## IUT Admission Test 2017-2018

Physics

01. The coordinate of a particle in meters is given by $x(t) = 16t - 3.0t^3$, where the time t is in seconds. What is the value of t when the particle is momentarily at rest?

(a) 0.75s   (b) 1.3s   (c) 5.3s   (d) 7.3s

Solution: (b); $x(t) = 16t - 3.0t^3$; $v = \frac{dx}{dt} = 16 - 9t^2$

A/C ; $16 - 9t^2 = 0 \Rightarrow t = 1.333 \approx 1.3$ Ans.

02. A plane has an air speed of 240 km/hr. What should be the plane's heading if it is to travel due north, relative to the earth in a wind blowing with a velocity of 100 km/hr. in an easterly direction?

(a) 24.62°   (b) 2.462°   (c) 2462°   (d) 42.62°

Solution: (a) ; $\sin\theta = \frac{100}{240} \Rightarrow \theta = 24.62°$ (Ans.)
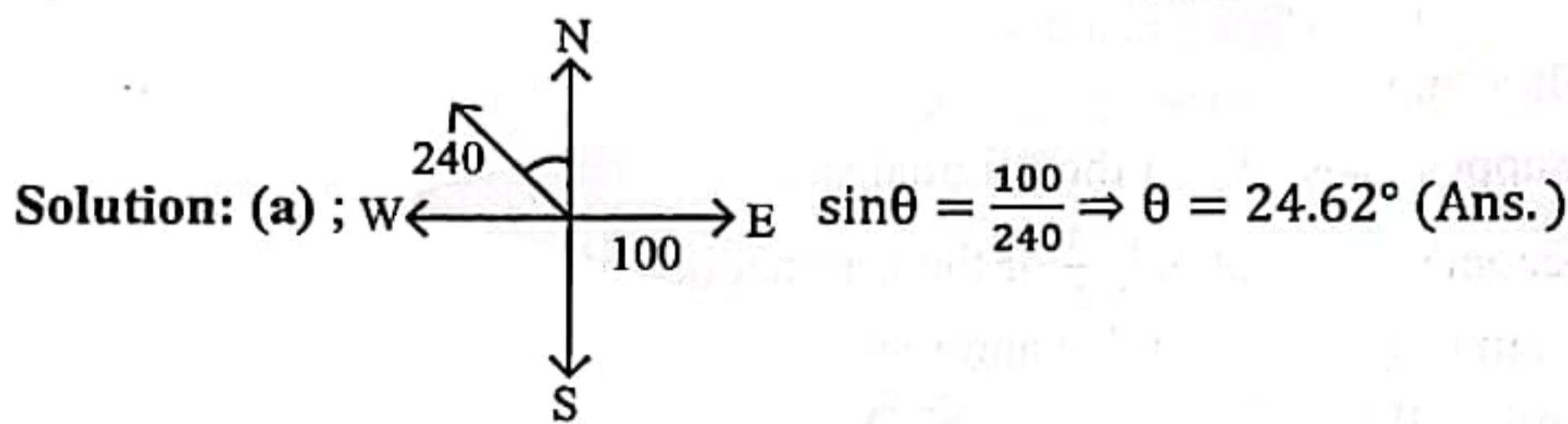
03. An elevator is accelerated upward at $2m/s^2$. If the elevator weighs 500kg, what is the tension in the supporting cable? ($g = 9.8m/s^2$).

(a) 20.604 kg   (b) 206.04 kg   (c) 602.04 kg   (d) 60.204 kg

Solution: (c) ; $F = m(g + a) = 500 \times (9.8 + 2)N = 5900$ $\therefore T = \frac{F}{g} = \frac{5900}{9.8} = 602.04$kg (Ans.)

04. A force of 100 N is required to stretch a steel wire 2.0 mm^2 in cross sectional area and 2.0 m long a distance of 0.50 mm. How much work is done?

(a) 0.25 N-m   (b) 0.025 N-m   (c) 2.5 N-m   (d) 25 N-m

Solution: (b); $W = \frac{1}{2}F \times l = \frac{1}{2} \times 100 \times 0.5 \times 10^{-3} = 0.025$ (Ans.)

05. A boy jogs around a horizontal circle with a constant speed. He travels one fourth of a revolution, a distance of 25 m along the circumference of the circle, in 5 s. The magnitude of his acceleration is:

(a) 0.31 m/s^2 (b) 1.3 m/s^2 (c) 1.6 m/s^2 (d) 3.9 m/s^2

Solution: (c) ; $2\pi r = 25 \times 4 = 100 \Rightarrow r = \frac{53}{\pi}$; $v = \frac{s}{t} = \frac{25}{5} = 5$; $a = \frac{v^2}{r} = 1.6$ (Ans.)

06. A car is being driven on a road having two distant circular bends B_1 and B_2 of radius R and 3R respectively. If S_1 is the speed of the car at the bend B_1 and S_2 is the speed at the bend B_2, what should the ratio S_1/S_2 be so that the centripetal forces at both bands are equal?

(a) 1   (b) $\sqrt{3}$ (c) $1/\sqrt{3}$ (d) $1/\sqrt{2}$

Solution: (c) ; $F_{c_1} = F_{c_2} \Rightarrow \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2} \Rightarrow \frac{v_1^2}{R} = \frac{v_2^2}{3R} \Rightarrow \frac{v_1}{v_2} = \frac{1}{\sqrt{3}}$ (Ans.)

07. In a spring balance, the length of the spring is 20 cm which can read from 0 to 60 N. Find the potential energy of the spring when it reads 40 N.

(a) 0.267 J   (b) 26.7 J   (c) 2.67 J   (d) 267 J

Solution: (c); $F = kx \Rightarrow k = \frac{F}{x} = \frac{60}{0.2}Nm^{-1} = 300Nm^2$

So, For 40N ; $x = \frac{F}{k} = \frac{40}{300} = 0.133$ $\therefore w = \frac{1}{2}kx^2 = \frac{1}{2} \times 300 \times 0.133^2 J = 2.67$ J (Ans.)

08. A proton collides with a neutron (mass almost identical to the proton) to form a deuteron. What will be the velocity of the deuteron if it is formed from a proton moving with velocity 7.0×10^6 m/s to the left and a neutron moving with velocity 4.0×10^6 m/s to the right? (Proton mass = 1.67×10^{-27} kg).

(a) 1.5×10^6 m/s towards left   (b) 15×10^6 m/s upward

(c) 15×10^5 m/s downward   (d) -1.5×10^6 m/s towards left

Solution: (a) ; $m_1 v_1 - m_2 v_2 = (m_1 + m_2)v \Rightarrow v = 1.5 \times 10^6 \, ms^{-1}$ to left

09. To what volume must a liter of oxygen be expanded if the molecules per unit volume is $12.0 \times 10^{11} cm^{-3}$? Diameter of the oxygen molecule = 3Å. Assume that the gas starts at STP. ($R = 8.3 \times 10^7$ dynes. cm. mole^{-1}K^{-1}, $N_0 = 6.02 \times 10^{23}$ mole^{-1}).

(a) $22.41 \times 10^{10} cm^3$ (b) $2.241 \times 10^{10} cm^3$ (c) $2.241 \times 10^{10} m^3$ (d) $4.482 \times 10^{10} cm^3$

Solution: (No correct answer) ; $10^3 \times 12 \times 10^{11} = \dfrac{V}{\frac{4}{3}\pi\left(\frac{3}{2}\times 10^{-2}\right)^3} \Rightarrow V = 1.696 \times 10^{10} cm^3$

10. A lens is used to produce a sharp image on a screen. When the right half of the lens is covered with on opaque material, how will image be affected?   [Ans: d]

(i) The right half of the image will disappear.

(ii) The left half of the image will disappear.

(iii) The image size will become approxntately $\frac{1}{2}$ of the original size.

(iv) The image brightness will become approxntately $\frac{1}{2}$ of the original brightness.

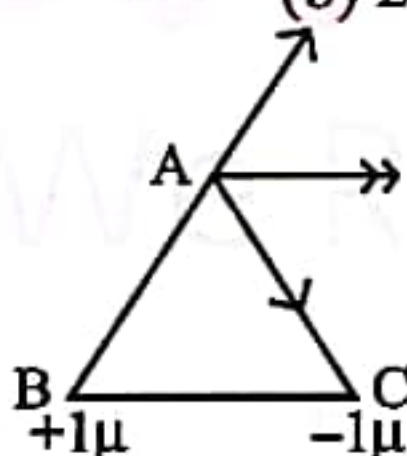(a) i, ii and iii only   (b) i and iii only   (c) ii and iv only   (d) iv only

11. Equal forces $\vec{F}$ act on isolated bodies P and Q. The mass of Q is three times that of P. The magnitude of acceleration of P is:

(a) Three times that of Q (b) 1/3 that of Q   (c) The same as Q   (d) 1/9 that of Q

Solution: (a) ; $F = ma \therefore m \propto \dfrac{1}{a}$

12. With two slits spaced d = 0.2mm apart, and a screen at a distance of D = 1.0 m, the third bright fringe is found to be displaced x = 7.5 mm from the central fringe. What is the wavelength, λ of the light used?
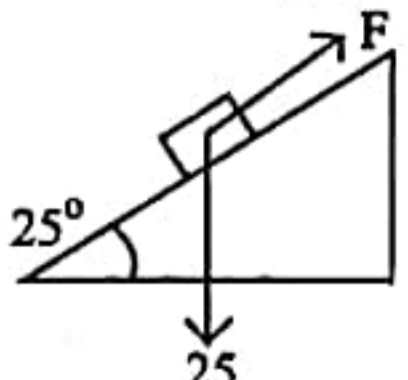
(a) 500 Å   (b) 500 nm   (c) 100 nm   (d) 100Å

Solution: (b) ; $x_n = \dfrac{n\lambda D}{d} \Rightarrow \lambda = \dfrac{x_n d}{nD} = \dfrac{7.5\times10^{-3}\times0.2\times10^{-3}}{3\times1} = 500 \times 10^{-9} m = 500nm$ (Ans.)

13. Two charges of $+1\,\mu C$ and $-1\mu C$ are placed at the corners of the base of an equilateral triangle. The length of a side of the triangle is 0.7 m. Find the electric field intensity at the apex of the triangle? [$K = 9 \times 10^9 Nm^2/c^2$]

(a) 36.8 kN   (b) 28.4 kN   (c) 18.4 kN   (d) 18.4 N

Solution: (d) ; For, $+1\,\mu C$; $E_1 = \dfrac{kq_1}{d^2} = \dfrac{9\times10^9\times1\times10^{-9}}{0.7^2} = 18.36$

So, $E^2 = E_1^2 + E_2^2 + 2E_1 E_2 \times \cos\theta$

$= 18.36^2 + 18.36^2 + 2 \times 18.36 \times 18.36 \times \cos120°J \Rightarrow E = 18.36N$ (Ans.)

14. A 25N check please is held at rest on a frictionless incline by a force that is parallel to the incline. If the incline is 25° above the horizontal the magnitude of the applied force is:

(a) 4.1N   (b) 4.6N   (c) 8.9N   (d) 11N

Solution: (d) ; $W\sin\theta = F \Rightarrow F = 25\sin25° = 10.56 \approx 11$ (Ans.)

15. A horizontal shove of at least 200N is required to start moving a 800N object initially at rest on a horizontal floor. The coefficient of static friction is:

(a) 0.25   (b) 0.125   (c) 0.50   (d) None of these

Solution: (a) ; $F = kR \Rightarrow k = \frac{F}{R} = \frac{200}{800} = 0.25$ (Ans.)

16. A tuning fork vibrating gently produces the note C. Another tuning fork produces the note C, an octave higher in pitch than C and slightly louder than C. The speed of the wave produced by the second fork, as compared with that produced by the first fork, is:

(a) 8 times as great   (b) 4 times as great   (c) 2 times as great   (d) the same

Solution: (d) ; Same medium, so speed will remain the same.

17. The weight of an object on the Moon is one sixth of its weight on Earth. The ratio of the kinetic energy of a body on Earth moving with speed V to that of same body moving with speed V on the Moon is: [Ans: c]

(a) 6:1   (b) 1:6   (c) 1:1   (d) 36:1

18. An electron is released from the rest at one point in a uniform electric field and moves a distance of 10 cm in 10^{-1}s. What is the voltage between the two points?

(a) 11.375 mV   (b) 1.1375 kV   (c) 11375 V   (d) 11.375 V

Solution: (a) ; $s = \frac{1}{2}at^2 \Rightarrow a = \frac{2s}{t^2} = 20ms^{-2}$

$E = \frac{ma}{q} = 1.1375 \times 10^{-10}$; $V = Ed = 1.1375 \times 10^{-11}V$ (Ans.)

19. Three devices are connected in parallel to a 12 V battery. Let the resistance of the devices be $R_1 = 2\Omega, R_2 = 3\Omega$, and $R_3 = 4\Omega$. What is the supplied current by the battery?

(a) 13 A   (b) 13 mA   (c) 26 A   (d) 2.6 A

Solution: (a) ; $R_{eq} = (2^{-1} + 3^{-1} + 4^{-1})^{-1} = \frac{12}{13}$ $\therefore I = \frac{12}{12/13} = 13A$ (Ans.)

20. In the figure, $C_1 = 6\mu F, C_2 = 3\mu F$ and $V_{ab} = 18V$. What is the value of equivalent capacitance?
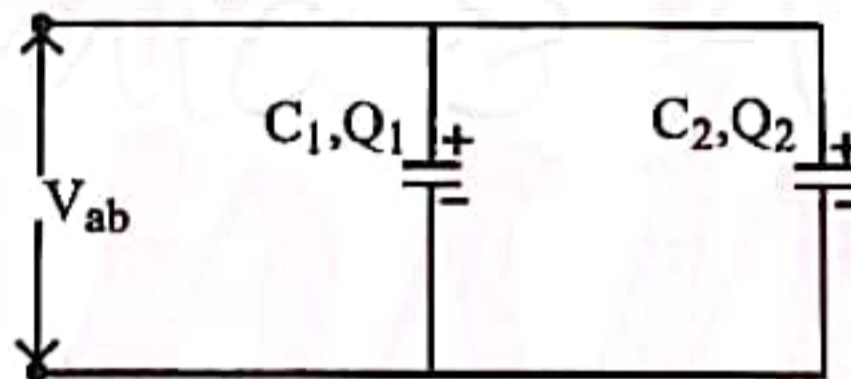
(a) 9 mF   (b) 9 μF   (c) 0.9 mF   (d) 9 F

Solution: (b) ; $C_{eq} = C_1 + C_2 = (6 + 3)\mu F = 9\mu F$ (Ans.)

21. How many nuclei are there in 1 kg aluminum?

(Atomic mass of aluminum = 26.98153 amu, 1 amu =1.66×10^{-27}kg)

(a) 2.23×10^{26}nuclei   (b) 2.23×10^{25}nuclei   (c) 4.46×10^{25}nuclei   (d) 4.46×10^{20}nuclei

Solution: (b); Mass of 1 atom is $26.981 \times 1.66 \times 10^{-27}$ kg

$\therefore$ In $26.981 \times 1.66 \times 10^{-7}$kg there is 1 nucleus

$\therefore$ In 1 kg $\frac{1}{26.981 \times 1.66 \times 10^{-27}} = 2.23 \times 10^{25}$muclei (Ans.)

22. A photon of wave length $\lambda = 0.400$ nm strikes an electron at rest and rebounds at an angle of $150°$ to its original direction. Find the wavelength of the photon after the collision. ($h = 6.63 \times 10^{-34}J - s$)

(a) 40.45 nm   (b) 404.5 cm   (c) 0.4045 nm   (d) 4.045 km

Solution: (c) ; $\lambda' = \frac{h}{m_o c}(1 - \cos\theta) + \lambda_0 = \frac{h}{m_o c}(1 - \cos150°) + 0.4 \times 10^{-9}$

$= 0.4045 \times 10^{-9}m = 0.4045nm$ (Ans.)

23. Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I, respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. The ratio of x_1/x_2 is-

   (a) 2   (b) 1/2   (c) $\sqrt{2}$ (d) $1/\sqrt{2}$

Solution: (c) ; $\frac{1}{2}kx^2 = \frac{1}{2}I\omega^2$; $\frac{x_1}{x_2} = \sqrt{\frac{I_1\omega_1^2}{I_2\omega_2^2}} = \sqrt{2}$

24. The pressure exerted on the ground by a man is greatest when-   [Ans: c]
   (a) He stands with both feet flat on the ground   (b) He stands flat on one foot
   (c) He stands on the toes of one foot   (d) He lies down on the ground.

25. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is-

   (a) -4 (b) 4   (c) $4\sqrt{2}$ (d) 8

Solution: (c) ; $x = 4(\cos\pi t + \sin\pi t) = 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\pi t + \frac{1}{\sqrt{2}}\sin\pi t\right)$

$= 4\sqrt{2}\left(\sin\frac{\pi}{4}\cos\pi t + \cos\frac{\pi}{4}\sin\pi t\right) = 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$ ∴Amplitude $4\sqrt{2}$ (Ans.)

26. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

   (a) 5%   (b) 20%   (c) Zero   (d) 0.5%

Solution: (b) ; $f' = \frac{v+v_0}{v} \times f = \frac{v+\frac{v}{5}}{v} \times f$; $f' = 1.2 \times f$

∴ increase $= \frac{f'-f}{f} \times 100\% = \frac{1.2f-f}{f} \times 100\% = 20\%$ (Ans.)

27. Two closed organ pipe sounded simultaneously give 5 beats per second between the fundamentals. If the shorter pipe is 1.1 m, find the length L_L of the longer pipe. (Speed of sound in air = 340 m/s)

   (a) 1.18 m   (b) 1.18 cm   (c) 11.8 cm   (d) 11.8 m

Solution: (a) ; $\lambda_s = 4 \times 1.1m = 4.4m$ ∴ $f_s = \frac{340}{4.4} = 77.27$ ∴ $L_L = \frac{3Lv}{4\times(f_s-5)\times4} = \frac{340}{4\times(77.27-5)} = 1.18m$

28. A real gas is changed slowly state 1 to state 2. During this process no work is done on or by the gas. This process must be-   [Ans: c]
   (a) Isothermal   (b) Adiabatic   (c) Isovolumic   (d) Isobaric

29. A Carnot heat engine operate between a hot reservoir at absolute temperature T_H and a cold reservoir at absolute temperature T_C. Its efficiency is-   [Ans: d]

   (a) $\frac{T_H}{T_C}$ (b) $\frac{T_C}{T_H}$ (c) $1 - \frac{T_H}{T_C}$ (d) $1 - \frac{T_C}{T_H}$

30. A small object has charge Q. Charge q is removed from it and placed on a second small object. The two objects are placed 1 m apart. For the force that each object exerts on the other to be a maximum, q should be-

   (a) 2Q   (b) Q   (c) Q/2   (d) Q/4

Solution: (c) ; $F = \frac{k(Q-q)\times q}{1} \Rightarrow F = k[Qq - q^2]$

for F_{max} ; $\frac{dF}{dq} = 0 \Rightarrow \frac{d}{dq}\{k(Qq - q^2)\} = 0 \Rightarrow k(Q - 2q) = 0 \Rightarrow Q = 2q \Rightarrow q = \frac{Q}{2}$ (Ans.)

31. A physics instructor is anteroom charges an electrostatic generator to 25 μC, then carries it into the lecture hall. The net electric flux in $N.m^2/C$ through the lecture hall wall is-

   (a) 0   (b) 25×10^{-6} (c) 2.2×10^5 (d) 2.8×10^6

Solution: (d) ; $\varphi = \frac{c}{\epsilon_o} = \frac{25\times10^{-6}}{8.85\times10^{-12}} = 2.8 \times 10^6$

32. A certain farsighted person has a minimum distance of distinct vision of 150 cm. He wishes to read clearly at distance of 25cm, what focal length glasses should he used and what is the type of the lens?

   (a) 300 m   (b) 30 m   (c) 30 cm   (d) 3.0 cm

Solution: (c) ; $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow \frac{1}{25}+\frac{1}{-150}=\frac{1}{f} \Rightarrow f=30cm$

33. An observer is moving with a velocity of 0.95C in a direction perpendicular to a rod of length L. The observer will measure the length of the rod to be-   [Ans: a]

   (a) Equal to L   (b) Less than L   (c) Greater than L   (d) Zero

34. Which of the following electromagnetic has photons with the greatest energy?   [Ans: d]

   (a) Blue light   (b) Yellow light   (c) Radio waves   (d) X-rays

35. In Compton scattering from stationary electrons the largest charge in wave length occurs when the photon is scattered through   [Ans: d]

   (a) 0°   (b) 90°   (c) 45°   (d) 180°

Chemistry

36. The half-life period for a first order reaction is 15 hours. How much reactant will remain unreacted at the end of 50 hours?

   (a) $\frac{3}{5}$ (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{1}{20}$

Solution: (b) ; $N = N_0 e^{-\lambda t} \Rightarrow N = N_0 e^{-\frac{\ln 2}{T_{1/2}} \times t} \Rightarrow \frac{N}{N_0} = e^{-\frac{\ln 2}{15} \times 50} \Rightarrow \frac{N}{N_0} = 0.099 \approx 0.1 = \frac{1}{10}$

37. Which is an example of Isoelectron?   [Ans: a]

   (a) $^{27}_{13}Al^{3+}, ^{16}_{8}O^{2-}$ (b) $^{14}_{7}N, ^{15}_{8}O^{2-}$ (c) $^{32}_{15}P, ^{32}_{16}S$ (d) $^{131}_{54}Xe, ^{59}_{27}Co$

38. Which is the strongest oxidizing agent of the following?   [Ans: a]

   (a) $K_2Cr_2O_7$ (b) $CuSO_4$ (c) $KMnO_4$ (d) $Fe_2(SO_4)_3$

39. Which of the following statement is not true?   [Ans: c]

   (a) Increase of atomic size decreases electron affinity.

   (b) Increase of nuclear charge increases electron affinity.

   (c) Increase of electron density in valence shell increases electron affinity.

   (d) Increase of suborbit increases ionisation energy.

40. In IR spectra, the stretching frequency of carboxylic -OH group arise at-   [Ans: a]

   (a) $3300 - 2500cm^{-1}$ (b) $1760 - 1690cm^{-1}$ (c) $1320 - 1210cm^{-1}$ (d) $1440 - 1395cm^{-1}$

41. At 11°C temperature and 749 mm pressure, the weight of a gaseous hydrocarbon is 0.11gm. If in that compound, there is 85.71% Carbon, then what is its molecular formula? (Volume =46.43 mL; molecular weight = 56).   [Ans: b]

   (a) CH_4 (b) C_4H_8 (c) C_2H_6 (d) CH_2

42. Which is incorrect?   [Ans: b]

   (a) Size of anion increase $F^- < Cl^- < Br^- < I^-$.

   (b) Covalent character increase $AgF < AgCl < AgBr < AgI$.

   (c) Ionic character Increase $AgF < AgCl < AgBr < AgI$.

   (d) Solubility decrease $AgF > AgCl > AgBr > AgI$.

43. The Lewis structure for the compound series ketone is-   [Ans: b]
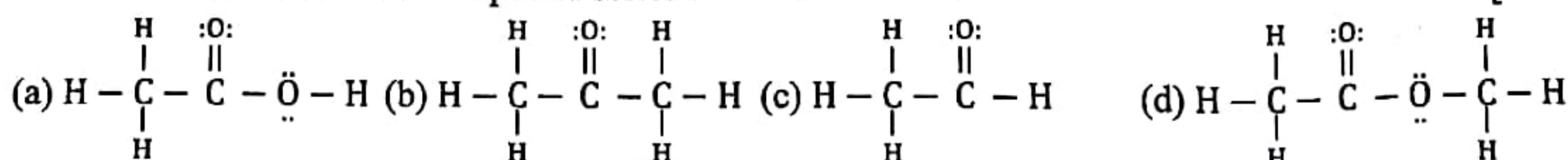
44. What is the formation enthalpy of sugar? If the combustion enthalpy of carbon, hydrogen, and sugar are $-406kJ, -284kJ$, and -5638.82 kJ, respectively. [Ans: c]
    (a) -4040.82 kJ/mol   (b) 6936.82 kJ/mol   (c) -2207.18 kJ/mol   (d) 1498.82 kJ/mol

45. Butane molecule is converted to 2-methyl propane by Isomerisation in the presence of the following environment. [Ans: d]
    (a) $AlCl_3, H_2SO_4, 300°C$. (b) $Al_2O_3, HCl, 150°C$.   (c) $Al_2O_3, H_2SO_4, 150°C$ (d) $AlCl_3, HCl, 300°C$.

46. Concentrated H_2O_2 solution is used as oxidant in rocket fuel and diluted H_2O_2 solution is used as hair bleach. A 30% H_2O_2 (w/w) aqueous solution has density 1.11 gm/mL. What is the molarity and mole fraction of the H_2O_2 solution? [Ans: b]
    (a) 7.97M, 0.185   (b) 9.79M, 0.158   (c) 9.79 M, 0.185   (d) 7.97 M, 0.158

47. Temporary hardness of water is caused due to the presence of which compounds in water? [Ans: a]
    (a) $CaCO_3$ (b) $CaCl_2$ and $MgCl_2$
    (c) $Ca(HCO_3)_2$ and $Mg(HCO_3)_2$ (d) Na_2CO_3

48. All of the following are the true statements concerning reaction orders except: [Ans: b]
    (a) The rate of a zero-order reaction is constant.
    (b) After three half-lives, a radioactive sample will have one-ninth of its original concentration.
    (c) The unit for the rate constant for first order reactions are s^{-1}
    (d) If doubling the concentration of a reactant doubles the rate of the reaction, then the reaction is first order in that reactant.

49. An exothermic reaction is at equilibrium. If temperature is increased, which of the following will take plae? [Ans: b]
    (a) The value of 'K' will increase.   (b) The value of 'K' will decrease.
    (c) The value of 'K' will not change.   (d) None of them.

50. Which one of the following is used to form a salt bridge in electrochemical cell? [Ans: a]
    (a) KNO_3 (b) HNO_3 (c) $AgNO_3$ (d) H_2SO_4

Mathematics

51. If $i^2 = -1$, then $\sqrt{(8 + 6i)}$ =?
    (a) $(3 + i)$ (b) $-(3 + i)$ (c) $\pm(3 + i)$ (d) $(3-i)$
    Solution: (c) ; Take the squares of the options and match them with the question.

52. If $\begin{bmatrix} a & b \\ a_1 & b_1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, then the values of a and b are?
    (a) a = 1 and b = 2   (b) a = 2 and b = 3   (c) a = 2 and b = 5   (d) a = 3 and b = 8
    Solution: (a) ; $\begin{bmatrix} a & b \\ a_1 & b_1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $3a - b = 1 ... (i)$ $-5a + 2b = -1 .... (ii)$
    (i) & (ii) $\Rightarrow$ a = 1 & b = 2

53. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and $B = A^{-1}$, then b_{23} =?
    (a) 1   (b) 2   (c) -1   (d) -2
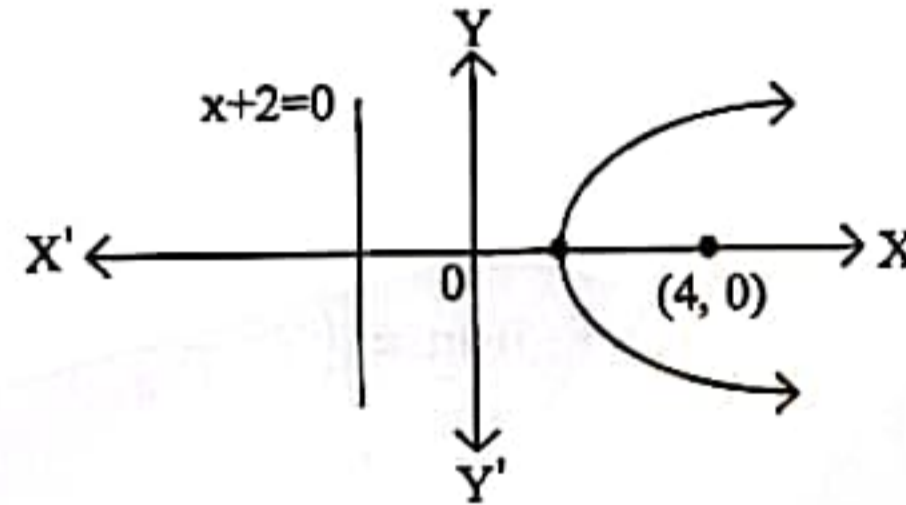   **Solution: (c)** ; Use calculator.

54. What is perimeter of $\triangle ABC$?
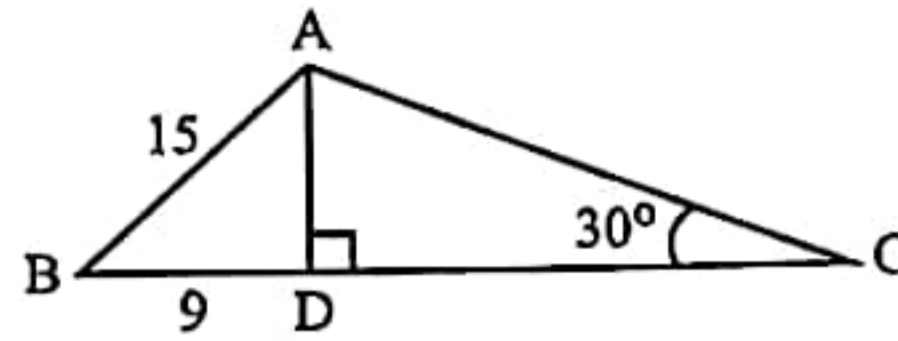
(a) 48 (b) $48 + 12\sqrt{2}$ (c) $60 + 6\sqrt{3}$ (d) $48 + 12\sqrt{3}$

Solution: (d) ; $AD = \sqrt{15^2 - 9^2} = 12$; $DC = AD\cot 30° = 12\sqrt{3}$

$AC = DC\sec 30° = 24$ ∴ $AB + BC + AC = 48 + 12\sqrt{3}$

55. From the figure below, equation of the parabola is-

(a) $y^2 = 4(x - 1)$ (b) $y^2 = 6(x - 2)$ (c) $y^2 = 4(x - 3)$ (d) $y^2 = 12(x - 1)$

Solution: (a) ; From fig. we find vertex $= \left(\frac{4+(-2)}{2}, 0\right) = (1, 0)$ ∴ eqⁿ $(y - 0)^2 = 4(x - 1)$ (Ans.)

56. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$

(a) $\frac{3}{11}$ (b) $\frac{5}{12}$ (c) $\frac{7}{11}$ (d) $\frac{7}{12}$

Solution: (b) ; $2(6 - r) - r = 0 \Rightarrow r = 4$ ∴ term $= {}^6C_4 \left(\frac{3}{2}\right)^2 \times (x^2)^2 \times \left(-\frac{1}{3x}\right)^4 = \frac{5}{12}$ (Ans.)

57. $\int x^x(1 + \ln x)dx = ?$

(a) $x^x + c$ (b) $x^{2x} + c$ (c) $x^{2x}\ln x + c$ (d) $\frac{1+\ln x}{x} + c$

Solution: (a) ; Let $x^x = z$; $x\ln x = \ln z \Rightarrow (1 + \ln x)dx = \frac{dz}{z}$

Now $\int x^x(1 + \ln x)dx \Rightarrow \int z\frac{dz}{z} = \int dz = z + c = x^x + c$ (Ans.)

58. $\lim\limits_{x\to 0} \frac{x}{\sqrt{1-\cos x}} = ?$

(a) 2 (b) $\frac{1}{4}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

Solution: (c) ; $\frac{0}{0}$ form

∴ $\lim\limits_{x\to 0} \frac{x}{\sqrt{1-\cos x}} = \lim\limits_{x\to 0} \frac{x}{\sqrt{2}\sin\frac{x}{2}} = \lim\limits_{x\to 0} \frac{1}{\sqrt{2}\cdot\frac{1}{2}\cos\frac{x}{2}}$ [L'Hospital] $= \sqrt{2}$

59. For what values of k, $3x - 4y = k$ will touch $x^2 + y^2 - 8x = 0$? [Ans: a]

(a) $-8, 32$ (b) $-32, 8$ (c) $8, 32$ (d) $81, 3$

Solution: (a) ; Center $(4,0)$, Radius $= 4$

∴ $\left|\frac{3\times4-k}{5}\right| = 4 \Rightarrow 12 - k = \pm 20 \Rightarrow k = -8, 32$

60. $\begin{vmatrix} 1+x & x & 2 \\ 3 & 2 & 1 \\ 2 & 5 & 7 \end{vmatrix} = 0$, x = ?

(a) $\frac{5}{2}$ (b) $-\frac{31}{10}$ (c) $\frac{3}{37}$ (d) None
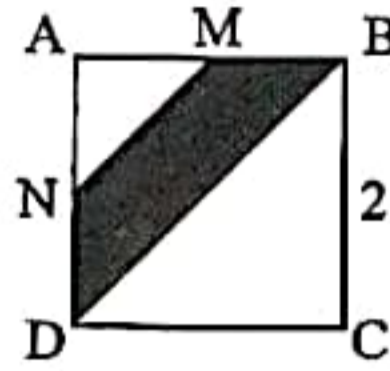
Solution: (No correct answer); Ans: $\frac{31}{10}$

পরিবর্তনের প্রত্যয়ে নিরন্তর পথচলা....

61. $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$ [Ans: d]
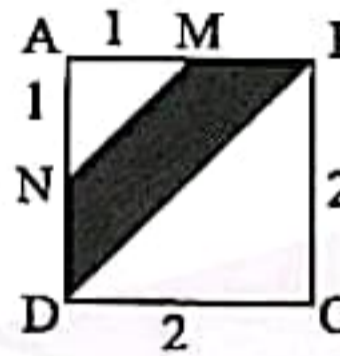
    (a) 1 (b) ω (c) ω^2 (d) 0

62. In the following figure, M and N are the midpoints of two of the sides of square ABCD. What is the area of the shaded region? [Ans: a]

    (a) 1.5 (b) 1.75 (c) 3.0 (d) $2\sqrt{2}$

    Solution: (a); $\therefore$ Area of shaded region $= \left(2^2 - \frac{1}{2} \times 2 \times 2 - \frac{1}{2} \times 1 \times 1\right) = 1.5$

63. Find the sum to infinity of the following series, $\frac{3}{(2)(4)} - \frac{5}{(4)(6)} + \frac{7}{(6)(8)} - \cdots$

    (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

    Solution: (a); General term of the series $= (-1)^{n+1}\frac{2n+1}{2n(2n+2)}$; $n \in \mathbb{N}$

    $\therefore$ Sum $= \int_1^\infty \frac{2n+1}{2n(2n+2)}dn = \frac{1}{4}\int_1^\infty \frac{2n+1}{n^2+n}dn = \frac{1}{4}[\ln(n^2+n)]_1^\infty = \frac{1}{4}[\ln(n) + \ln(n+1)]_1^\infty$

    $\lim_{x\to\infty}\ln(n) = 0; \lim_{x\to\infty}\ln(n+1) = 0$

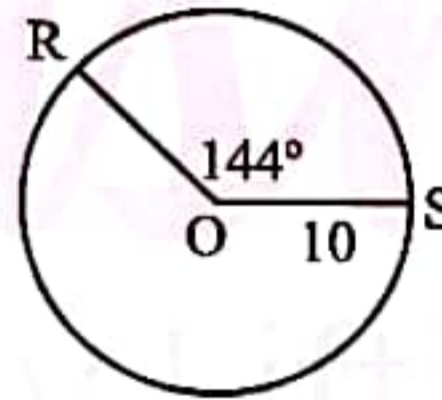64. $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2(x - \pi/3) = ?$ [Ans: b]

    (a) $-\frac{3}{4}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{4}\text{co}3x$ (d) $-\frac{3}{2}\cos 3x$

65. In the following circle, what is the length of arc RS? [Ans: c]

    (a) 8 (b) 20 (c) 8π (d) 20π

66. Find the area of the triangle whose vertices are the origin and focii of the ellipse $16(x-2)^2 + 25(y-3)^2 = 400$.

    (a) 9 sq. units (b) 12 sq. units (c) 16 sq. units (d) 10 sq. units

    Solution: (a); $16(x-2)^2 + 25(y-3)^2 = 400 \Rightarrow \frac{(x-2)^2}{5^2} + \frac{(y-3)^2}{4^2} = 1$

    $\therefore$ focus: $(5,3); (-1,3)$ and origin $(0,0)$

    $\therefore$ Area $= \frac{1}{2}\begin{vmatrix} 5 & 3 & 1 \\ -1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 9$ sq. unit (Ans.)

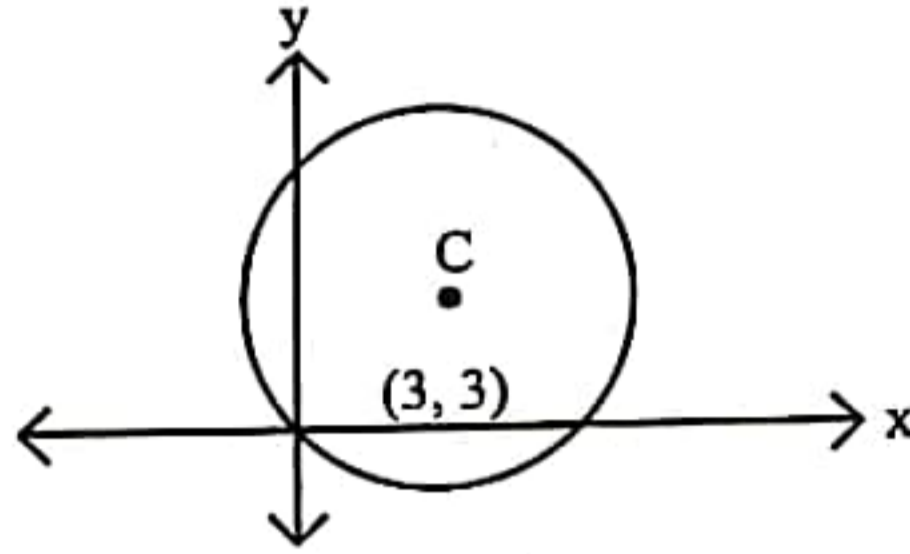67. $0.5+0.05+0.005+\ldots$ The sum of the series up to infinity is- [Ans: d]

    (a) $\frac{1}{3}$ (b) $\frac{7}{9}$ (c) $\frac{1}{9}$ (d) $\frac{5}{9}$

68. There are 10 blue and 15 red marbles in a box. A boy picks up two marbles at the random one by one. The probability of both being of the same color is-

(a) $\frac{1}{2}$ (b) $\frac{4}{5}$ (c) $\frac{3}{20}$ (d) $\frac{7}{20}$

Solution: (a); Probability $= \frac{^{10}C_2 + ^{15}C_2}{^{25}C_2} = \frac{1}{2}$

69. In the following circle, the area is $K\pi$, what is the value of K?

(a) 3   (b) 6   (c) 9   (d) 18

Solution: (d) ; $r = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ $\therefore K\pi = 18\pi \Rightarrow K = 18$

70. If $y + x = x^{-y}$, find $\frac{dy}{dx}$

(a) $\frac{x^2 + 2y}{2x + \log x}$ (b) $\frac{x + 2y + 5}{(x+y) + \log x}$ (c) $\frac{xy + y^2 + 1}{x + \log x}$ (d) $-\frac{xy + x + y^2}{x[1 + (x+y)\log x]}$

Solution: (d) ; $\ln(x + y) = -y\ln y \Rightarrow \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = \frac{-y}{x} - \ln x \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{x+y} + \ln x\right) = \frac{-y}{x} - \frac{1}{x+y} \Rightarrow \frac{dy}{dx} = \frac{-(xy + x + y^2)}{\{1 + (x+y)\ln x\}}$ (Ans.)

71. There are 100 people on a line. Shawon is the 37th person and Naomi is the 67th person. If a person on line is chosen at random, what is the probability that the person is standing between Shawon and Naomi?

(a) $\frac{1}{100}$ (b) $\frac{29}{100}$ (c) $\frac{3}{10}$ (d) $\frac{31}{100}$ [Ans: b]

72. A equation of a straight line having slope m and x-intercept b is-

(a) $y = m(x - b)$ (b) $x = my + b$ (c) $y = mx + b$ (d) $y = mx - b$

Solution: (a) ; Let st. line be $y = mx + c$

$\Rightarrow mx = y - c \Rightarrow x = \frac{y}{m} - \frac{c}{m}$ but $\frac{c}{m} = b \Rightarrow c = bm$ $\therefore y = mx - bm \Rightarrow y = m(x - b)$ (Ans.)

73. If two roots of the equation $x^2 + bx + a = 0$, are equal and one root of the equation $x^2 + ax + 8 = 0$ is 4, the value of b will be-

(a) 4   (b) $2\sqrt{6}i$ (c) $1 - i$ (d) $1 + i$

Solution: (b) ; The other roots of $x^2 + ax + 8 = 0$ is $\frac{8}{4} = 2$ $\therefore a = -(4 + 2) = -6$

Now, $b^2 - 4a = 0 \Rightarrow b^2 = 4a \Rightarrow b^2 = -24 \Rightarrow b = 2\sqrt{6}i$ (Ans.)

74. A circular Garden is bounded by East Avenue and Diagonal Road as shown in the figure given below. Diagonal Road intersects North Street at C and East Avenue at D. Diagonal Road is tangential to the Circular Garden at B. Find the equation of the diagonal Road.
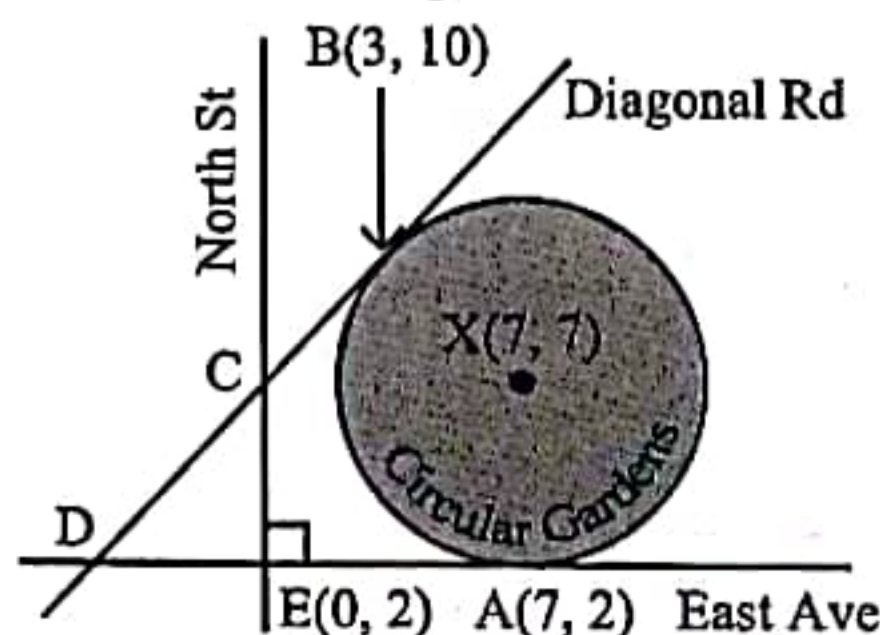
(a) $3x - 4y + 18 = 0$ (b) $x - 3y + 18 = 0$ (c) $7x - 10y + 18 = 0$ (d) $10x - 7y + 18 = 0$

Solution: (No correct answer); Radius $= \sqrt{(7-3)^2 + (7-10)^2} = 5$

$\therefore$ equation of circle $= (x-7)^2 + (y-7)^2 = 25$

$\therefore 2x - 14 + (2y - 14)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{14-2x}{2y-14} \therefore \frac{dy}{dx}\Big|_{(3,10)} = \frac{4}{3}$

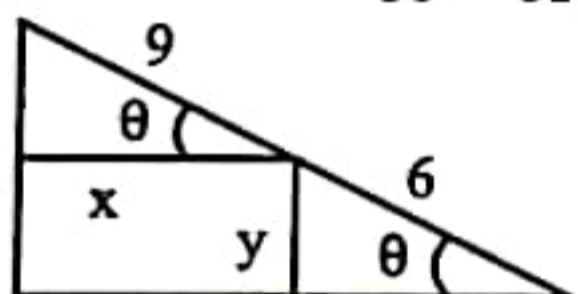$\therefore$ Equation of diagonal road $= y - 10 = \frac{4}{3}(x-3) \Rightarrow 4x - 3y + 18 = 0$

75. When $x = 1$, the function $x^3 - 3x^2 + 7 = 0$ is:

(a) Increasing   (b) Maximum   (c) Decreasing   (d) Minimum

Solution: (c) ; at $x = 1$ $\frac{dy}{dx} = -3 < 0$ $\therefore$ decreesing

76. A rod AB of length 15cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point P(x, y) is taken on the rod in such a way that AP = 6cm. If the rod moves with its ends always touching the coordinate axes, find the equation of the locus of the point P.
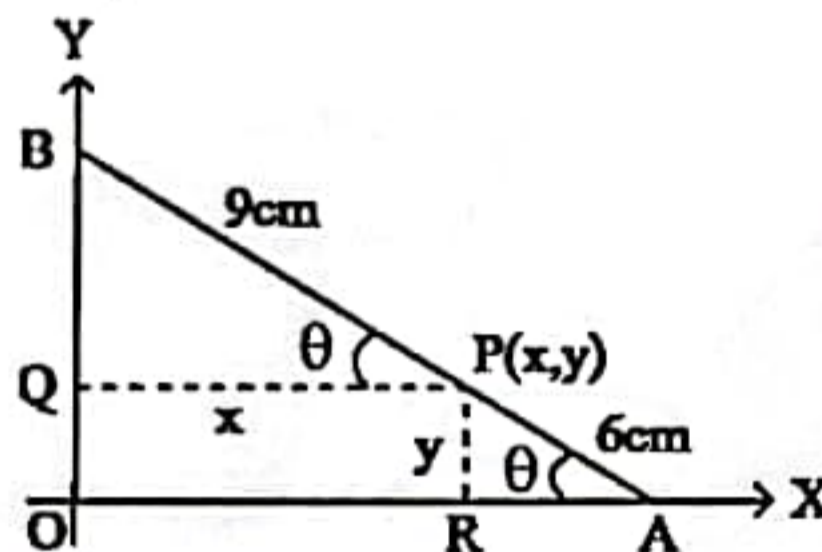
(a) $\frac{x^2}{81} + \frac{y^2}{36} = 1$ (b) $\frac{x^2}{36} + \frac{y^2}{81} = 1$ (c) $\frac{x^2}{36} + \frac{y^2}{9} = 1$ (d) $\frac{x^2}{9} + \frac{y^2}{36} = 1$

Solution: (b) ;

$y = 6\sin\theta \dots$ (i); $x = 9\cos\theta \dots$ (ii)

$\frac{(i)^2}{6} + \frac{(ii)^2}{9} \Rightarrow \frac{x^2}{81} + \frac{y^2}{36} = 1$ (Ans.)

77. $\int \frac{dx}{x + \sqrt{x}} = ?$

(a) $\ln(\sqrt{x} + 1) + c$ (b) $\tan^{-1}(\sqrt{x} + 1) + c$ (c) $2\ln(\sqrt{x} + 1) + c$ (d) $2\tan^{-1}(\sqrt{x} + 1) + c$

Solution: (c) ; Let $1 + \sqrt{x} = z \Rightarrow \frac{1}{2\sqrt{x}}dx = dz \Rightarrow \frac{1}{\sqrt{x}}dx = 2dz$

Now, $\int \frac{dx}{x + \sqrt{x}} = \int \frac{dx}{\sqrt{x}(1 + \sqrt{x})} = \int \frac{dx}{\sqrt{x}} \cdot \frac{1}{1 + \sqrt{x}} = 2\int \frac{dz}{z} = 2\ln z + c = 2\ln(\sqrt{x} + 1) + c$ (Ans.)

78. If $y = \sec^2(\tan^{-1} x)$, then $\frac{dy}{dx} = ?$

(a) $\frac{x}{1 + x^2}$ (b) $\frac{x}{\sqrt{1 + x^2}}$ (c) x (d) $2x$

Solution: (d) ; $y = \sec^2(\tan^{-1} x) \Rightarrow y = 1 + \{\tan(\tan^{-1} x)\}^2 \Rightarrow y = 1 + x^2$; $\frac{dy}{dx} = 2x$ (Ans.)

79. The figure given below shows triangle AOC inscribed in the region cut from the parabola $y = x^2$ by the line $y = a^2$. Find the limit of the ratio if the area of the triangle to the area of the parabolic region as a approaches zero.
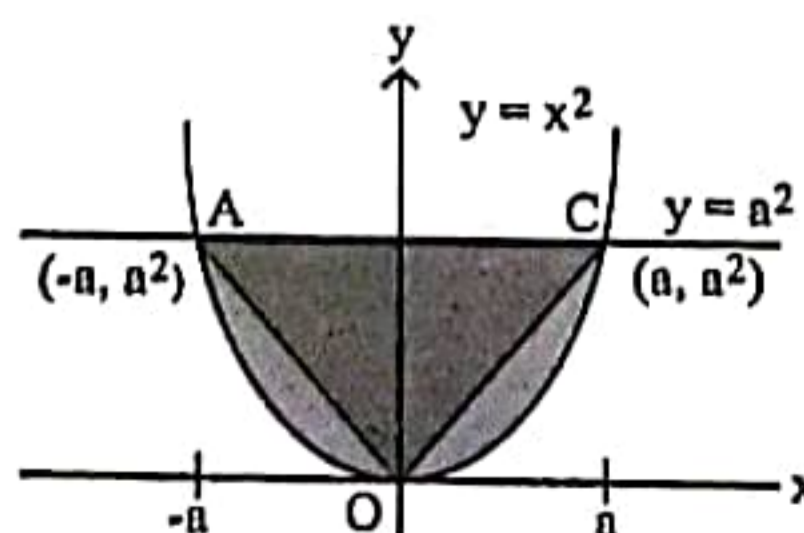
(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{5}{6}$ (d) $\frac{6}{7}$

Solution: (a) ; $y = x^2$; $x = \sqrt{y}$

Now, $2\int_0^{a^2} dx = 2\int_0^{a^2} \sqrt{y}\, dy = 2\left[\frac{2}{3} y^{3/2}\right]_0^{a^2} = \frac{4}{3} a^3$

For triangle area $= \frac{1}{2} \times 2a \times a^2 = a^3$ ∴ Ratio $= \frac{a^3}{\frac{4}{3}a^3} = \frac{3}{4}$ (Ans.)

80. A small ball from the ejector device at O is ejected in such a way that it passes through the small aperture at A and strikes the contact point at B as shown in the figure below. The coordinates of A and B are (2, 2) and (3, 1), respectively. If the player controls the angel θ and velocity v, then the value of θ that gives him a success is:
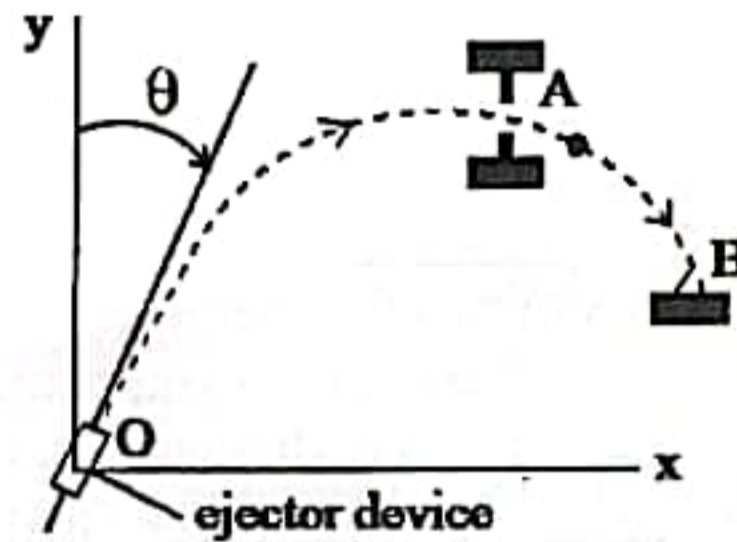
(a) 29.7°   (b) 66.8°   (c) 23.2°   (d) 74.1°

Solution: (c) ; Let, equation of the parabola, $y = ax - bx^2$; $a = \tan\alpha$; $b = \dfrac{g}{2u^2 \cos^2\alpha}$
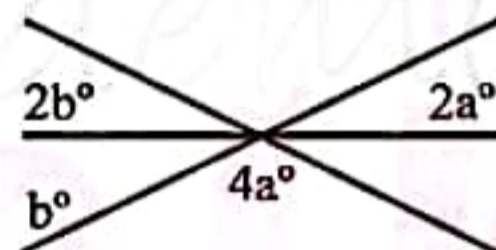
$(2,2) \Rightarrow 2 = 2a - 4b$

$(3,1) \Rightarrow 1 = 3a - 9b$

∴ $a = \frac{7}{3}$, $b = \frac{2}{3}$

∴ $\alpha = \tan^{-1}\frac{7}{3}$ ∴ $\theta = 90 - \tan^{-1}\frac{7}{3} = 23.19° \approx 23.2°$

81. In the figure below, what is the value of b?

(a) 9   (b) 18   (c) 27   (d) 36

Solution: (d) ; $b = 2a \Rightarrow b - 2a = 0$ .... (i)

$4a + 2b + 2a + 4a + 2b + b = 360$ ..... (ii)

(i) & (ii) $\Rightarrow a = 18, b = 36$ (Ans.)

82. What is the total number of terms of $(x + 3y + 5z)^{20}$?

(a) 231   (b) 250   (c) 501   (d) 102

Solution: (a) ; $^{n+r-1}C_{r-1} = {}^{20+3-1}C_{3-1} = 231$

83. For what values of a, roots of $ax^2 + 3x + 4 = 0$ will be complex?   [Ans: c]

(a) $a = \frac{9}{16}$ (b) $a < \frac{9}{16}$ (c) $a > \frac{9}{16}$ (d) $a \geq \frac{9}{16}$

84. What is the minimum value of $x^2 - 2x + 5$?   [Ans: c]

(a) 3   (b) $\frac{11}{4}$ (c) 4   (d) 5

85. For what values of k, roots of $(k - 1)x^2 - (k + 2)x + 4 = 0$ will be real and equal?   [Ans: a]

(a) 2, 10   (b) 12,10   (c) 2, 1   (d) None

English

Question 86-90:

Choose the appropriate word for the blank space to complete the sentence of the following passage:

For the first time, scientists have directly detected gravitational waves, ripples in space-time, in addition to light from the spectacular collision of two neutron stars. This marks the first time that a cosmic event has been viewed in both gravitational waves and light.

The discovery was made using the U.S-based Laser Interferometer Gravitational-Wave Observatory (LIGO); the Europe-based Virgo detector; and some 70 ground and space-based observatories.

Neutron stars are the smallest, densest stars known to exist and are formed when massive stars explode in supernovas. As these neutron stars spiraled together, they emitted gravitational waves that _____ for about 100 seconds; when they collided; a_____ light in the form of gamma rays was emitted and seen on Earth about two seconds after the gravitational waves. In the days and weeks following the smashup, other forms of light or electromagnetic radiation _____ including X-ray, ultraviolet, optical, infrared, and radio waves were detected.

The observations have given astronomers a(n) _____ opportunity to probe a collision of two neutron stars. For example, observations made by the U.S. Gemini Observatory, the European Very Large Telescope, and the Hubble Space Telescope_____ signatures of recently synthesized material, including gold and platinum, solving a decades-long mystery of where about half of all elements heavier then iron are produced.

The LIGO-Virgo results are published today in the journal Physical Review Letters; additional papers from the LIGO and Virgo_____ and the astronomical community have been either submitted or accepted for publication in various journals.

86.	(a) stable	(b) ephemeral	(c) detectable	(d) invisible	[Ans: c]
87.	(a) durable	(b) permanent	(c) visible	(d) temporary	[Ans: d]
88.	(a) unprecedented	(b) enormous	(c) astronomical	(d) significant	[Ans: a]
89.	(a) discover	(b) uncloak	(c) gather	(d) provide	[Ans: b]
90.	(a) dealings	(b) divisions	(c) collaborations	(d) league	[Ans: c]

Question 91-95:

Choose the word or phrase which is most nearly opposite of the meaning of the given word.

91. Turbulence   [Ans: b]
(a) Immunity   (b) Tranquility   (c) Meditation   (d) Coordination

92. Equivocal   [Ans: a]
(a) Clear   (b) Open to many interpretations
(c) Unsure   (d) Indefinite

93. Guile   [Ans: a]
(a) Innocence   (b) Clever   (c) Tricky   (d) Sophisticated

94. Voracious   [Ans: c]
(a) Hungry   (b) Greedy   (c) Satisfied   (d) Starving

95. Acrimony   [Ans: d]
(a) Bitterness   (b) Ill-will   (c) Animosity   (d) Civility

Question 96-100:

Choose the word or phrase which is most nearly similar of the meaning of the given word.

96. Auspicious   [Ans: a]
(a) Lucky   (b) Guileless   (c) Unpromising   (d) Cryptic

97. Vociferous   [Ans: a]
(a) Offensively loud   (b) Satisfied   (c) Hushed   (d) Shy

98. Dogmatic   [Ans: c]
(a) Ambiguous   (b) Willing   (c) Stubborn   (d) Skeptical

99. Poignant   [Ans: a]
(a) Heart touching   (b) Cheerful   (c) Indifferent   (d) Calm

100. Galvanize   [Ans: c]
(a) Discourage   (b) Comfort
(c) Excite   (d) Coat (iron or steal) with a protective layer of copper.