্বারা প্রকাশ করা যায়। সুক্ষ্মকোণদ্বয় এর মান নির্ণয় কর।  
সমীকরণটিঃ  $\sin \theta + 2 \cos \theta = 1$  [CUET'09-10]  
সমাধান:  $\sin \theta + 2 \cos \theta = 1 \Rightarrow (\sin \theta - 1)^2 = 4 \cos^2 \theta \Rightarrow \sin^2 \theta - 2 \sin \theta + 1 = 4 - 4 \sin^2 \theta$   
 $\Rightarrow 5 \sin^2 \theta - 2 \sin \theta - 3 = 0 \Rightarrow 5 \sin^2 \theta - 5 \sin \theta + 3 \sin \theta - 3 = 0$   
 $(\sin \theta - 1)(5 \sin \theta + 3) = 0 \therefore \theta = 90^\circ, \sin^{-1} \frac{-3}{5} \therefore$  এরূপ কোন সমকোণী ত্রিভুজ সম্ভব নয়। (Ans.)
06.  $\cos \theta = 0$  হলে  $\theta$  এর সাধারণ সমাধান কী? [BUTEX'09-10]  
সমাধান:  $\theta = (2n + 1)\frac{\pi}{2} [n \in \mathbb{Z}]$
07. যদি  $\sin \theta = 1$  হয়, তবে  $\theta$  এর মান কত? [BUTEX'09-10]  
Solution:  $\theta = (4n + 1)\frac{\pi}{2} [n \in \mathbb{Z}]$ ;  $x = n\pi \pm \frac{\pi}{3} \therefore x = \frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{2\pi}{3}, \frac{5\pi}{8}, \frac{7\pi}{8}$  (Ans.)
08. সমাধান করঃ  $1 + \sin 2\phi + \sin 2\theta = \cos(2\phi + 2\theta) [0 \leq \phi, \theta \leq 90^\circ]$  [BUET'05-06]  
সমাধান:  $1 + \sin 2\phi + \sin 2\theta = \cos(2\phi + 2\theta) \Rightarrow 2 \sin(\phi + \theta) \cos(\phi - \theta) + 1 - \cos 2(\phi + \theta) = 0$   
 $\Rightarrow 2 \sin(\phi + \theta) \cos(\phi - \theta) + 2 \sin^2(\phi + \theta) = 0 \Rightarrow 2 \sin(\phi + \theta) [\cos(\phi - \theta) + \sin(\phi + \theta) = 0]$   
 $\therefore 2 \sin(\phi + \theta) = 0$   $\left| \begin{array}{l} \cos(\phi - \theta) + \sin(\phi + \theta) \neq 0 \\ [0 \leq \phi, \theta \leq 90^\circ \text{ এর মধ্যে } \phi \text{ ও } \theta \text{ এর কোন} \\ \text{মানের জন্যেই } \cos(\phi - \theta) = 0 \text{ হতে পারে না]} \end{array} \right.$   
 $\therefore \phi + \theta = n\pi \therefore 0 \leq \phi, \theta \leq 90^\circ$   
 $\therefore \phi + \theta = 0^\circ, 180^\circ \therefore \phi = \theta = 0^\circ$  অথবা,  $\phi = \theta = 90^\circ$  (Ans.)
09. সমাধান করঃ  $\cos \theta - \cos 7\theta = \sin 4\theta$ . [BUTEX'02-03]  
সমাধান:  $\cos v - \cos 7\theta = \sin 4\theta$  বা,  $2 \sin 4\theta \sin 3\theta - \sin v = 0$  বা,  $\sin 4\theta (2 \sin 3\theta - 1) = 0$   
বা,  $\sin 4\theta = 0$  বা,  $4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$  (Ans.)  
অথবা,  $2 \sin 3\theta - 1 = 0$  বা,  $\sin 3\theta = \frac{1}{2} = \sin \frac{\pi}{6}$  বা,  $3\theta = n\pi + (-1)^n \frac{\pi}{6} \therefore \theta = \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$ . (Ans.)

### Question Type-07: $\sin \theta, \cos \theta$ ইত্যাদি ত্রিকোণমিতিক অনুপাত গুণ আকারে থাকলে

#### Formula & Concept:

$\sin$  এবং  $\cos$  গুণ আকারে থাকলে  $2 \sin A \cos B; 2 \cos A \sin B; 2 \cos A \cos B; 2 \sin A \sin B$ ; এর সূত্রগুলো ব্যবহার করলে সহজ হবে।

#### MCQ

01.  $2(\sin \theta \cos \theta + \sqrt{3}) = \sqrt{3} \cos \theta + 4 \sin \theta; 0 < \theta < \frac{\pi}{2}$  এর সমাধান বের কর। [CUET'15-16]  
(a)  $\frac{\pi}{3}$  (b)  $\frac{\pi}{6}$  (c)  $\frac{\pi}{2}$  (d)  $\frac{\pi}{4}$   
সমাধান: (a);  $2(\sin \theta \cos \theta + \sqrt{3}) = \sqrt{3} \cos \theta + 4 \sin \theta \Rightarrow 2 \sin \theta \cos \theta - 4 \sin \theta + 2\sqrt{3} - \sqrt{3} \cos \theta = 0$   
 $\Rightarrow 2 \sin \theta (\cos \theta - 2) - \sqrt{3}(\cos \theta - 2) = 0 \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} [\cos \theta \neq 2] \Rightarrow \theta = \frac{\pi}{3}$
02.  $\cos(\pi\sqrt{x-4}) \cos(\pi\sqrt{x}) = 1$  এর কয়টি সমাধান পাওয়া যাবে? [RUET'14-15]  
(a) 0 (b) 1 (c) 2 (d)  $> 2$  (e) None  
সমাধান: (b);  $x = 4$  হলে,  $\cos 0^\circ \cdot \cos(2\pi) = 1$
03.  $2(\sin \theta \cos \theta + \sqrt{3}) = \sqrt{3} \cos \theta + 3 \sin \theta$  এবং  $0 < \theta < 360^\circ$  তবে,  $\theta = ?$  [RUET'11-12]  
(a)  $30^\circ, 150^\circ$  (b)  $90^\circ, 0^\circ$  (c)  $60^\circ, 120^\circ$  (d)  $45^\circ, 135^\circ$  (e) None  
সমাধান: (c);  $2(\sin \theta \cos \theta + \sqrt{3}) = \sqrt{3} \cos \theta + 3 \sin \theta; \theta = 60^\circ$  ও  $120^\circ$  উভয়ের জন্য L.H.S. = R.H.S. হয়।





## Written

04. সমাধান করঃ  $2(\sin x \cos x + \sqrt{3}) = \sqrt{3} \cos x + 4 \sin x, 0 < x < \pi$ .

[RUET'17-18]

সমাধান:  $2 \sin x \cos x + 2\sqrt{3} - \sqrt{3} \cos x - 4 \sin x = 0$

$\Rightarrow 2 \sin x (\cos x - 2) - \sqrt{3} (\cos x - 2) = 0 \Rightarrow (2 \sin x - \sqrt{3})(\cos x - 2) = 0$

$\therefore 2 \sin x - \sqrt{3} = 0$  অথবা  $\cos x - 2 = 0$ ; কিন্তু  $\cos x - 2 \neq 0$

$\therefore \sin x = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \therefore x = n\pi + (-1)^n \frac{\pi}{3}$  [যেখানে,  $n \in \mathbb{Z}$ ]

$n = 0$  হলে,  $x = \frac{\pi}{3}$ ;  $n = 1$  হলে,  $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$x = 2$  হলে,  $x = 2\pi + \frac{\pi}{3} = \frac{7\pi}{3} \therefore$  নির্ণেয় সমাধান  $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$  (Ans.)

05. সমাধান করঃ  $4 \cos x \cos 2x \cos 3x = 1; 0 < x < \pi$

[BUETex'03-04, CUET'08-09]

সমাধান:  $4 \cos x \cos 2x \cos 3x = 1 \Rightarrow 2 \cos 2x (\cos 4x + \cos 2x) = 1$

$\Rightarrow 2 \cos 2x \cos 4x + 2 \cos^2 2x - 1 = 0 \Rightarrow 2 \cos 2x \cos 4x + \cos 4x = 0 \Rightarrow \cos 4x (2 \cos 2x + 1) = 0$

সুতরাং,  $\cos 4x = 0 \Rightarrow 4x = (2n + 1) \frac{\pi}{2}$

$\therefore x = (2n + 1) \frac{\pi}{8}$  অথবা,  $2 \cos 2x + 1 = 0 \Rightarrow \cos 2x = -\frac{1}{2} = \cos \frac{2\pi}{3} \therefore 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3} \therefore x = \frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{2\pi}{3}, \frac{5\pi}{8}, \frac{7\pi}{8}$  (Ans.)

06. সমাধান করঃ  $4 \cos x \cos 2x \cos 3x = 1; 0 < x < \pi$

[KUET'06-07]

সমাধান:  $4 \cos x \cos 2x \cos 3x = 1 \Rightarrow 2 \cos 2x (2 \cos 3x \cos x) = 1$

$\Rightarrow 2 \cos 2x (\cos 4x + \cos 2x) = 1 \Rightarrow 2 \cos 2x \cos 4x + 2 \cos^2 2x = 1$

$\Rightarrow 2 \cos 2x \cos 4x + 2 \cos^2 2x - 1 = 0$

$\Rightarrow 2 \cos 2x \cos 4x + \cos 4x = 0$

$\Rightarrow \cos 4x (2 \cos 2x + 1) = 0;$

$\cos 4x = 0 \Rightarrow x = (2n + 1) \frac{\pi}{8} \dots \dots (i) \quad n \in \mathbb{Z}$

আবার,  $\cos 2x = -\frac{1}{2} = \cos \frac{2\pi}{3} \Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$

$\therefore x = n\pi \pm \frac{\pi}{3} \dots \dots (ii)$

(i) ও (ii) হতে পাই

যখন,  $n = 0, x = \frac{\pi}{8}, \frac{\pi}{3}$

$n = 1, x = \frac{3\pi}{8}, \frac{2\pi}{3}$

$n = 2, x = \frac{5\pi}{8}$

$n = 3, x = \frac{7\pi}{8}$  (Ans.)

Question Type-08:  $\sin \theta, \cos \theta, \tan \theta, \sec \theta$  এর দ্বিঘাতরাশি সম্বলিত পদ থাকলে

## Formula &amp; Concept:

এক্ষেত্রে পূর্ণ বর্গ রাশি বানাতে হবে অথবা দ্বিঘাত সমীকরণের ন্যায় সমাধান করতে হবে।

## MCQ

01. If  $2 \cos^2 \theta - \sin \theta = 1$  then the value of  $\theta$  is-

[IUT'14-15]

(a)  $\frac{\pi}{6}$  and  $\frac{5\pi}{6}$

(b)  $\frac{3\pi}{2}$

(c) Both a and b

(d) None of these

Solution: (c);  $2 \cos^2 \theta - \sin \theta = 1 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0 \Rightarrow \sin \theta = \frac{1}{2}, -1 = \sin \frac{\pi}{6}, \sin \left(-\frac{\pi}{2}\right)$

$\therefore \theta = n\pi + (-1)^n \frac{\pi}{6}, n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$  Putting  $n = 0, 1$  we get,  $\theta = \frac{\pi}{6}, -\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

02.  $\cot^2 \theta - 2\sqrt{2} \operatorname{cosec} \theta + 3 = 0$  হলে,  $\theta$  এর মান কত?

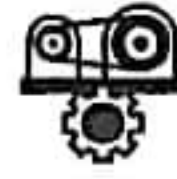
[Ans: c] [CUET'11-12]

(a)  $\pm 45^\circ$

(b)  $\pm 135^\circ$

(c)  $135^\circ$

(d) None of these



## Written

03.  $a \cos^2 x + b \sin^2 x = c$  হলে  $\tan x$  এর মান নির্ণয় কর।

[BUTEX'20-21]

সমাধান:  $a \cos^2 x + b \sin^2 x = c \Rightarrow a + b \tan^2 x = c \sec^2 x$  [উভয়পক্ষকে  $\sec^2 x$  দ্বারা গুণ করে]

$$\Rightarrow a + b \tan^2 x = c(1 + \tan^2 x) \Rightarrow (b - c) \tan^2 x = c - a \therefore \tan x = \pm \sqrt{\frac{c-a}{b-c}}$$

04. ABC ত্রিভুজ থেকে প্রমাণ কর যে,  $a^2(\sin^2 B - \sin^2 C) + b^2(\sin^2 C - \sin^2 A) + c^2(\sin^2 A - \sin^2 B) = 0$

[RUET'06-07]

সমাধান: L.H.S =  $a^2 \left( \frac{b^2}{4R^2} - \frac{c^2}{4R^2} \right) + b^2 \left( \frac{c^2}{4R^2} - \frac{a^2}{4R^2} \right) + c^2 \left( \frac{a^2}{4R^2} - \frac{b^2}{4R^2} \right) = 0$

05. সমাধান কর:  $\sec^2 \frac{x}{2} - 2\sqrt{2} \tan \frac{x}{2} = 0$

[BUTEX'05-06]

সমাধান:  $\sec^2 \frac{x}{2} - 2\sqrt{2} \tan \frac{x}{2} = 0 \Rightarrow \frac{1}{\cos^2 \frac{x}{2}} - 2\sqrt{2} \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = 0 \Rightarrow \frac{1 - 2\sqrt{2} \cos \frac{x}{2} \sin \frac{x}{2}}{\cos^2 \frac{x}{2}} = 0$

$$\Rightarrow \frac{1 - \sqrt{2} \sin x}{\cos^2 \frac{x}{2}} = 0 \Rightarrow 1 - \sqrt{2} \sin x = 0 \Rightarrow \sqrt{2} \sin x = 1$$

$$\Rightarrow \sin x = \sin \frac{\pi}{4} \Rightarrow x = n\pi + (-1)^n \frac{\pi}{4} [n \in \mathbb{Z}] \text{ (Ans.)}$$

06. সমাধান কর:  $\tan^2 \theta = 3 \operatorname{cosec}^2 \theta - 1$  for  $0 \leq \theta \leq 2\pi$

[KUET'03-04]

সমাধান:  $\tan^2 \theta = 3 \operatorname{cosec}^2 \theta - 1 \Rightarrow \sec^2 \theta = 3 \operatorname{cosec}^2 \theta \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = 3 \Rightarrow \tan^2 \theta = 3$

$$\Rightarrow \tan \theta = \sqrt{3} = \tan \frac{\pi}{3} \therefore \theta = n\pi + \frac{\pi}{3} \mid \tan \theta = -\sqrt{3} \Rightarrow \theta = n\pi - \frac{\pi}{3}$$

$$n = 0 \text{ হলে, } \theta = \frac{\pi}{3}, n = 1 \text{ হলে } \therefore \theta = \frac{4\pi}{3} \text{ for } n = 2, 3, \dots \theta > 2\pi \therefore \theta = \frac{\pi}{3}, 4\frac{\pi}{3}, 2\frac{\pi}{3}, 5\frac{\pi}{3} \text{ (Ans.)}$$

### Question Type-09: ত্রিকোণমিতিক সমীকরণ সমাধানে বহুপদী সমীকরণের সূত্রের ব্যবহার সংক্রান্ত

#### Formula & Concept:

$$ax^2 + bx + c = 0 \text{ দ্বিঘাত বহুপদী সমীকরণের মূলদ্বয় } \alpha, \beta \text{ হলে, } \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

অনেক সময় দ্বিঘাত ত্রিকোণমিতিক সমীকরণ সমাধান বা সমীকরণ সংক্রান্ত প্রমাণের জন্য উপরোক্ত সূত্রটি ব্যবহার করা হয়।

## Written

01.  $a \tan \theta + b \sec \theta = c$  সমীকরণের মূলদ্বয়  $\alpha, \beta$  হলে প্রমাণ কর যে,  $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$

[BUET'19-20]

সমাধান:  $a \tan \theta + b \sec \theta = c \Rightarrow a \tan \theta - c = -b \sec \theta \Rightarrow a^2 \tan^2 \theta + c^2 - 2ca \tan \theta = b^2 + b^2 \tan^2 \theta$

$$\Rightarrow (a^2 - b^2) \tan^2 \theta - 2ca \tan \theta + c^2 - b^2 = 0$$

$$\Rightarrow \tan \alpha + \tan \beta = \frac{2ca}{a^2 - b^2}; \tan \alpha \tan \beta = \frac{c^2 - b^2}{a^2 - b^2}$$

$$\text{L.H.S} = \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{2ca}{a^2 - b^2}}{1 - \frac{c^2 - b^2}{a^2 - b^2}}$$

$$= \frac{2ca}{a^2 - b^2 - c^2 + b^2} = \frac{2ca}{a^2 - c^2} = \text{R.H.S. (Proved)}$$

