IUT Admission Test 2020-2021

Full Marks: 100 **MCQ** **Time: 2:00**

English: MCQ (15 × 1 = 15)

Read the following passage carefully and pick up the correct answer (Questions 1 to 4).

Newton's surprising success at developing the laws of motion, as well as the development and refinement of other physical laws, led to the idea of scientific determinism. The first expression of this principle was in the beginning of the nineteenth century by Laplace, a French scientist. Laplace argued that if one knew the position and velocity of all the particles in the universe at a given time, the laws of physics would be able to predict the future state of the universe.

Scientific determinism held sway over a great many scientists until the early twentieth century, when the quantum mechanics revolution occurred. Quantum mechanics introduced the world to the idea of the uncertainty principle, which stated that it was impossible to accurately measure both the position and the velocity of a particle at one time. Because Laplace's omniscience could never occur, even in theory, the principle of scientific determinism was thrown into doubt. However, quantum mechanics does allow for a reduced form of scientific determinism. Even though physicists are unable to know precisely where a particle is and what its velocity is, they can determine certain probabilities about its position and velocity. These probabilities are called wave functions. By use of a formula known as the Schrödinger equation, a scientist with the wave function of a particle at a given time can calculate the particle's future wave function. These calculations can give the particle's position or velocity, but not both. Thus, the physicist is in possession of exactly half of the information needed to satisfy Laplace's view of determinism. Unfortunately, under modem physics theories, that is far as any researcher can go in predicting the future.

1. The passage suggests that if scientific determinism were true **[Ans: a]**
   (a) scientists would, in theory, be able to predict the future
   (b) all the particles in the universe would have a measurable position and velocity
   (c) the theory of quantum mechanics would be false
   (d) Schrödinger's equation could be used to calculate any particle's position
2. According to the passage, wave functions **[Ans: c]**
   (a) allow scientists to determine the position and velocity of a particle
   (b) are determined by the Schrödinger equation
   (c) provide a range of possible locations and velocities for a particle
   (d) allow a scientist to calculate the future state of the universe.
3. Which of the following best describes the organization of the passage? **[Ans: c]**
   (a) A paradox is introduced, competing explanations are offered, and a final resolution is reached.
   (b) Two opposing theories are introduced, critiqued, and reconciled.
   (c) An idea is introduced, its validity is questioned, and its application qualified.
   (d) A theory is introduced, its mathematical basis is examined, and it is rejected.

4. Which of the following, if true, would most strengthen the author's conclusion in the passage's final sentence? **[Ans: d]**
(a) Some physics believe quantum mechanics will eventually be discarded in favor of a new theory.
(b) Physics till use Newton's laws of motion to calculate the velocities and positions of planets and stars.
(c) Even if position and velocity of a particle were known, predicting the future would be impossible because there are too many ether variables to calculate.
(d) There is little to no chance that the modem theory of quantum mechanics will be overturned by another theory.

Fill in the blanks with the most appropriate words (Questions 5 to 9)

According to (5.) .... of the big-bang theory, the equations used in big-bang calculation are tested by the science (6.) ....as the ultimate reality of the universe. They say that even after these equations are shown to (7.) .... with observational facts, they are retained by big bangers because of an irrational (8.) .... that the theory must corrects (9.) .... of the facts.

5.	(a) Critics	(b) Plaudits	(c) Advocates	(d) Scholars	**[Ans: a]**
6.	(a) Mediocre	(b) Elites	(c) Geeks	(d) Personnel	**[Ans: a]**
7.	(a) Be discordant	(b) Harmonize	(c) Adhere	(d) Subsidize	**[Ans: a]**
8.	(a) Austere	(b) Tolerance	(c) Acrimony	(d) Prejudice	**[Ans: b]**
9.	(a) Frugal	(b) Mindful	(c) Regardless	(d) Inconsiderate	**[Ans: d]**

10. Select the word with similar denotation **Colossal** **[Ans: b]**
(a) diminutive (b) comic (c) poxy (d) Damage

11. Select the word with similar denotation **Conducive** **[Ans: b]**
(a) adverse (b) opportune (c) inopportune (d) inimical

Questions 12 to 15 are based on the opposite meaning of a given word, choose the best answer

12. **discreet** **[Ans: c]**
(a) prudent (b) continuous (c) rash (d) canny

13. **heinous** **[Ans: d]**
(a) vicious (b) atrocious (c) nefarious (d) laudable

14. **eschew** **[Ans: b]**
(a) abstain (b) embrace (c) abjure (d) abandon

15. **maroon** **[Ans: d]**
(a) strand (b) abandon (c) desert (d) None of these

Chemistry: MCQ (15 × 1 = 15)

Short Syllabus

16. The dimension of an examination hall is 20 m × 10m × 5 m. How many kilograms of air is present in the room? The temperature of air in the room is 30°C and molecular weight of air is 29.
(a) 1166.46 (b) 1661.46 (c) 1266.46 (d) None of these

Solution: (a); $V = (20 \times 10 \times 5)\ m^3 = 1000\ m^3$; $P = 1.01325 \times 10^5$ Pa
$T = 30°C = (30 + 273)\ K = 303\ K$; $M = 29$ g/mole
$\therefore n = \frac{PV}{RT} \Rightarrow W = \frac{MPV}{RT} = \frac{29 \times 1.01325 \times 10^5 \times 1000}{8.314 \times 303} gm = 1.1664 \times 10^6 gm = 1166.46$ kg

17. A Jar test was conducted with alum and it was found that the optimum alum dosage obtained when 50 mL alum solution containing 1.0gm/L is added into 2 L of water. The concentration of alum dosage was:
(a) 25 ppm (b) 2.5 ppm (c) 30 ppm (d) 12.5 ppm

Solution: (a); Density of solution, $\rho = 1g/L$; $V = 50mL \therefore W = V\rho = (50 \times 10^{-3})\ g = 50$ mg
Concentration $= \frac{50}{2} mg/L = 25\ mg/L \equiv 25$ ppm

20. An unknown gas with a volume of 500 mL takes 320 sec to effuse through a thin hole. In the same temperature and pressure, the same volume of chlorine takes 240 sec to effuse. The molecular mass of the unknown gas is:
(a) 110.25 (b) 126.22 (c) 175.21 (d) 150.35

Solution: (b); From Graham's diffusion law: $\frac{r_1}{r_2} = \frac{t_2}{t_1} = \sqrt{\frac{M_2}{M_1}} \Rightarrow M_{unknown} = \left(\frac{t_{unknown}}{t}\right)^2 \times M_{Cl_2}$

$= \left(\frac{320}{240}\right)^2 \times 71\,gm/mol = 126.22\,gm/mol$

21. Which one of the following is false?
(a) Ca and Ca^{+2} have same number of protons
(b) O_2 molecule has two covalent bonds
(c) Fe^{+2} and Fe^{+3} ions have equal number of electrons
(d) Hydrogen can have both positive and negative valency

Solution: (c); $Fe^{2+}(26-2) = [Ar]3d^6 4s^0$; $Fe^{3+}(26-3) = [Ar]3d^5 4s^0$

22. 727°C the equilibrium constant for the reaction $2SO_2(g) + O_2(g) = 2SO_3(g)$ is K_p 3.50 atm^{-1}. If the total pressure in the reaction flask is 1.00 atm, and the partial pressure of O_2 at equilibrium is 0.10 atm, calculate the partial pressure of SO_2.
(a) 0.57 atm (b) 0.33 atm (c) 0.43 atm (d) 0.10 atm

Solution: (a); $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g); K_p = \frac{P_{SO_3}^2}{P_{SO_2}^2 \times P_{O_2}} \Rightarrow 3.5 = \left(\frac{P_{SO_3}}{P_{SO_2}}\right)^2 \times \frac{1}{0.1} \Rightarrow \frac{P_{SO_3}}{P_{SO_2}} = 0.5916$

$P_{SO_3} + P_{SO_2} + P_{O_2} = 1$

$\Rightarrow P_{SO_2} \times 0.5916 + P_{SO_2} + 0.1 = 1 \Rightarrow P_{SO_2}(1.5916) = 0.9 \Rightarrow P_{SO_2} = 0.56568\ atm \approx 0.57\ atm$

24. Calculate the pH of 0.1 M CH_3COOH. The dissociation constant of acetic acid is 1.8×10^{-5}.
(a) 1.80 (b) 2.17 (c) 3.15 (d) 2.87

Solution: (d);
$\Rightarrow [H^+] = \sqrt{K_a \times C} = \sqrt{1.8 \times 10^{-5} \times 0.1}\ M$
$= 1.3416 \times 10^{-3}\ M$

$pH = -\log[H^+]$
$= -\log(1.3416 \times 10^{-3})$
$= 2.8723$ (Ans.)

25. Which is true for aromatic compound?
(a) C-C bond length 1.34 Å (b) C-C bond length 1.53 Å
(c) C-C bond length 1.39 Å (d) C-C bond length 1.34 Å

Solution: (c);

 C − C bond length−0.139 nm; $\geqq C-C \leqq$ C − C bond length −0.154 nm

$\rangle C=C \langle$ C − C bond length−0.134 nm; $-C\equiv C-$ C − C bond length −0.120 nm

27. A mixture is prepared by mixing 1.50 mole Toluene and 3.50 mole Benzene. If the vapor pressure of pure Benzene and Toluene are 40.55 kPa and 20.24 kPa, respectively at 0°C, then the total vapor pressure of the mixing will be:
(a) 42.3 kPa (b) 40.50 kPa (c) 34.46 kPa (d) 35.40 Pa

Solution: (c); $P = 40.55 \times \frac{3.5}{1.5+3.5} + 20.24 \times \frac{1.5}{1.5+3.5}\ kPa = 34.46\ kPa$

28. According to the order of electronegativity, which of the following is correct? **[Ans: d]**
(a) F > Cl > I > Br (b) Br < F < Cl < I (c) I > Br > Cl > F (d) I < Br < Cl < F

29. What type of the reaction of the following? $H-CHO + NaOH = CH_3OH + HCOONa$ **[Ans: c]**
(a) Hexamin reaction (b) Grignard reaction (c) Cannizzaro reaction (d) None of these

Extra Syllabus

18. Water is heated to boiling under a pressure of 1.0 atm. When an electric current of 0.50 A from a 12 V supply is passed for 300 s through a resistance in thermal contact with it, it is found that 0.798 g of water is vaporized. Calculate the molar enthalpy change at the boiling point.
(a) 1.8 kJ (b) 41 $kJmole^{-1}$ (c) 38 $kJmole^{-1}$ (d) 48 $Jmole^{-1}$
Solution: (b); $P = 1\ atm = 1.01325 \times 10^5 Pa$; $m = 0.7989 = 0.798 \times 10^{-3}$ kg
$I = 0.5\ A, V = 12\ V, t = 300\ s$; $m = \frac{0.798}{18}$ mole $= 0.04433$ mole
$\therefore$ Change in molar enthalpy Change $= \frac{VIt}{n} = \frac{12\times0.5\times300}{0.04433}\ Jmol^{-1} = 40.6\ kJmol^{-1} \approx 41\ kJmol^{-1}$

19. ${}^8_3Li$ decays to ${}^8_4Be$. What type of decays is this? **[Ans: b]**
(a) positron emission (b) beta (c) alpha (d) gamma

23. For a certain first order reaction half-life is 100s. How long will it take for the reaction to be completed 75%?
(a) 693 s (b) 200 s (c) 400 s (d) 159 s
Solution: (b); For completing 75% of the reaction, $N = N_o - N_o \times 0.75 = 0.25\ N_o$
$\ln\frac{N_o}{N} = \lambda t \Rightarrow \ln\frac{N_o}{N} = \frac{\ln 2}{T_{\frac{1}{2}}} \Rightarrow \ln\frac{1}{0.25} = \frac{\ln 2}{100} \times t \Rightarrow \ln 4 = \frac{\ln 2}{100} \times t \Rightarrow 2 \times 100 = t \Rightarrow t = 200\ s$

26. In an experiment, 0.774 gm of CO_2 and 0.445 gm of H_2O was found to produce after combustion of 0.50 gm of a fuel. The empirical formula of the fuel is:
(a) $C_8H_{16}O_5$ (b) $C_8H_{23}O_7$ (c) $C_2H_3O_5$ (d) $C_6H_{12}O_7$
Solution: (b); $n_{CO_2} = \frac{0.774}{44}$ mole $= 0.0176$ mole; $n_{H_2O} = \frac{0.445}{18}$ mole $= 0.024722$ mole
$\frac{n_{CO_2}}{n_{H_2O}} = 0.71154239, \frac{n_C}{n_H} = \frac{0.0176}{2\times0.024722} = 0.36$
Option check: for (b) $C_8H_{23}O_7, \frac{n_C}{n_H} = \frac{8}{23} \approx 0.35$, Best answer is (b) $C_8H_{23}O_7$

30. Which one of the following is the Benedict solution: **[Ans: c]**
(a) A Water solution of glucose sodium carbonate and copper sulphate
(b) Water solution of sodium citrate, sodium carbonate and glucose
(c) Water solution of sodium carbonate, copper sulphate and sodium citrate
(d) Water solution of sodium carbonate, copper sulphate and potassium citrate

Physics: MCQ (35 × 1 = 35)

Short Syllabus

31. Length of a simple pendulum $l(100.0 \pm 0.5)$ cm, and time period $T = (2.00 \pm 0.01)$ s. Determine the percentage of error in acceleration due to gravity 'g'.
(a) ±1.5% (b) ±2.0% (c) ±1.05% (d) ±1.75%
Solution: (a); $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow \frac{T^2}{4\pi^2} = \frac{L}{g} \Rightarrow g = \frac{4\pi^2 L}{T^2}; \frac{\Delta g}{g} \times 100\% = \frac{\Delta L}{L} \times 100\% + 2\frac{\Delta T}{T} \times 100\%$
$= \pm\left(\frac{0.5}{100} \times 100\% + 2 \times \frac{0.01}{2} \times 100\%\right) = \pm(0.5\% + 1\%) = \pm1.5\%$

34. The recoil velocity of a gun of mass of 8 kg is 10 ms^{-1} when a bullet of mass of 10 g leaves from the gun. After penetrating 0.3 m inside the target the bullet stops. Calculate the applied resistance of the bullet?
(a) 1.067×10^5N (b) 1.067×10^6N (c) 1.067×10^7N (d) 1.067×10^8N
Solution: (b); $MV + M_bV_b = 0 \Rightarrow V_b = \frac{8\times10}{10\times10^{-3}} ms^{-1} = 8 \times 10^3 ms^{-1} \therefore F_k = \frac{\frac{1}{2}m_b v_b^2}{d} = 1.0667 \times 10^6$ N

35. A tennis ball coming with velocity, $v_1 = 16 ms^{-1}$ is sent back by a racket in the opposite direction with velocity, $v_2 = 20 ms^{-1}$. If the change of kinetic energy of the ball is $\Delta E = 9.25 J$, then calculate the change of momentum of the ball.

(a) 5.626 kg ms^{-1} (b) 6.626 kg ms^{-1} (c) 7.626 kg ms^{-1} (d) 4.626 kg ms^{-1}

Solution: (d); $\frac{1}{2}m(v_f^2 - v_i^2) = \Delta E \Rightarrow \frac{1}{2} m(20^2 - 16^2) = 9.25 \Rightarrow m = 0.12847$ kg

$\therefore P = m\left(v_f - (-v_i)\right) = m\,(v_f + v_i) = 0.12847\,(16 + 20)\ kgms^{-1} = 4.625\ kgms^{-1}$

36. A space craft from the earth is moving towards the moon, find a location from the earth where at the gravitational force is zero. [Mass of the earth = 6×10^{24} kg, Mass of the moon = 7.4×10^{22} kg, distance between the earth and the moon = 3.8×10^8 m]

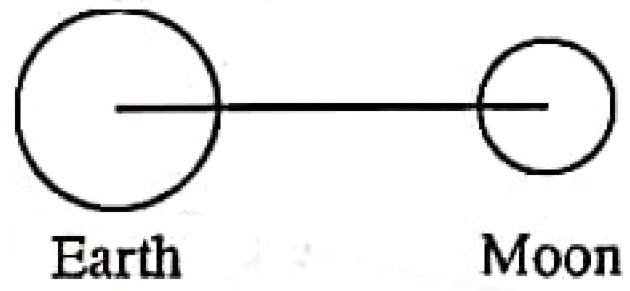
(a) 3.12×10^8 m (b) 3.42×10^8 m (c) 3.02×10^8 m (d) 3.42×10^7 m

Solution: (b); $\frac{GM_E m}{x^2} = \frac{GM_{Moon} m}{(d-x)^2} \Rightarrow \frac{6\times10^{24}}{x^2} = \frac{7.4\times10^{22}}{(d-x)^2}$

$\Rightarrow \frac{d-x}{x} = \frac{1}{9} \Rightarrow 9d - 9x = x \Rightarrow 9d = 10x$

$\Rightarrow x = \frac{9d}{10} = \frac{9}{10} \times 3.8 \times 10^8 m$

$= 3.4217 \times 10^8$ m



$M_E = 6 \times 10^{24}$ kg

$M_{Moon} = 7.4 \times 10^{22}$ kg

$d = 3.8 \times 10^8$ m

37. What is the height above the earth where the value of acceleration due to gravity is 40% of that on the earth's surface? (Radius of the earth, $R = 6.38 \times 10^6 m$)

(a) 4.7×10^6 m (b) 4.7×10^5 m (c) 3.7×10^6 m (d) 3.7×10^5 m

Solution: (c); $0.4 = \frac{R^2}{(R+h)^2} \Rightarrow \frac{R+h}{R} = \frac{1}{\sqrt{0.4}} \Rightarrow 1 + \frac{h}{R} = \frac{1}{\sqrt{0.4}} \Rightarrow h = 3.7 \times 10^6$ m

38. A drop of water is falling through air. The terminal velocity of the drop is $1.2 \times 10^{-2} ms^{-1}$ and co-efficient of viscosity of air, $\eta = 1.8 \times 10^{-5} Nsm^{-2}$ What is the diameter of the drop?

(a) 1.99×10^{-5} m (b) 2.99×10^{-5} m (c) 1.49×10^{-5} m (d) 2.49×10^{-5} m

Solution: (a); $v = \frac{2}{9}\frac{r^2 g(\rho_w - \rho_{air})}{\eta} \Rightarrow \sqrt{\frac{9v\eta}{2g(\rho_w)}} = r\ [\because \rho_{air} \ll \rho_w] \Rightarrow 2r = 1.99182 \times 10^{-5}$ m

39. When a person of mass 70 kg enters into a car, then center of gravity of the car descends by 0.4 cm. If mass of the car is 1000 kg, then calculate the frequency of vibration of the car when it is empty.

(a) $1.085\ s^{-1}$ (b) $2.085\ s^{-1}$ (c) $2.485\ s^{-1}$ (d) $1.185\ s^{-1}$

Solution: (b); $mg = ke \Rightarrow 70 \times 9.8 = k \times (0.4 \times 10^{-2}) \Rightarrow k = 1.715 \times 10^5\ Nm^{-1}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{car}}} = 2.085\ s^{-1}$

41. In a cylinder there is $0.001\ m^3$ gas at 300 K temperature and at 10^5 Pa pressure. The gas is expanded isothermally first and later on it is expanded again adiabatically. In each case ratio of expansion is 1:2. Calculate the total amount of work in expansion. ($\gamma = 1.4$)

(a) 3072.17 J (b) 3172.17 J (c) 3272.17 J (d) 3372.17 J

Solution: (No Answer); $W = W_1 + W_2 = 0.04 \times 8.314 \times 300 \ln 2 + \frac{0.04\times8.314\times(300-227)}{0.4} = 130$ J

$\therefore n = \frac{PV}{RT} = \frac{0.001\times10^5}{8.314\times300}$ mole $= 0.04$ mole; $TV^{\gamma-1} =$ constant $\Rightarrow 300 \times \left(\frac{1}{2}\right)^{0.4} = T_2 \Rightarrow T_2 = 227.357$ K

42. Internal resistance of a battery is $1\,\Omega$. 1% error is found if the electromotive force of the battery is measured by a voltmeter. What is the resistance of the voltmeter?

(a) $89\,\Omega$ (b) $87\,\Omega$ (c) $99\,\Omega$ (d) $97\,\Omega$

Solution: (c); $(1 - 0.01)E = IR \Rightarrow 0.99\,I(R + r) = IR \Rightarrow 0.99\,(R + r) = R \Rightarrow 0.99 \times 1 = 0.01\,R$

$\Rightarrow R = 99\,\Omega$

43. In the fourth arm S of a post office box has a wire of length of 1 m and cross-sectional area of 1×10^{-6} m^2 is connected Now, the galvanometer gives zero deflection when 10 Ω plug from arm Q, 1000 Ω plug from arm P and 2025 Ω plug from arm R are removed from the box. Determine the specific resistance.
(a) 30.25×10^{-6} Ωm (b) 10.25×10^{-6} Ωm (c) 40.25×10^{-6} Ωm (d) 20.25×10^{-6} Ωm
Solution: (d); $\frac{P}{Q} = \frac{R}{S} \Rightarrow S = \frac{Q \times R}{P} = \frac{10 \times 2025}{1000}\ \Omega = 20.25\ \Omega$
$\therefore \rho = \frac{SA}{L} = \frac{20.25 \times 10^{-6}}{1}\ \Omega m = 20.25 \times 10^{-6}\ \Omega m$

47. Fraunhofer diffraction is observed on a screen placed at a distance of 0.8 m from two parallel slits separated by 0.1×10^{-3} m when the slits are illuminated by light of wavelength of 5.46×10^{-7}m. What is the distance of the 3rd bright fringe from the central bright fringe?
(a) 1.11×10^{-2} m (b) 1.21×10^{-2} m (c) 1.31×10^{-2} m (d) 1.41×10^{-2}m
Solution: (c); $y = \frac{3\lambda D}{a} = \frac{3 \times 5.46 \times 10^{-7} \times 0.8}{0.1 \times 10^{-3}} m = 1.31 \times 10^{-2}$ m

48. Each gram of ^{266}Ra emits 3.5×10^{10} alpha particles per second. How many years are the half-life of radium?
(a) 1677 years (b) 1777 years (c) 1857 years (d) 1947 years
Solution: (a); $\frac{dN}{dt} = \lambda N_o, N_o = \frac{1}{226} \times 6.02 \times 10^{23} \Rightarrow 3.5 \times 10^{10} = \frac{\ln 2}{t_{\frac{1}{2}}} \times \frac{1}{226} \times 6.02 \times 10^{23}$
$\Rightarrow T_{\frac{1}{2}} = 5.275 \times 10^{10}s = 1672.78$ years ≈ 1677 years

49. A n-p-n transistor is kept in common emitter connection. Current grain of the transistor $\beta = 100$. What will be the change in emitter current if the collector current is changed by 1 mA?
(a) 3.11 mA (b) 1.01 mA (c) 2.11 mA (d) 2.01 mA
Solution: (b); $\alpha = \frac{100}{101}\ \left[\because \beta = \frac{\alpha}{1+\alpha}\right] \Rightarrow \frac{\Delta I_C}{\Delta I_E} = \frac{100}{101} \Rightarrow \Delta I_E = \frac{101}{100} \times 1\ mA = 1.01\ mA$

51. A 0.25 kg ball hits a brick wall with a velocity of 30 ms^{-1} and bounces back at the same speed. If the ball is in contact with the wall for 0.1 s, what is the value of the force exerted by the wall on the ball?
(a) 100 N (b) −150 N (c) −300 N (d) 0 N
Solution: (b); $Ft = m\Delta v \Rightarrow F = -\frac{0.25 \times (30+30)}{0.1}\ \therefore F = -150N$

52. An object has a mass of 36 kg and weight 360 N at the surface of the Earth. If this object is transported to an altitude that is twice the Earth's radius, what is the objects mass and weight, respectively?
(a) 9 kg and 90 N (b) 36 kg and 90 N (c) 4 kg and 90 N (d) 36 kg and 40 N
Solution: (b); $g' = \frac{GM}{R^2} = \frac{GM}{4R^2} \Rightarrow \frac{W'}{W} = \frac{g'}{g} = \frac{\frac{GM}{4R^2}}{\frac{GM}{R^2}} = \frac{1}{4} \Rightarrow W' = \frac{360}{4} \Rightarrow W' = 90N$
Here, mass of the object remains same.

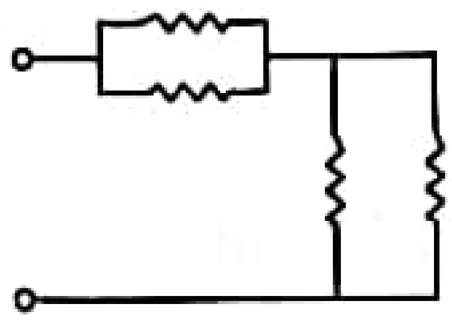
53. Cart 1 (2 kg) and Cart 2 (2.5 kg) run along a frictionless, level, one-dimensional track. Cart 2 is initially at rest, and Cart 1 is traveling 0.6 ms^{-1} toward the right when it encounters cart 2. What is the efficiency of the collision with respect to kinetic energy?
(a) 16 % (b) 65 % (c) 80 % (d) 25 %
Solution: (No Answer); Here, this is inelastic collision, $m_1u_1 + m_2u_2 = v(m_1 + m_2)$
$\Rightarrow v = \frac{2 \times 0.6}{4.5} = 0.267\ ms^{-1}$ ∴ Efficiency of kinetic Energy, $\frac{\frac{1}{2}(m_1+m_2)v^2}{\frac{1}{2}m_1v^2} = \frac{4.5 \times 0.267^2}{2 \times 0.6^2} \times 100\% = 44.44\%$

57. The intensity of the waves from a point source at a distance d from the source is I. What is the intensity at a distance 2d from the source?

(a) $\frac{I}{2}$ (b) $\frac{I}{4}$ (c) 4I (d) 2I

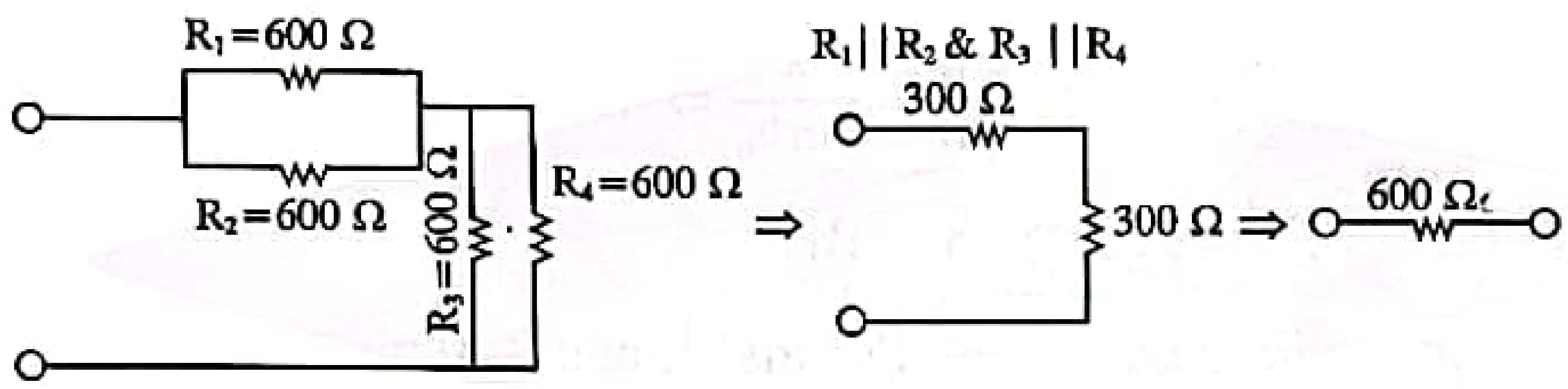
Solution: (b); $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} = \frac{d^2}{4d^2} \therefore I_2 = \frac{1}{4}$

59. What is the equivalent resistance of the circuits if each has a resistance of 600 Ω ?



(a) 60 Ω (b) 1200 Ω (c) 600 Ω (d) 175 Ω

Solution: (c);



60. Two charges $Q_1 = 2.4 \times 10^{-10}$C and $Q_2 = 9.2 \times 10^{-10}$ C are near each other, and charge Q_1 exerts a force F_1 on Q_2. How does F_1 change if the distance between Q_1 and Q_2 is increased by a factor of 4?

(a) Decreases by a factor of F (b) Increases by a factor of 16

(c) Decreases by a factor of 16 (d) Increases by a factor of 4

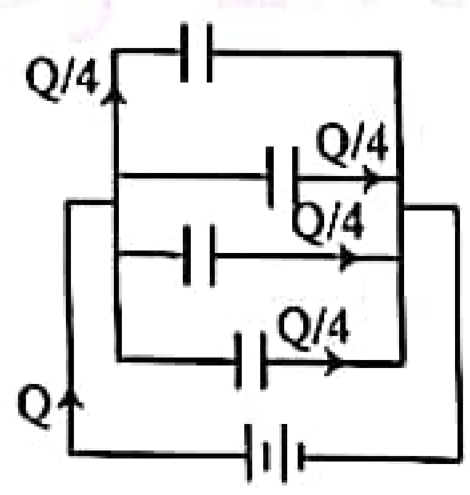
Solution: (c); Here, $F_1 = C.\frac{Q_1 \times Q_2}{d^2}$ Now, $F_1' = C \times \frac{Q_1 \times Q_2}{(4d)^2} = \frac{1}{16} C \times \frac{Q_1 \times Q_2}{d^2} = \frac{1}{16}F$

Hence, it decreases by a factor of 16

61. Four identical capacitors are connected in parallel to a battery. If a total charge of Q flows from the battery, how much charge does each capacitor carry?

(a) $\frac{Q}{4}$ (b) Q (c) 4 Q (d) 16 Q

Solution: (a); Here, current flow equally separates as 4 capacitors are identical.



62. Compared to the initial value, what is the resulting pressure for an ideal gas that is compressed isothermally so one-third of its initial volume ?

(a) Equal (b) Three times larger

(c) Larger, but less than three times larger (d) More than three times larger

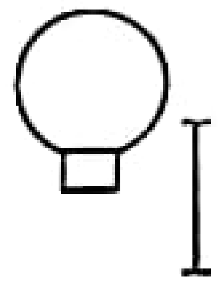
Solution: (b); Three times longer $\Rightarrow P_1 = P_2V_2 \Rightarrow P_1 \times V_1 = P_2 \times \frac{V_1}{3} \Rightarrow P_2 = 3P_1$

Extra Syllabus

32. A parachutist fell after releasing himself at 50 m down without any resistance. When the parachute was opened then the rate of decrease of acceleration was $2\ ms^{-2}$ and he fell on the ground with velocity of $3\ ms^{-1}$. At what height he was released?

(a) 295.75 m (b) 297.75 m (c) 290.75 m (d) 292.75 m

Solution: (d); $v_i = \sqrt{2gh} = 31.30495\ ms^{-1}, a = 2\ ms^{-2}$ and $v_f = 3 ms^{-1}$



$\therefore h_1 = \frac{v_i^2 - v_f^2}{2a} = 242.75$ m and $h = (50 + 242.75)$ m $= 292.75$ m

33. At what angle a projectile is to be projected so that its horizontal range is equal to the maximum height it reaches?

(a) 75.96° (b) 77.96° (c) 79.96° (d) 73.96°

Solution: (a); $\frac{v_0^2 \sin 2\theta_0}{g} = R, \frac{v_0^2 \sin 2\theta_0}{2g} = H \therefore \frac{R}{H} = \frac{4}{\tan\theta_0} \Rightarrow 4H = R\tan\theta_0$

$\Rightarrow \theta_0 = \tan^{-1}\left(\frac{4H}{R}\right) = \tan^{-1}(4) = 75.96°\ [H = R]$

40. Velocities of sound in two media P and Q are $300\ ms^{-1}$, and $340\ ms^{-1}$ respectively. If the difference of wavelength of the two sounds in 0.2 m, then for 50 complete vibrations of a tuning fork. What distance the sound will travel in medium Q?

(a) 71 m (b) 75 m (c) 81 m (d) 85 m

Solution: (d); $\Delta\lambda = 0.2 \Rightarrow \lambda Q - \lambda P = 0.2 \Rightarrow \frac{1}{f}(V_Q - V_P) = 0.2 \Rightarrow f = \frac{340-300}{0.2} \therefore f = 200HZ$

Now, $\lambda_Q - \lambda_P = \left(\frac{7}{6} - 1\right)\lambda_P - \frac{\lambda_P}{6}, \quad \frac{\lambda_P}{6} = 0.2 \Rightarrow \lambda_p = 1.2\ m, \lambda_Q = 1.4\ m$

$\therefore$ For 50 complete vibrations, $d = \lambda_Q \times 50 = 70$ m

44. Magnetic flux in a coil is changing according to the following equation: $\varphi = (4t^2 + 2t - 10)$ Wb, where t is measured in s. If the resistance of the coil is 5 Ω then at $t = 2s$ calculate the induced current in the coil.

(a) 3.2 A (b) 3.4 A (c) 3.6 A (d) 3.8 A

Solution: (c); $\epsilon = -\frac{d\phi}{dt} = -(8t + 2) = -(8 \times 2 + 2)V = -18\ V \therefore I = \frac{E}{R} = \frac{18}{5} A = 3.6\ A$

45. A proton of 8 MeV energy is applied perpendicularly in uniform magnetic field of 5.0 T. Calculate the effective force on the proton. $[M_p = 1.6 \times 10^{-27} kg, \text{charge} = 1.6 \times 10^{-19} C]$

(a) 3.2×10^{-14} N (b) 3.2×10^{-13} N (c) 3.2×10^{-12} N (d) 3.2×10^{-11} N

Solution: (d); $F = qvB = qB\sqrt{\frac{2E_k}{m}} = 3.2 \times 10^{-11}$ N

46. The focal lengths of the objective and the eye-piece of a compound microscope are 5 mm and 6 cm respectively. The distance of the image formed by the objective is at a distance of 25 cm from it. The final virtual image is situated at a distance of 30 cm from the eye-piece. Find the total magnification of the image.

(a) 294 (b) 296 (c) 298 (d) 300

Solution:(a); $f_e = 6$ cm, $f_o = 5$ mm $= 0.5$ cm; $v_o = 25$ cm, $v_e = 30$ cm

$\therefore \frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o} \Rightarrow u_o = 0.5102$ cm and $\frac{1}{u_e} - \frac{1}{v_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{v_e} + \frac{1}{f_e} \Rightarrow u_e = 5$ cm

$\therefore m = m_1 \times m_2 = \left|\frac{v_e}{u_e}\right| \times \left|\frac{v_o}{u_o}\right| = 294$

50. It is evident from the incoming light rays from a quasar that the quasar is receding at the speed of $2.7 \times 10^8\ ms^{-1}$. Calculate the distance of the quasar from the earth $[H = 72kms^{-1} / MPc]$.

(a) 1.16×10^{24} km (b) 1.16×10^{25} km (c) 1.16×10^{26} km (d) 1.16×10^{27} km

Solution: (c); From Hubble's theory, $v = Hd \Rightarrow d = \frac{v}{H} = \frac{2.7\times10^8 ms^{-1}}{72\ kms^{-1}/MPc} = \frac{2.7\times10^8 ms^{-1}\times MPc}{72\ kms^{-1}}$

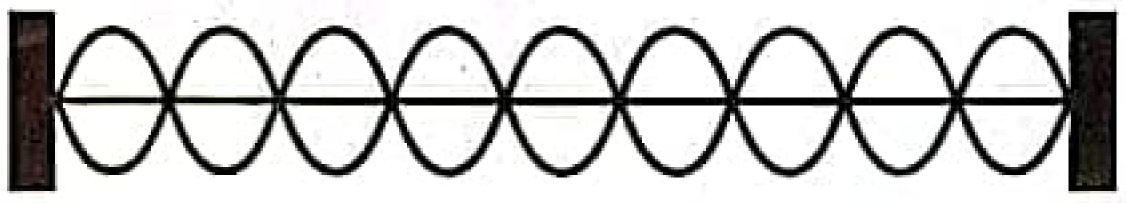
$= \frac{2.7\times10^8 ms^{-1}\times 3.084\times10^{19} km}{72\ kms^{-1}}$ $[\because 1MPc = 3.014 \times 10^{19} km] = 1.16 \times 10^{26} km$

55. A 2.5 g string, 0.75 m long, is under tension. The string produces a 700 Hz tone when it vibrates in the third harmonic. What is the wavelength of the tone in the air? (Use the speed of sound in air $v = 344\ ms^{-1}$)

(a) 0.65 m (b) 0.57 m (c) 0.33 m (d) 0.5 m

Solution: (d); $v = f\lambda = \lambda = \frac{v}{f} = \frac{344}{700} \therefore \lambda = 0.5m$

56. A string, 2 m in length, is fixed at both ends and tightened until the wave speed is $92\ ms^{-1}$. What is the frequency of the standing wave shown?



(a) 46 Hz (b) 33 Hz (c) 240 Hz (d) 138 Hz

Solution: (No Answer); $\lambda = \frac{2L}{n} \Rightarrow \lambda = \frac{2\times2}{9} = \frac{4}{9}m \therefore v = f\lambda \therefore f = \frac{92}{\frac{4}{9}} = 207$ Hz

58. Assume that the sound level of a whisper is 20 dB and a shout is 90 dB. How many times greater is the intensity of a shout than a whisper, given that the decibel level of a sound wave is related to the intensity I of the wave by: $dB = 10 \log\frac{I}{I_0}$, where $I_0 = 10^{-12}\ Wm^{-2}$

(a) 7 (b) 7000 (c) 7×10^6 (d) 10×10^6

Solution: (d); $20 = 10 \times \log\frac{I_1}{I_0} \Rightarrow 10^2 = \frac{I_1}{I_0} \Rightarrow I_1 = 10^{-10}Wm^{-2}$ and $90 = 10\log\frac{I_2}{I_0} \Rightarrow 9 = \log\frac{I_2}{I_0}$

$\Rightarrow 10^9 I_0 = I_2 \Rightarrow I_2 = 10^{-3}Wm^{-2} \therefore \frac{I_2}{I_1} = \frac{10^{-3}}{10^{-10}} = 10^7 = 10^7 = 10 \times 10^6$

63. Which expression describes the critical angle for the interface of water with air? (Use the index of refraction for water $\mu_w = 1.33$ and the index of refraction for air $\mu_a = 1$)

(a) $\sin^{-1}\left(\frac{1}{3}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$ (c) $\sin^{-1}\left(\frac{2}{3}\right)$ (d) $\sin^{-1}\left(\frac{4}{3}\right)$

Solution: (b); $\frac{\mu_a}{\mu_w} = \frac{\sin\theta c}{\sin 90°} \Rightarrow \theta_c = \sin^{-1}\left(\frac{3}{4}\right)$

64. The object is placed 60 cm from a spherical convex mirror. If the mirror forms a virtual image 20 cm from the mirror. what is magnitude of the mirror's radius of curvature?

(a) 7.5 cm (b) 15 cm (c) 30 cm (d) 60 cm

Solution: (d); As we know, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{f} = \frac{1}{60} - \frac{1}{20} = \frac{-1}{30} \therefore f = -30$ cm $\therefore r = 2|f| = 2 \times 30 = 60$cm

65. Using a mirror with a focal length of 10 m, an object is viewed at various distances. What is its magnification and orientation when the object is 5 m in front of the mirror?

(a) Twice at large and upright (b) Twice at large and inverted

(c) Twice at large and upright (d) Same size and inverted

Solution: (a, c); $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ [Here, we normally use concave mirror] $\Rightarrow \frac{1}{10} = \frac{1}{5} + \frac{1}{v} \Rightarrow -\frac{1}{10} = \frac{1}{v} \Rightarrow v = -10m$ and $m = -\frac{v}{u} = -\frac{(-10)}{5} = 2$ [Here $m > 2$ it means it is 2 times longer] it means this reflection is imaginary & upright.

Old Syllabus

54. Two 10 cm long aluminum rods and five 8 cm long steel rods are at 5°C temperature and are joined together to form a 60 cm long rod. What is the increase in the length of the joined rod when the temperature is raised to 80°C? (Use a coefficient of linear expansion for aluminum = $2.4 \times 10^{-5}\ K^{-1}$ and coefficient of linear expansion for steel = $1.2 \times 10^{-5} K^{-1}$

(a) 0.3 mm (b) 0.5 mm (c) 1.8 mm (d) 0.72 mm

Solution: (d); $\Delta L_1 = L_1 \alpha\, \Delta\theta = (2 \times 10) \times 2.4 \times 10^{-5} \times (80 - 5)$ cm

$= 20 \times 2.4 \times 10^{-5} \times 75$ cm $= 0.036$ cm $= 0.36$ mm

$\therefore \Delta L_2 = L_2\, \alpha\, \Delta\theta = (8 \times 5) \times 1.2 \times 10^{-5} \times (80 - 5)$ cm $= 0.36$ mm

$\therefore \Delta L = \Delta L_1 + \Delta L_2 = 0.72$ mm

Mathematics: MCQ (35 × 1 = 35)

Short Syllabus

66. If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix} + \begin{bmatrix}1 & y\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, then what is the value of $(x - y)$?

(a) −8 (b) 2 (c) 10 (d) −6

Solution: (c); $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix} + \begin{bmatrix}1 & y\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix} \Rightarrow \begin{bmatrix}6 & 8\\10 & 2x\end{bmatrix} + \begin{bmatrix}1 & y\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}7 & 8+y\\10 & 2x+1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$

$\therefore 8 + y = 0 \therefore y = -8 \quad \therefore 2x + 1 = 5 \Rightarrow x = 2 \therefore x - y = 2 - (-8) = 10$

69. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground in 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. The distance covered by the bird is,

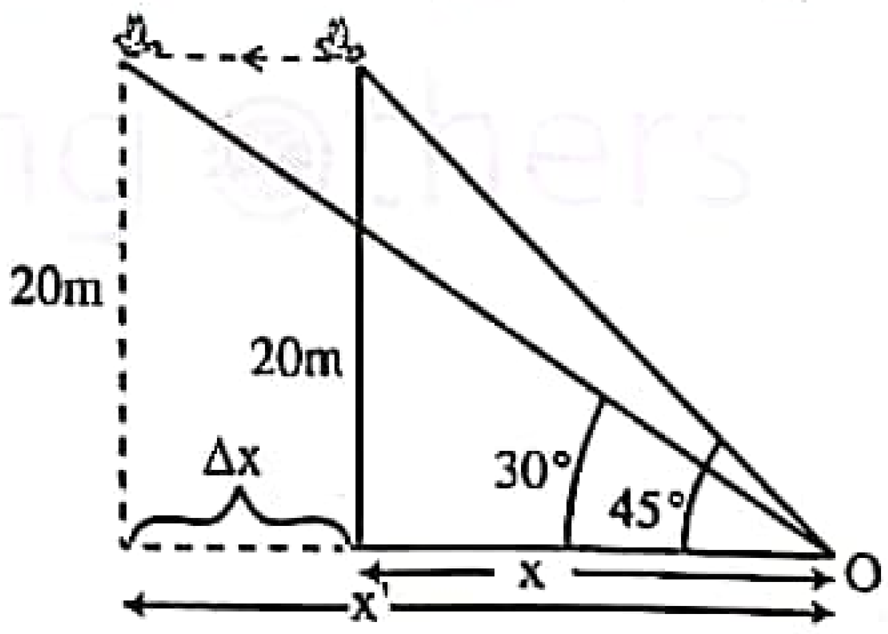
(a) $40(\sqrt{2} - 1)$ (b) $40(\sqrt{3} - \sqrt{2})$ (c) $20\sqrt{2}$ (d) $20(\sqrt{3} - 1)$

Solution: (d); $\tan 45° = \frac{20}{x} \Rightarrow x = 20$m

$\tan 30° = \frac{20}{x'} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{x'}$

$\therefore x' = 20\sqrt{3}$

$\therefore \Delta x\, x' - x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$m



70. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx$ is equal to:

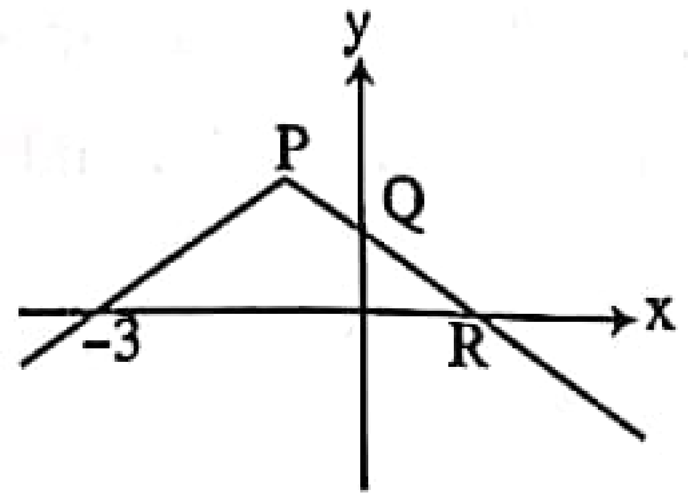
(a) $(x - 1)e^{x+\frac{1}{x}} + C$ (b) $xe^{x+\frac{1}{x}} + C$ (c) $(x + 1)e^{x+\frac{1}{x}} + C$ (d) $-xe^{x+\frac{1}{x}} + C$

Solution: (b); Option test: $f(x) = xe^{x+\frac{1}{x}}$

$\therefore f'(x) = x.e^{x+\frac{1}{x}}.\left(1 - \frac{1}{x^2}\right) + e^{x+\frac{1}{x}}.1 = e^{x+\frac{1}{x}}\left(x - \frac{1}{x} + 1\right) = e^{x+\frac{1}{x}}\left(1 + x - \frac{1}{x}\right)$

$\therefore \int f'(x)dx = f(x) + c = xe^{x+\frac{1}{x}} + c$

71. Following figure shows the graph of $y = f(x), x \in \mathbb{R}$. The graph consists of two-line segments that meet at the point P. The graph cut the y-axis at the point Q and the x-axis at the points $(-3, 0)$ and R. Find the coordinates of the points Q and R.



(a) Q (1, 0), R (1, 1) (b) Q (0, 1), R (1, 0) (c) Q (1, 1), R (1, 0) (d) Q (1, 1), R (1, 1)

Solution: (b); Point R lies on x-axis $\therefore y = 0$ and point Q lies on y-axis $\therefore x = 0$

Here, 1 one has filled the conditions $\therefore Q(0,1), R(1,0)$

73. What is the value of x where, $3^{2x} = 5^{x+1}$?

(a) 1.74 (b) 6.84 (c) 2.74 (d) 3.84

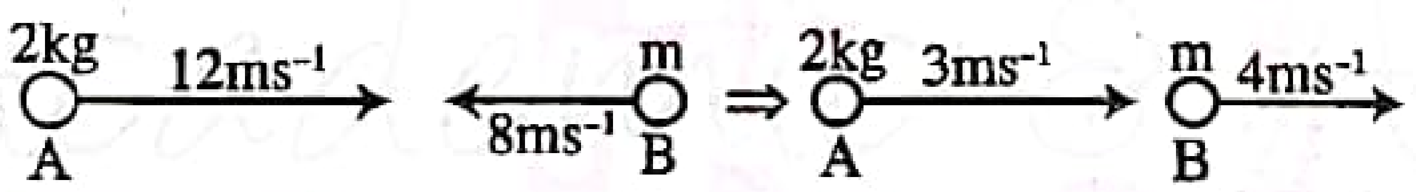
Solution: (c); $3^{2x} = 5^{x+1} \Rightarrow 2x\ln 3 = (x+1)\ln 5 \Rightarrow 2x\ln 3 = x\ln 5 + \ln 5$

$\Rightarrow x(2\ln 3 - \ln 5) = \ln 5 \Rightarrow x = \frac{\ln 5}{2\ln 3 - \ln 5} \Rightarrow x = 2.738 \approx 2.74$

74. A particle A of mass 2 kg is moving along a straight horizontal line with speed 12 ms^{-1}. Another particle B of mass m kg is moving along the same straight line, in the opposite direction to A, with speed 8 ms^{-1}. The particles collide. The direction of motion of A is unchanged by the collision. Immediately after the collision, A is moving with speed 3 ms^{-1} and B is moving with speed 4 ms^{-1} Find the value of m.

(a) 2.5 kg (b) 1.5 kg (c) 1.0 kg (d) 5.2 kg

Solution: (b);



Here, the conservation of momentum, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow 2 \times 12 - m \times 8 = 2 \times 3 + m \times 4$

$\Rightarrow 24 - 8m = 6 + 4m \Rightarrow 18 = 12m \therefore m = 1.5$ kg

N.B: The direction of B is not clarified in the question so assumed its final direction as that of a at that time.

77. Alal and Dulal shopped at the same store. Alal bought 5 kg of apples and 2 kg of bananas and paid altogether 22 Tk Dulal bought 4 kg of apples and 6 kg of bananas and paid together 33 Tk. Find the cost of 1 kg of bananas.

(a) 3.5 TK (b) 4.5 Tk (c) 6.0 TK (d) 7.66 Tk

Solution: (a); Let, the price of apples be x tk/kg and price of bananas be y tk/kg.

Now $5x + 2y = 22$ ... (i), $4x + 6y = 33$ ... (ii)

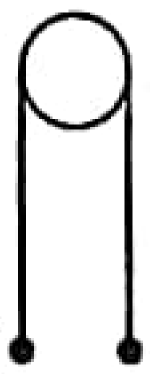
Solving (i) & (ii) $\Rightarrow x = 3\text{tk/ky}, y = \frac{7}{2} = 3.5$ tk/kg $\therefore$ price of 1kg of bananas is 3.5 tk.

78. What is the value of, $\lim_{x\to\infty} \frac{x}{x^2+1}$?

(a) 2 (b) 0 (c) 3 (d) 0.5

Solution: (b); $\lim_{x\to\infty} \frac{x^2.\frac{1}{x}}{x^2\left(1+\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{0}{1} = 0$

79. Two particles A and B have masses 5 m and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in the following figure. The system is released from rest. After release, A descends with acceleration $\left(\frac{1}{4}\right)$ g Find the tension in the string as A descends.



(a) $\left(\frac{17}{4}\right)$ mg (b) $\left(\frac{15}{4}\right)$ mg (c) $\left(\frac{19}{15}\right)$ mg (d) $\left(\frac{4}{15}\right)$ mg

Solution: (b); $a = \frac{m_1-m_2}{m_1+m_2}g \Rightarrow \frac{1}{4}g = \frac{5m-km}{5m+km}g \therefore k = 3 \therefore T_A = \frac{2m_1m_2g}{m_1+m_2} = \frac{2\times5\times3\times m^2g}{8m} = \frac{15}{4}mg$

80. α and β are two positive acute angles and $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3-\cos 2\beta}$.
Which of the following relation is correct ?
(a) $\tan\alpha = \pm\tan 2\beta$ (b) $\tan\alpha = \sqrt{3}\tan\beta$ (c) $\tan\alpha = \pm\sqrt{3}\cot\beta$ (d) $\tan\alpha = \pm\sqrt{2}\tan\beta$

Solution: (d); $\frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{3\frac{1-\tan^2\beta}{1+\tan^2\beta}-1}{3-\frac{1-\tan^2\beta}{1+\tan^2\beta}} \Rightarrow \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{3-3\tan^2\beta-1-\tan^2\beta}{3+3\tan^2\beta-1+\tan^2\beta}$

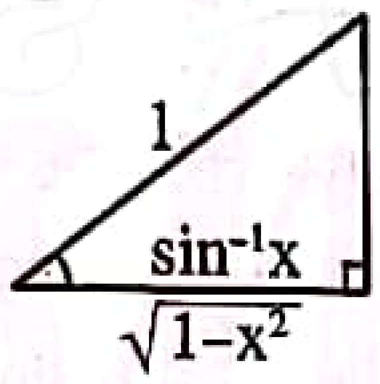
$\Rightarrow \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{2(1-2\tan^2\beta)}{2(1+2\tan^2\beta)} \Rightarrow \frac{1+\tan^2\alpha}{1-2\tan^2\alpha} = \frac{1+2\tan^2 B}{1-2\tan^2 B}$

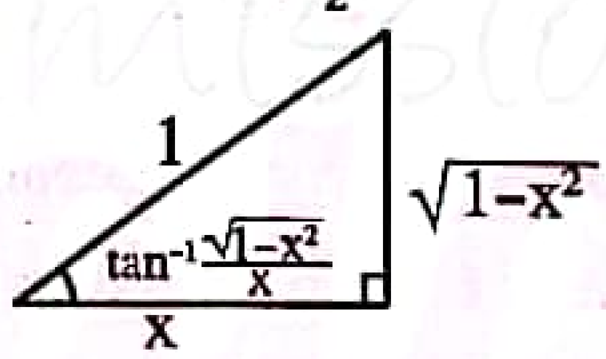
$\Rightarrow \tan^2\alpha = 2\tan^2\beta$ (componendo-dividendo) $\therefore \tan\alpha = \pm\sqrt{2}\tan\beta$

81. What is the value of, $\cos\tan^{-1}\cot\sin^{-1}x$?
(a) x (b) −x (c) $\frac{\pi}{2}+x$ (d) $\frac{\pi}{2}-x$

Solution: (a); $\cos\tan^{-1}\cot\cot^{-1}\frac{\sqrt{1-x^2}}{x}$

$= \cos\tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cos\cos^{-1}\frac{x}{1} = x$





82. If A is a 3 × 3 invertible matrix and $\det(A^{-1}) = (\det A)^{k+2}$, then the value of k is-
(a) 1 (b) 0 (c) −1 (d) −3
Solution: (d); We know, $\det(A^{-1}) = (\det A)^{-1}$
$\therefore$ Comparing, $-1 = k+2 \therefore k = -3$

84. If real part $\left(\frac{2z+1}{z+3i}\right) = 2$ when $z = x+iy$ them locus of z is-
(a) Circle whose diameter is 4 (b) Straight line whose slope is −1
(c) Circle whose radius is 1 (d) Straight line whose slope is $\frac{1}{6}$

Solution: (d); Here, $z = x+iy$ so, $\frac{2z+1}{z+3i} = \frac{2x+2iy+1}{x+(y+3)i} = \frac{\{(2x+1)+i2y\}\{x-(y+3)i\}}{x^2+(y+3)^2}$

$= \frac{x(2x+1)+2y(y+3)+i\{2xy-(2x+1)(y+3)\}}{x^2+(y+3)^2}$

Real part, $\left(\frac{2z+1}{z+3i}\right) = \frac{x(2x+1)+2y(y+3)}{x^2+(y+3)^2} = 2$

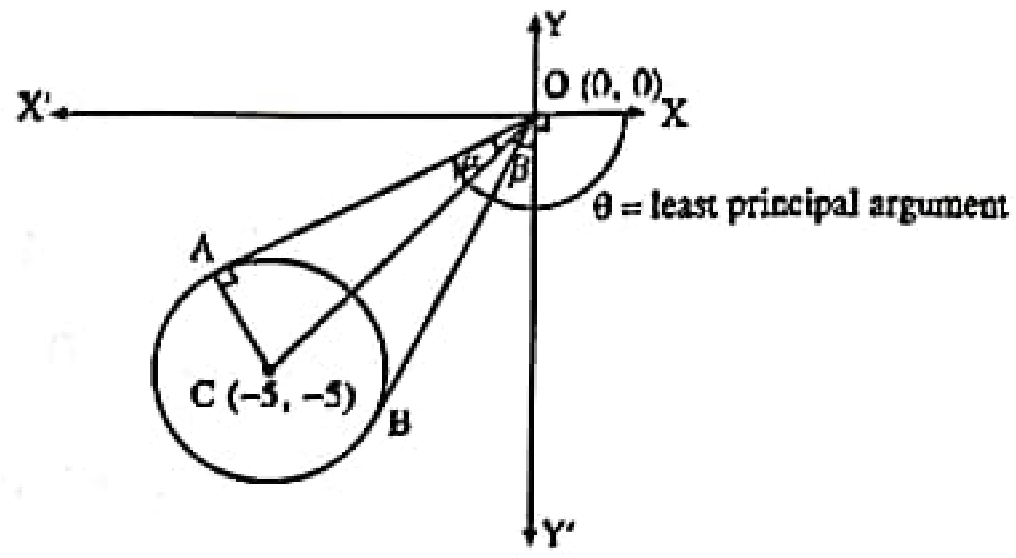
$\Rightarrow 2x^2+x+2y^2+6y = 2x^2+2y^2+12y+18 \Rightarrow x-6y = 18$

Which represents a straight line with slope $= -\frac{1}{-6} = \frac{1}{6}$ (Ans).

86. If a complex number z satisfies $|2z + 10 + 10i| \leq 5\sqrt{3} - 5$, then the least principal argument of z is-

(a) $-\frac{5\pi}{6}$ (b) $-\frac{11\pi}{12}$ (c) $-\frac{3\pi}{4}$ (d) $-\frac{2\pi}{3}$

Solution: (a); $|2z + 10 + 10i| \leq 5\sqrt{3} - 5$



$\Rightarrow |z + 5 + 5i| \leq \frac{5\sqrt{3}-5}{2} \Rightarrow |x + iy + 5 + 5i| \leq \frac{5\sqrt{3}-5}{2}$

$\Rightarrow |(x+5) + i(y+5)| \leq \frac{5\sqrt{3}-5}{2} \Rightarrow \sqrt{(x+5)^2 + (y+5)^2} \leq \frac{5\sqrt{3}-5}{2}$

$\therefore (x+5)^2 + (y+5)^2 \leq \left(\frac{5\sqrt{3}-5}{2}\right)^2$ .... (i)

(i) indicates the inner side (including the perimeter) of a circle with center at $C(-5, -5)$ and radius, $r = \frac{5\sqrt{3}-5}{2}$.

Here, $OC = \sqrt{(0+5)^2 + (0+5)^2} = 5\sqrt{2}$ units and $AC = r = \frac{5\sqrt{3}-5}{2}$

Again, $\beta = \tan^{-1}\left|\frac{-5}{-5}\right| = \frac{\pi}{4}$ and $\alpha = \sin^{-1}\frac{AC}{OC} = \sin^{-1}\left(\frac{\frac{5\sqrt{3}-5}{2}}{5\sqrt{2}}\right) = \frac{\pi}{12}$

Now, least principal argument, $\angle XOA = -\left[\frac{\pi}{2} + \alpha + \beta\right] = -\left[\frac{\pi}{2} + \frac{\pi}{12} + \frac{\pi}{4}\right] = -\frac{5\pi}{6}$ **(Ans.)**

87. If $\tan x = n \tan y, n \in R^+$, then the maximum value of $\sec^2(x - y)$ is equal to-

(a) $\frac{(n+1)^2}{2n}$ (b) $\frac{(n+1)^2}{n}$ (c) $\frac{(n+1)^2}{2}$ (d) $\frac{(n+1)^2}{4n}$

Solution: (d); Say, $z = \sec^2(x-y) = 1 + \tan^2(x-y) = 1 + \left(\frac{\tan x - \tan y}{1 + \tan x . \tan y}\right)^2$

$= 1 + \left(\frac{n \tan y - \tan y}{1 + n.\tan y.\tan y}\right)^2$ [$\because \tan x = n.\tan y$]

$\Rightarrow z = 1 + \left(\frac{(n-1)\tan y}{1 + n.\tan^2 y}\right)^2 = 1 + (n-1)^2\left(\frac{\tan y}{1 + n.\tan^2 y}\right)^2 \Rightarrow z = 1 + (n-1)^2\left(\frac{t}{1+nt^2}\right)^2$ [Putting, $\tan y = z$]

Now, for maximum and minimum, $\frac{dz}{dt} = 0 \Rightarrow 0 + 2(n-1)^2\left(\frac{t}{1+nt^2}\right).\frac{(1+nt^2).1 - t.(0+2nt)}{(1+nt^2)^2} = 0$

$\Rightarrow t(1 + nt^2 - 2nt^2) = 0 \Rightarrow t(1 - nt^2) = 0$ [$\because t \neq 0$] $\Rightarrow 1 - nt^2 = 0 \therefore t^2 = \frac{1}{n}$

$\therefore z_{max} = 1 + (n-1)^2\frac{t^2}{(1+nt^2)^2} = 1 + (n-1)^2 \times \frac{\frac{1}{n}}{\left(1+n.\frac{1}{n}\right)^2}$

$= 1 + \frac{(n-1)^2}{4n} = \frac{4n + (n-1)^2}{4n} = \frac{(n-1)^2 + 4.n.1}{4n} = \frac{(n+1)^2}{4n}$ **(Ans.)**

88. The equation $\tan^4 x - 2\sec^2 x + a = 0$ will have at least one solution if-

(a) $1 < a \leq 4$ (b) $a \geq 2$ (c) $a \leq 3$ (d) None of these

Solution: (c); $\tan^4 x - 2(1 + \tan^2 x) + a = 0 \Rightarrow \tan^4 x - 2\tan^2 x - 2 + a = 0$

$D = (-2)^2 - 4.1(a-2) \geq 0 \Rightarrow 4 - 4(a-2) \geq 0 \Rightarrow 1 - a + 2 \geq 0 \therefore a \leq 3$

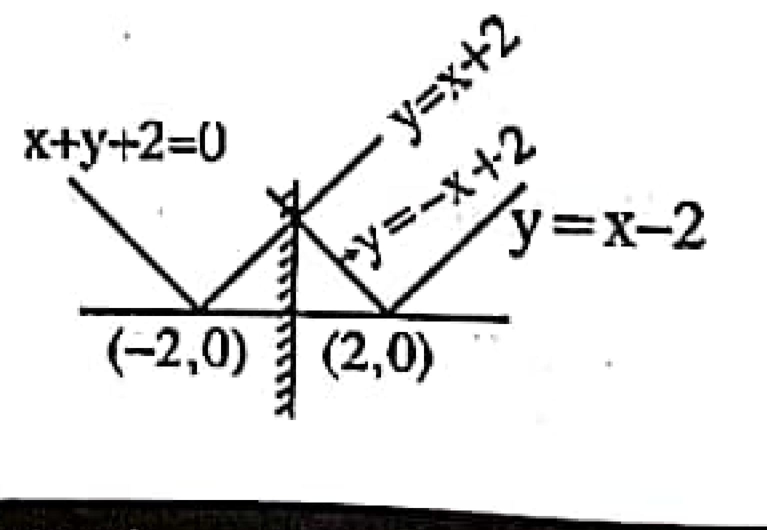
90. The combined equation of straight lines that can be obtained by reflecting the lines $y = |x - 2|$ in the y-axis is-

(a) $y^2 + x^2 + 4x + 4 = 0$ (b) $y^2 + x^2 - 4x + 4 = 0$

(c) $y^2 - x^2 + 4x - 4 = 0$ (d) $y^2 - x^2 - 4x - 4 = 0$

Solution: (d); Equation of the straight line, $y = |x - 2| \therefore y = \pm(x - 2)$

Equation of the reflected straight lines with respect to y axis, $y = \pm(-x - 2)$



$\therefore x + y + 2 = 0$ and $x - y + 2 = 0$

$\therefore$ Combined equation: $(x + 2 + y)(x + 2 - y) = 0$

$\Rightarrow (x+2)^2 - y^2 = 0 \Rightarrow x^2 + 4x + 4 - y^2 = 0 \therefore y^2 - x^2 - 4x - 4 = 0$

91. The locus of the center of the circles such that the point (2, 3) is the midpoint of the chord: 5x+2y=16 is

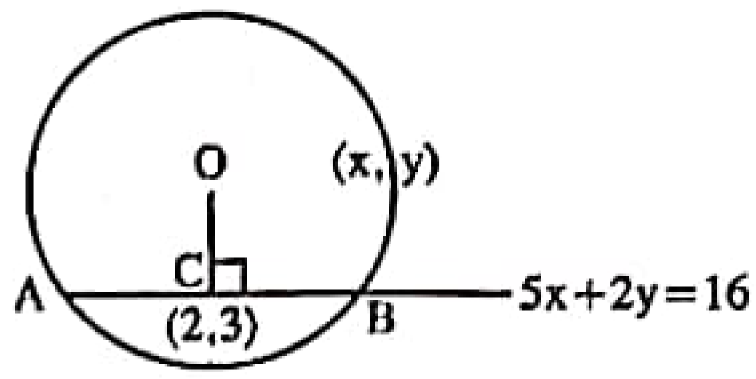
(a) $2x - 5y = 11$ (b) $2x + 5y - 11 = 0$ (c) $2x + 5y + 11 = 0$ (d) None of these

Solution:(d);

OC; $2x - 5y + k = 0$

Passing through the point $(2, 3)$,

$\therefore k = 11 \therefore 2x - 5y + 11 = 0$



Alternative: Slope of AB or $5x + 2y = 16$ is $m_{AB} = -\frac{5}{2}$ Say, $O(x, y)$ is the variable coordinate of the center.

Now, $OC \perp AB \therefore m_{OC} \times m_{AB} = -1 \Rightarrow \left(\frac{y-3}{x-2}\right) \times \left(-\frac{5}{2}\right) = -1 \Rightarrow 5y - 15 = 2x - 4 \therefore 2x - 5y + 11 = 0.$

93. $\lim_{x\to\infty} \frac{e^{\frac{1}{x^2}}-1}{2\tan^{-1}(x^2)-\pi}$ is equal to-

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Solution: (d); $\lim_{x\to\infty} \frac{\left(e^{\frac{1}{x^2}}-1\right)}{2\tan^{-1}(x^2)-\pi}\left[\frac{0}{0} \text{ form}\right] = \lim_{x\to\infty} \frac{e^{\frac{1}{x^2}}\left(-\frac{2}{x^3}\right)-0}{2.\frac{1}{1+x^4}.2x-0}$ [L'Hôpital's Rule]

$= \lim_{x\to\infty} \frac{-2e^{\frac{1}{x^2}}(1+x^4)}{2.2x^4} = \lim_{x\to\infty} -\frac{1}{2}e^{\frac{1}{x^2}}\left(\frac{1}{x^4}+1\right) = -\frac{1}{2}\, e^{\frac{1}{\infty}}\left(\frac{1}{\infty}+1\right) = -\frac{1}{2}e^0(0+1) = -\frac{1}{2}$ **(Ans.)**

94. The area of the region enclosed between the curves $x = y^2 - 1$ and $x = |y|\sqrt{1-y^2}$ is-

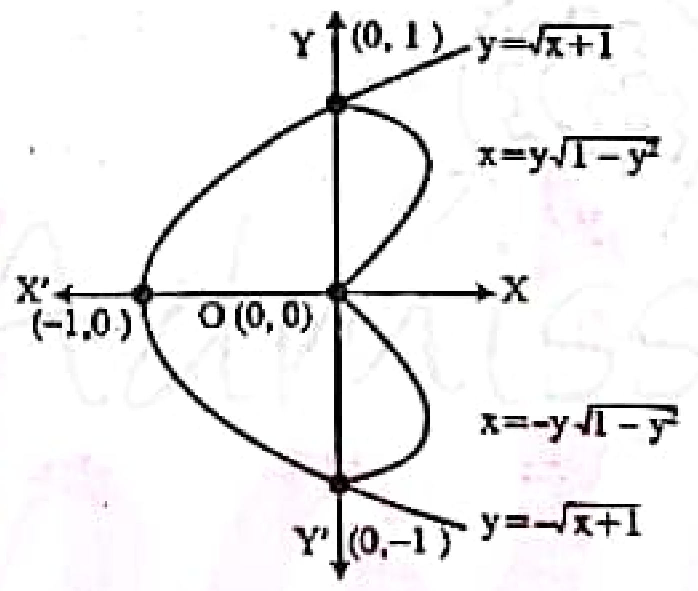
(a) 1 sq. units (b) $\frac{3}{4}$ sq. units (c) $\frac{2}{3}$ sq. units (d) 2 sq. units

Solution: (d); $x = y^2 - 1$

$\therefore y = \pm\sqrt{x+1}$

$I = 2\left[\int_{-1}^{0} \sqrt{x+1}\,dx + \int_0^1 y\sqrt{1-y^2}dy\right]$

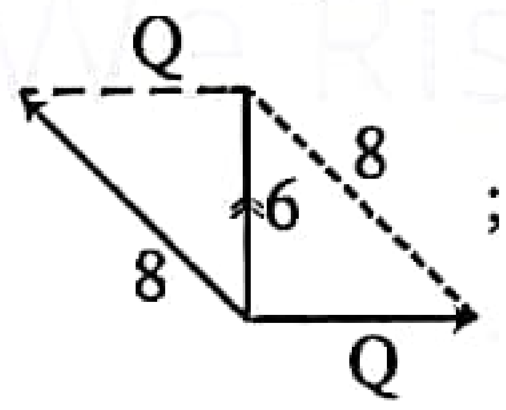
$= 2 \times \left(\frac{2}{3}+\frac{1}{3}\right) = 2 \times 1 = 2$ sq. unit



97. The resultant of two forces 8 N and Q acting at a point is 6 N, acting at right angle to the direction of Q. The value of Q is-

(a) $3\sqrt{7}$ N (b) $3\sqrt{5}$ (c) $2\sqrt{7}$ N (d) $2\sqrt{5}$ N

Solution: (c);

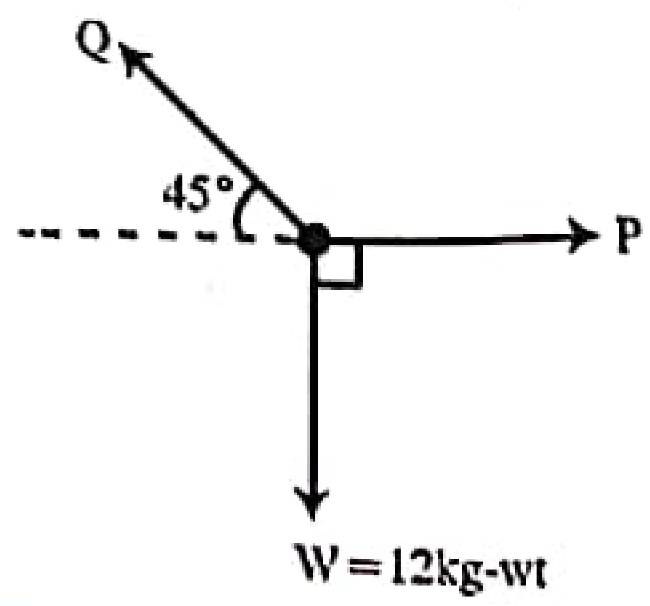


; $Q = \sqrt{8^2 - 6^2}$ N $= \sqrt{64-36}$ N $= \sqrt{28}$ N $= 2\sqrt{7}$ N

98. A hanging body weighing 12 kg is kept in a position by applying two forces P and Q on it. P is acting horizontally while Q is acting by making an angle of 45° with the horizontal. The magnitude of the Q is-

(a) $\frac{12}{\sqrt{3}}$ kg − wt (b) $12\sqrt{3}$ kg − wt

(c) $6\sqrt{3}$ kg − wt (d) $12\sqrt{2}$ kg − wt



Solution: (d); Here, Lami's Theorem, $= \frac{W}{\sin(180°-45°)} = \frac{Q}{\sin 90°}$

$\Rightarrow \frac{W}{\sin 135°} = Q \Rightarrow \frac{12}{\frac{1}{\sqrt{2}}} = Q \therefore Q = 12\sqrt{2}$ kg-wt

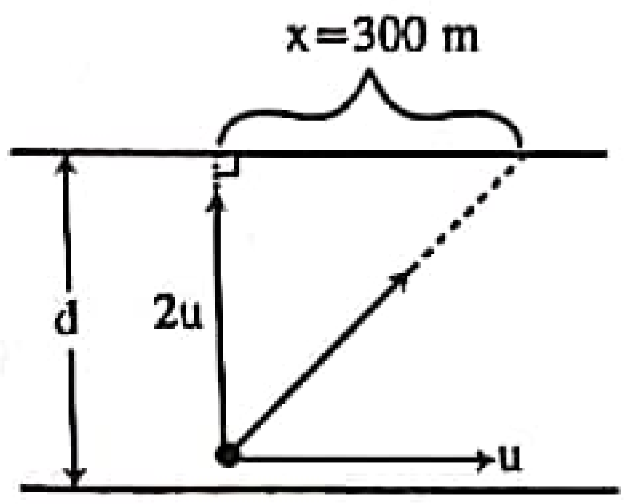
99. Suppose, your swimming velocity is twice of current in a river. You wish to cross the river perpendicularly having the current. But you reached 300 m far from the opposite of your starting point. Then width of the river is-

(a) 150 m (b) 300 m (c) 600 m (d) None of these

Solution: (c); $x = (u + 2u\cos 90°) \times t \Rightarrow 300 = u \times \frac{d}{2u\sin 90°}$

$\Rightarrow 300 = \frac{d}{2} \therefore d = 600m$

Shortcut: IF $\alpha = 90°; \frac{x}{d} = \frac{u}{2u} \Rightarrow d = 2x = 2 \times 300 = 600m$



Extra Syllabus

67. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a-

(a) Parallelogram, which is neither a rhombus nor a rectangle (b) Square
(c) Rectangle, but not a square (d) Rhombus, but not a square

Solution: (a); Coordinate of $P(-2,-1), Q(4,0), R(3,3), S(-3,2)$

$PQ^2 = (-2-4)^2 + (-1-0)^2 = 37;$

$QR^2 = (4-3)^2 + (0-3)^2 = 10$

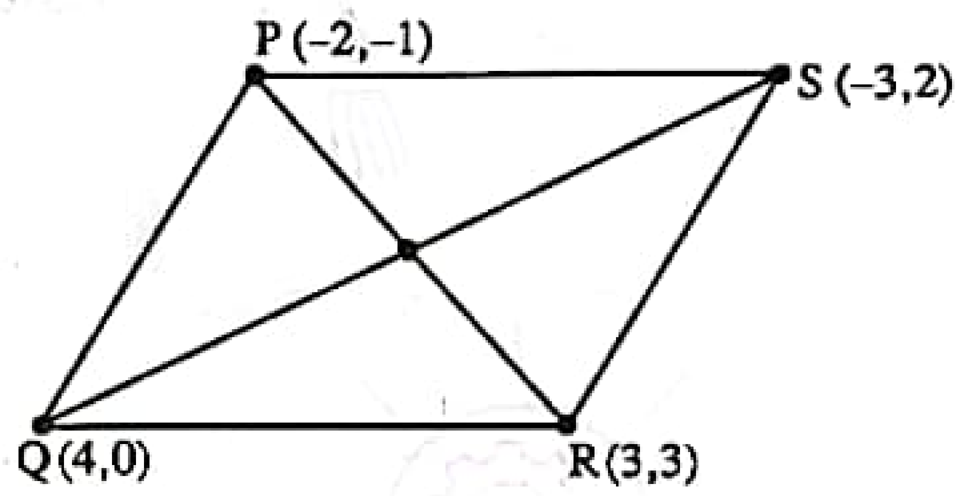
$RS^2 = (3+3)^2 + (3+3)^2 = 37;$

$SP^2 = (-3+2)^2 + (2+1)^2 = 10;$

$PR^2 = (-2-3)^2 + (-1-3) = 41$

$\therefore PQ = RS$ and $QR = SP$ and $PQ^2 + QR^2 \neq PR^2$

$\therefore$ it's not a rectangle, it's a parallelogram



72. Relative to a fixed origin O, the point A has position vector $\hat{i} - 3\hat{j} + 2\hat{k}$ and the point B has position vector $-2\hat{i} + 2\hat{j} - \hat{k}$ The points A and B lie on a straight-line l.

The point C has position vector $2\hat{i} + p\hat{j} - 4\hat{k}$ with respect to O, where p is a constant. Given that AC is perpendicular to 1. Find the value of p.

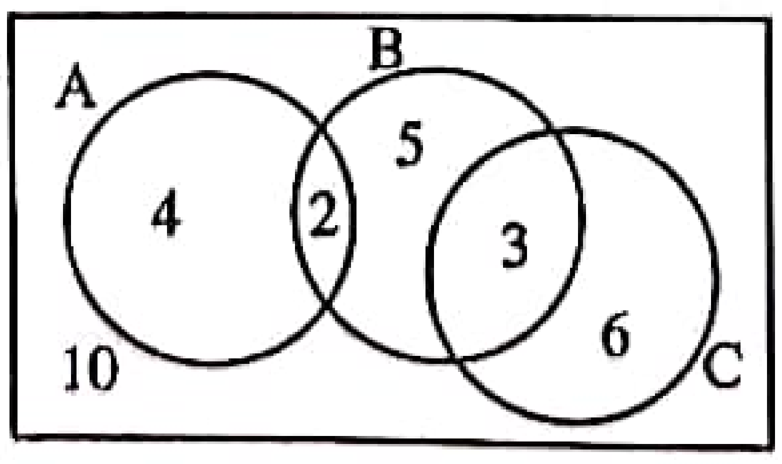
(a) $p = 16$ (b) $p = 3$ (c) $p = -12$ (d) $p = -6$

Solution: (d); $\overrightarrow{AB} = (-2-1)\hat{i} + (2+3)\hat{j} + (-1-2)\hat{k} = -3\hat{i} + 5\hat{j} - 3\hat{k}$

$\therefore \overrightarrow{AC} = (2-1)\hat{i} + (p+3)\hat{j} + (-4-2)\hat{k} = \hat{i} + (3+p)\hat{j} - 6\hat{k}$ As $\overrightarrow{AB} \perp \overrightarrow{AC} \therefore \overrightarrow{AB}.\overrightarrow{AC} = 0$

$\Rightarrow (-3).1 + 5(p+3) + (-3).(-6) = 0 \Rightarrow -3 + 5p + 15 + 18 = 0 \Rightarrow 5p = -30 \therefore p = -6$

75. The following figure represents a Venn diagram that shows the number of students in a class who read any of 3 popular magazines A, B and C. One of these students is selected at random. Find the probability that the student reads more than one magazine.



(a) $\frac{1}{7}$ (b) $\frac{1}{9.5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{20}$

Solution: (c); n (more than one magazine readers) $= 2 + 3 = 5$, n (total) $= 30 \therefore$ Probability $= \frac{5}{30} = \frac{1}{6}$

83. IUT debate club consist of 10 boys and 5 girls. A team of 4 members have to select from this club including the selection of a captain (from the team members). If the team has to include at most one girl, the number of ways selecting the team is-

(a) 3260 (b) 3240 (c) 2865 (d) 3060

	Boys (10)	Girls(5)	
Solution: (b); (i)	4	0	Total no. of was to make a team of 4 members
(ii)	3	1	

$= {}^{10}C_4 \times {}^5C_0 + {}^{10}C_3 \times {}^5C_1 = 210 + 600 = 810$ A captain can be selected among 4 members in ${}^4C_1 = 4$ ways $\therefore$ Total no of ways $= 810 \times 4 = 3240$

85. The sum of $1 + n\left(1 - \frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1 - \frac{1}{x}\right)^2 + \cdots \infty$ will be-

(a) $\left(1 - \frac{1}{x}\right)^n$ (b) x^n (c) $\left(1 + \frac{1}{x}\right)^n$ (d) None of these

Solution: (d); Let, $1 - \frac{1}{x} = y \therefore 1 + ny + \frac{n(n+1)}{2!}y^2 + \ \dots\infty = (1+y)^2 = \left(1 + 1 - \frac{1}{x}\right)^n = \left(2 - \frac{1}{x}\right)^2$

92. The period of $\frac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}$, $n \in N$, is 6π, then n is equal to-

(a) 3 (b) 2 (c) 6 (d) 1

Solution: (c); Here, the function is, $f(x) = \frac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}$

Numerator is $\cos(\sin(nx))$. We know, the fundamental period of $\cos(\sin x)$ is π.

$\therefore$ Fundamental period of $\cos(\sin(nx))$ is $\frac{\pi}{n}$. Denominator is $\tan\frac{x}{n}$

We know, the fundamental period of $\tan x$ is π $\therefore$ Fundamental period of $\tan\left(\frac{x}{n}\right)$ is $\frac{\pi}{\frac{1}{n}} = n\pi$

$\therefore$ Fandamental period of f(x) is LCM of $\frac{\pi}{n}$ and $n\pi$.

$\therefore$ Fundamental period of $f(x) = \frac{\text{LCM of }(\pi, n\pi)}{\text{HCF of }(n,1)} = \frac{n\pi}{1} = n\pi$.

A/C to the question, $n\pi = 6\pi \therefore n = 6$ **(Ans.)**

95. The altitude of a parallelepiped whose three coterminous edges are vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 3\hat{k}$, with $\vec{A}$ and $\vec{B}$ as sides of the base of the parallelepiped is-

(a) $\frac{4\sqrt{3}}{5}$ (b) $\frac{4\sqrt{31}}{7}$ (c) $\frac{2\sqrt{38}}{19}$ (d) $\frac{4\sqrt{37}}{15}$

Solution: (c); $\vec{A} \times \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} + 4\hat{j} - \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & -1 \end{vmatrix}$

$= \hat{j}(-1-4) - \hat{j}(-1-2) + \hat{k}(4-2) = -5\hat{i} + 3\hat{j} + 2\hat{k}$

Area of base, $S = |\vec{A} \times \vec{B}| = \sqrt{(-5)^2 + 3^2 + 2^2} = \sqrt{38}$ sq. units

Volume of parallelepiped, $V = \vec{C}.(\vec{A} \times \vec{B})$

$\therefore V = (\hat{i} + \hat{j} + 3\hat{k}).(-5\hat{i} + 3\hat{j} + 2\hat{k}) = 1(-5) + 1.3 + 3.2 = 4$ cubic units $\therefore$ Altitude, $h = \frac{V}{S} = \frac{4}{\sqrt{38}} = \frac{2\sqrt{38}}{19}$

96. A natural number x is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is-

(a) $\frac{1}{10}$ (b) $\frac{11}{20}$ (c) $\frac{11}{50}$ (d) None of these

Solution: (b); $x + \frac{100}{x} > 50 \Rightarrow x^2 - 50x + 100 > 0 \Rightarrow x^2 - 50x + 100 + 525 - 525 > 0$

$\Rightarrow x^2 - 50x + 625 > 525 \Rightarrow (x-25)^2 > 525 \Rightarrow x - 25 < -\sqrt{525}$ or, $x - 25 > \sqrt{525}$

$\Rightarrow x < 25 - \sqrt{525}$ or, $x > 25 + \sqrt{525}$

$\Rightarrow x < 25 - 22.91$ or, $x > 25 + 22.91 \Rightarrow x < 2.09$ or, $x > 47.91$. As only integer values are considered. We can say that $x \le 2$ or, $x \ge 48$ Let, E be the event for favorable cases and S be the sample space.

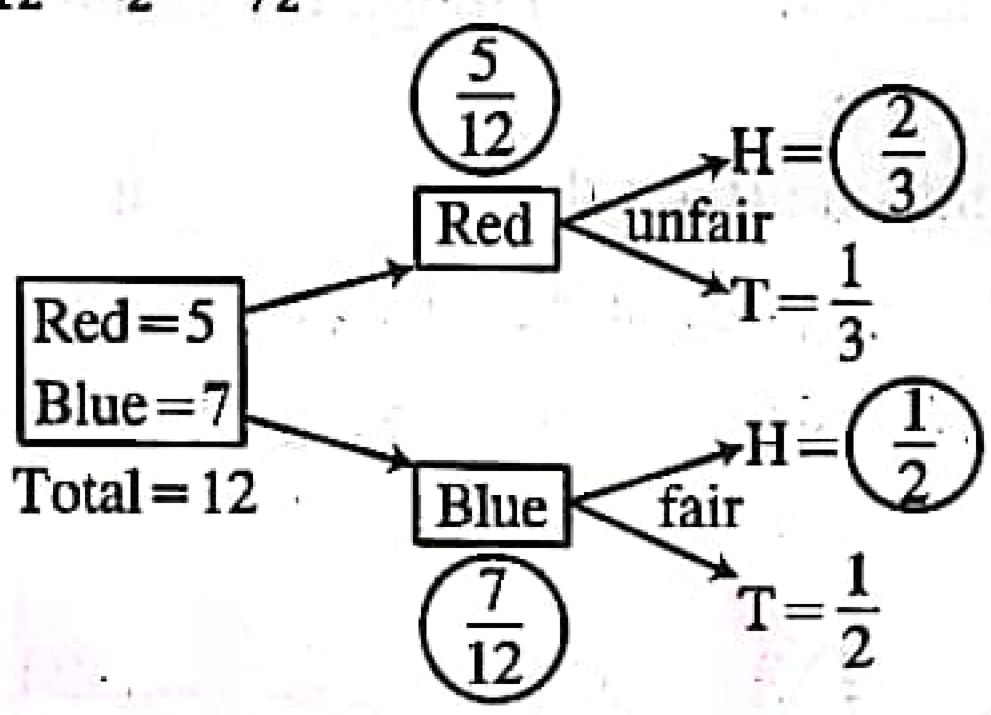
$E = \{1, 2, 48, 49, 50, \cdots\cdots, 100\}$ ∴ $n(E) = 55$ and $n(S) = 100$

Hence the required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{55}{100} = \frac{11}{20}$

100. An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its color is noted and then the ball is returned to the bag. When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun. When a blue ball is selected a fair coin is spun. Find the probability that she obtains a head.

(a) 0.569 (b) 0.24 (c) 0.965 (d) 0.659

Solution: (a); $P(H) = \frac{5}{12} \times \frac{2}{3} + \frac{7}{12} \times \frac{1}{2} = \frac{41}{72} = 0.569$



Old Syllabus

68. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is-

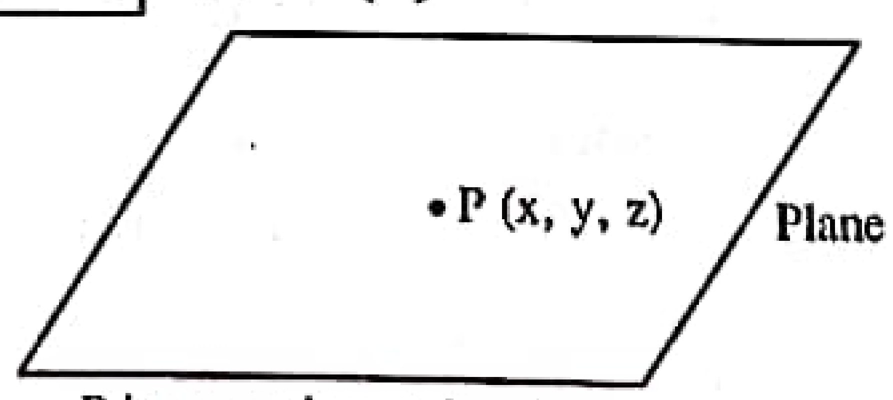
(a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$ (c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

Solution: (c); [Note: The general equation of straight line in xy plane (2D) can be denoted by the equation $Ax + By = C$. Similarly, the general equation of a plane in 3D space can be denoted by the equation,

$Ax + By + Cz = D$ ... ... (1)

(1) Seems to have 4 arbitrary constants (A, B, C, D) but, dividing (1) by A we get,

$x + \frac{B}{A}y + \frac{C}{A}z = \frac{D}{A}$ ∴ $\boxed{x + ay + bz = c}$ ... ... (2)



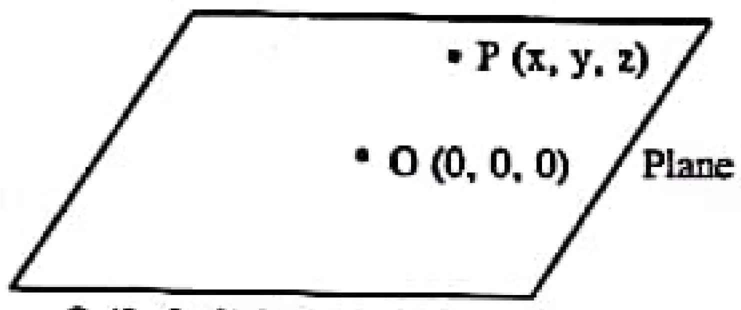
P is any point on the plane

(2) can also be considered as the general equation of a plane in 3D space. (2) has 3 arbitrary constants. Hence, if 3 points on the plane is known, the equation on of the plane can be obtained using (2).

Again, if the plane passes through the origin O (0,0,0)

(2) $\Rightarrow 0 + 0 + 0 = C$ ∴ $\boxed{C = 0}$

∴ The equation of a plane passing through the origin can be written as $\boxed{x + ay + bz = 0}$ ……… (3)



O (0, 0, 0) is the origin

Using these Concepts, we can solve problem no-68. Let's solve the problem]

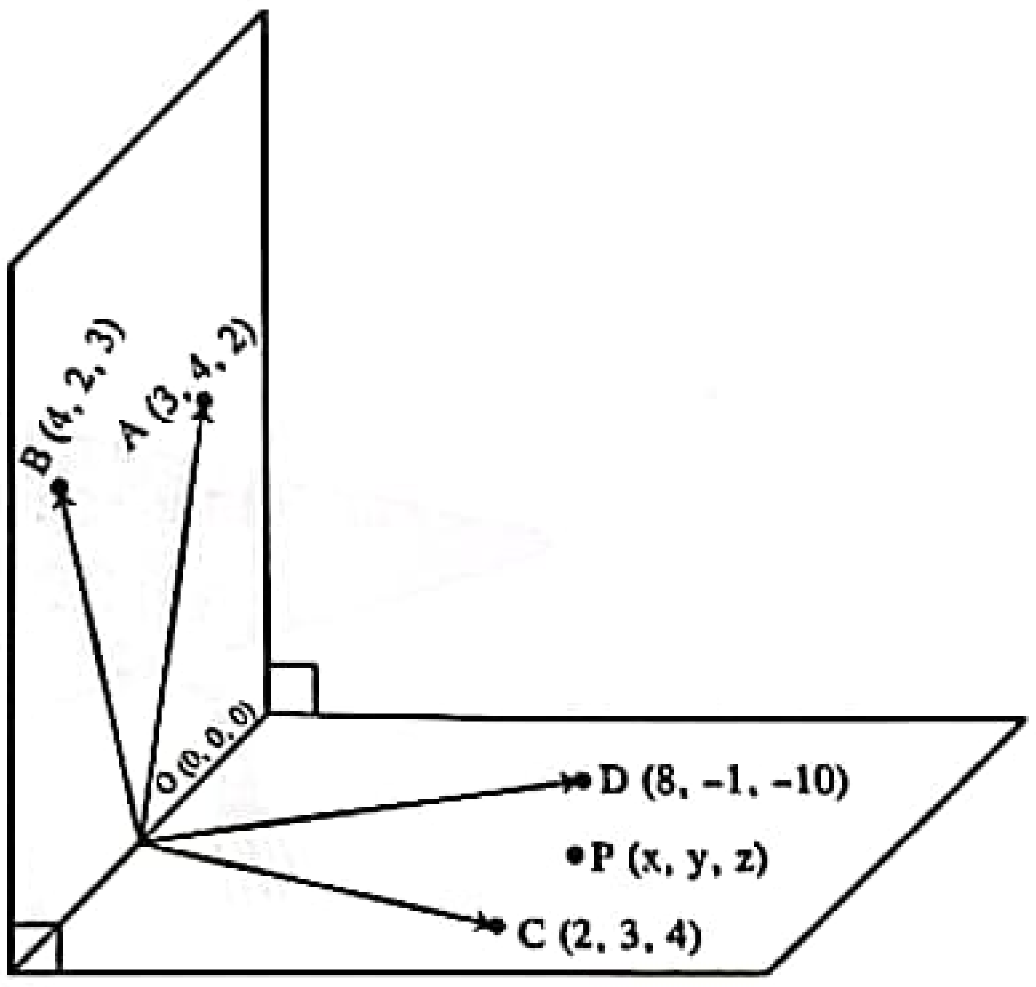
$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ denotes a straight line (OA) that pane through origin O (0, 0, 0) and A (3, 4, 2)

$\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ denotes a straight line (OB) that pane through origin O (0, 0, 0) and B (4, 2, 3)

Now, $\overrightarrow{OA} = 3\hat{j} + 4\hat{j} + 2\hat{k}$ and $\overrightarrow{OB} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

as our required plane is perpendicular to both $\overrightarrow{OA}$ and $\overrightarrow{OB}$

So, $\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = \hat{i}(12-4) - \hat{j}(9-8) + \hat{k}(6-16) = 8\hat{i} - \hat{j} - 10\hat{k}$



Say, D (8, −1, −10)

So, $\frac{x}{8} = \frac{y}{-1} = \frac{z}{-10}$ is a straight line (OD) that is perpendicular to both $\overrightarrow{OA}$ and $\overrightarrow{OB}$. This straight line also passes through the origin O (0, 0, 0) and D (8, −1, −16)

Now, $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ denotes a straight line (OC) that passes through origin (0, 0, 0) and C (2, 3, 4). Our required plane must contain the straight lines. OC and OD or must pass through the origin O (0, 0, 0), C (2, 3, 4) and D (8, −1, −10). As the plane passes through the origin, say the equation of the plane is,

$x + ay + bz = 0$ ……… (1)

(1) passes through C (2, 3, 4), we get, $2 + 3a + 4b = 0$ ……… (2)

(1) also passes through D (8 − 1 − 10), we get, $8 - a - 10b = 0$ ……… (3)

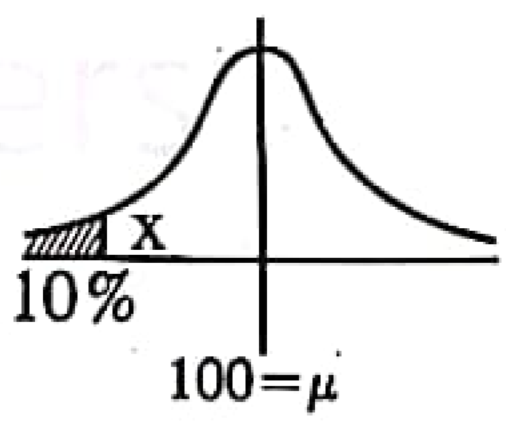
Solving (2) & (3) we get, $a = -2$ and $b = 1$

Putting the value of a and b in (1) we get equation of the required plane is, $x - 2y + z = 0$ **(Ans.)**

76. The length of time, L hours, that a laptop will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours. Find the value of d such that $P(L < d) = 0.10$.

(a) 78.78 (b) 81.98 (c) 89.78 (d) 80.78

Solution: (d); Insufficient Data, Out of H. S. C syllabus



Necessary data to solve this problem: for $P = 0.1, z = -1.2816$

Here, $\mu = 100$, $\sigma = 15$, $z = -1.2816$ Now, $z = \frac{d-\mu}{\sigma}$

$\Rightarrow d = \mu + z\sigma = 100 + (-1.2816) \times 15 = 80.776$ ∴ $d \approx 80.78$ **(Ans.)**

89. If in a triangle, $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$ (r_1, r_2, r_3 denote the radii of the escribed circles opposite to the angles A, B and C respectively), then the triangle is-

(a) right angled (b) isosceles (c) equilateral (d) None of these

Solution: (a); $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2 \Rightarrow \left(1 - \frac{s-b}{s-a}\right)\left(1 - \frac{s-c}{s-a}\right) = 2 \left[\text{Note, } r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}\right]$

$\Rightarrow (s - a - s + b)(s - a - s + c) = 2(s - a)^2 \Rightarrow (b - a)(c - a) = 2\left(\frac{a+b+c}{2} - a\right)^2$

$\Rightarrow bc - ab - ac + a^2 = 2.\frac{(a+c-a)^2}{4} \Rightarrow 2bc - 2ab - 2ac + 2a^2 = b^2 + c^2 + a^2 + 2bc - 2ca - 2ab$

$\Rightarrow a^2 = b^2 + c^2$ ∴ $\angle A = \frac{\pi}{2}$ ∴ the triangle is right angled.