IUT Admission Test 2021-2022

Full Marks: 100 | **MCQ** | **Time: 2:00**

Physics: MCQ ($35 \times 1 = 35$)

Short Syllabus

01. A force $\vec{F} = (40\hat{i} + 30\hat{j})$ N is acting on an object of mass 10 kg. What is the magnitude of the acceleration of the object? Assume other forces are involved.

(a) 6 ms^{-2} (b) 5 ms^{-2} (c) 7 ms^{-2} (d) 4 ms^{-2}

Solution: (b); $\vec{F} = (40\hat{i} + 30\hat{j})$N and m = 10kg

$\therefore \vec{a} = \frac{\vec{F}}{m} = (4\hat{i} + 3\hat{j})ms^{-2} \therefore |\vec{a}| = \sqrt{16+9}\, ms^{-2} = 5\ ms^{-2}$

02. When 7500 kg diesel engine pulls a 25000 kg train along straight and level rails, it produces an acceleration of 2.5 ms^{-2}. What will be the acceleration of the engine when the load of the train becomes 45000 kg? (Neglect the friction).

(a) 1.55 ms^{-2} (b) 2.30 ms^{-2} (c) 1.85 ms^{-2} (d) 2.15 ms^{-2}

Solution: (a); $F = (m + M)a_1 \ldots\ldots\ldots$ (i)

$F = (m + M')a_2 \ldots\ldots\ldots$ (ii)

(i) ÷ (ii) $\Rightarrow \frac{a_1(m+M)}{a_2(m+M')} = 1 \Rightarrow a_2 = \frac{a_1 \times (m+M)}{m+M'} = 2.5 \times \frac{(25000+7500)}{(45000+7500)} ms^{-2} = 1.547619\ ms^{-2} \approx 1.55\ ms^{-2}$

03. An 80 W electric fan is rotating at 300 rpm. How much torque is being produced by the electric motor of the fan? Neglect all losses.

(a) 3.55 Nm (b) 2.85 Nm (c) 2.55 Nm (d) 2.98 Nm

Solution: (c); $P = \tau\omega \Rightarrow 80 = \tau \times \frac{300 \times 2\pi}{60} \Rightarrow \tau = \frac{80 \times 60}{300 \times 2\pi}$ Nm = 2.55 Nm

04. A flywheel is being driven from rest by an electric motor with a constant torque of 25 Nm. The moment of inertia of the fly wheel is 5 kgm^2. What is the kinetic energy of the flywheel after 10 sec?

(a) 6250 J (b) 7250 J (c) 5550 J (d) 7238 J

Solution: (a); $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{25}{5} rads^{-2} = 5\ rads^{-2}$

$\omega = \alpha t = (5 \times 10) rads^{-1} = 50\ rads^{-1} \therefore E_k = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 5 \times (50)^2$ J = 6250 J

05. A 12 V and 50 W lamp is connected to a 12 volt battery with internal resistance 0.1 Ω. What is the actual voltage applied to the lamp terminals?

(a) 11.60 V (b) 11.35 V (c) 10.95 V (d) 11.85 V

Solution: (a); $R = \frac{V^2}{P} = \frac{12^2}{50}\ \Omega = 2.88\ \Omega \therefore V = IR = \frac{E}{R+r} \times R = \frac{12}{2.88+0.1} \times 2.88$ V = 11.60 V

06. A solenoidal vector field is given as $\vec{V} = (5x + 2y)\hat{i} + (my - z)\hat{j} + (x - 4z)\hat{k}$. What is the value of m?

(a) −2 (b) −1 (c) 1 (d) 3

Solution: (b); $\vec{V} = (5x + 2y)\hat{i} + (my - z)\hat{j} + (x - 4z)\hat{k}$

$\therefore \vec{\nabla}.\vec{V} = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\{(5x + 2y)\hat{i} + (my - z)\hat{j} + (x - 4z)\hat{k}\} = 5 + m - 4 = 1 + m$

For being solenoidal, $\vec{\nabla}.\vec{V} = 0 \Rightarrow 1 + m = 0 \Rightarrow m = -1$

07. In a construction site, a lifting machine on the ground is used to lift bricks to the 8th floor which is 25 m high. At a time, the machine can lift a 2000 kg load in 1 min. The power supplied to the engine of the lift is 15 HP. What is the efficiency of the lifting machine? (Use $g = 9.81\ ms^{-2}$ and $1\ HP = 746\ W$).

(a) 87% (b) 73% (c) 83% (d) 76%

Solution: (b); $P_{out} = \frac{2000\times9.81\times25}{1\times60} W = 8175\ W = 10.96\ HP$

$\eta = \frac{P_{out}}{P_{in}} = \frac{10.96}{15} \times 100\% = 73.06\% \approx 73\%$

08. To measure the spring constant of a Hookean spring a 500 g mass is hooked up at its end and length of the stretched spring is measured. Then the mass is increased to 550 g and observed that the spring is stretched 10 cm more. What is the spring constant?

(a) $4.05\ Nm^{-1}$ (b) $4.08\ Nm^{-1}$ (c) $3.95\ Nm^{-1}$ (d) $4.91\ Nm^{-1}$

Solution: (d); $\Delta mg = k\Delta x \therefore k = \frac{\Delta mg}{\Delta x} = \frac{0.05\times9.8}{0.1} = 4.9\ Nm^{-1}$

09. A coal-fired power plant that operates at an efficiency of 38% generates 750 MW of electric power. How much heat does the plant discharge to the environment in one day (24 h)?

(a) 1.057×10^{14} J/day (b) 2.570×10^{14} J/day (c) 1.570×10^{14} J/day (d) 2.057×10^{14} J/day

Solution: (a); $0.38 = \frac{750\times10^6}{P_{in}} \Rightarrow P_{in} = 1973.6842\ MW \therefore Q_2 = Q_1 - W = Q_1 - Q_1 \times \eta = Q_1(1-\eta)$

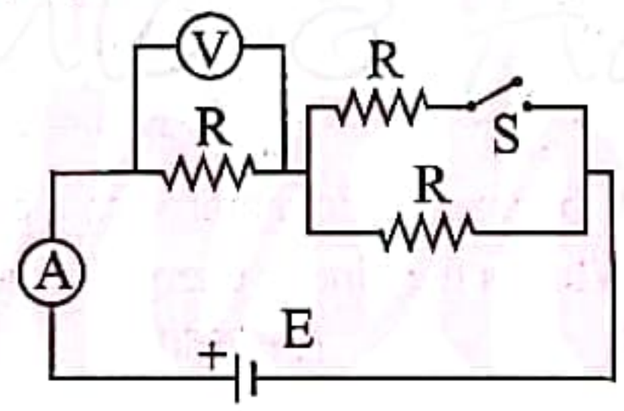
$= P_{in} \times t \times (1-\eta) = 1973.6842 \times 10^6 \times 24 \times 60 \times 60 \times (1-0.38) J/day = 1.057 \times 10^{14}\ J/day$

10. An old battery with an emf of 9 V has a terminal voltage of 8.2 V when it is supplying a current of 2 mA. What is the internal resistance of the battery?

(a) 420 Ω (b) 400 Ω (c) 450 Ω (d) 390 Ω

Solution: (b); $Ir = (9-8.2)V = 0.8V \Rightarrow r \times 2 \times 10^{-3} = 0.8\ V \Rightarrow r = 400\ \Omega$

11. When the switch S in the following figure is closed, the reading of the voltmeter, V will be-



(a) 0.25 E (b) 0.50 E (c) 0.67 E (d) 0.57 E

Solution: (c); $V = E \times \frac{R}{R+\frac{R}{2}} = E \times \frac{1}{\frac{3}{2}} = E \times \frac{2}{3} = 0.67\ E$

12. A thermometer is made using a carbon resistor having a temperature coefficient of resistivity of $-0.00050\ \Omega°C^{-1}$. The variation of resistance with temperature in the linear region is used to measure the temperature. On a cold winter day when the temperature is 8°C, the resistance is 216.8 Ω. What is the temperature on a summer day when the resistance is 213.5 Ω?

(a) 33.4°C (b) 36.4°C (c) 34.4°C (d) 38.4°C

Solution: (d); $R_1 = R_0(1 + \alpha\,\theta_1)$ ..... (i)

$R_2 = R_0(1 + \alpha\,\theta_2)$ ..... (ii)

(ii) ÷ (i) $\Rightarrow \frac{R_2}{R_1} = \frac{1+\alpha\,\theta_2}{1+\alpha\,\theta_2} \Rightarrow \frac{213.5}{216.8} = \frac{1-5\times10^{-4}\times\theta_2}{1-5\times10^{-4}\times8} \Rightarrow \theta_2 = 38.32^{\circ}C \approx 38.4°C$

13. Suppose that NASA wants to explore a distant solar system 200 light years away from the earth. What will be the aging of the astronauts traveling in a spaceship with a speed of 0.99c as measured from the spaceship's frame of reference, where c is the speed of light in vacuum?
(a) 14 Years (b) 28 Years (c) 19 Years (d) 22 Years

Solution: (b); $t_0 = t\sqrt{1-\frac{v^2}{c^2}} = 200\sqrt{1-0.99^2}$ Years $= 28.2$ Years ≈ 28 Years

14. An npn transistor in CE configuration has $\alpha = 0.995$. What is the base current if the collector current is 4 mA?
(a) 18 μA (b) 23 μA (c) 28 μA (d) 20 μA

Solution: (d); $\alpha = \frac{I_C}{I_E} \Rightarrow I_E = \frac{I_C}{\alpha} = \frac{4}{0.995}\text{mA} = 4.0201\text{ mA} \therefore I_B = I_E - I_C = 0.02\text{ mA} = 20\ \mu\text{A}$

15. An automobile battery is charged by a constant current of 2 A for 10 hours. The terminal voltage of the battery is $V = (11 + 0.5t)$ V for $t \geq 0$, where time t is in hours. What is the total energy delivered to the battery during this time?
(a) 299 Wh (b) 258 Wh (c) 270 Wh (d) 285 Wh

Solution: (c); $I = 2A, t = 10$ hours $\therefore dW = VIdt = (11 + 0.5t) \times 2\,dt = (22 + t)dt$

$\therefore W = \int_0^{10}(22 + t)dt = \left[22t + \frac{t^2}{2}\right]_0^{10} = 22 \times 10 + \frac{10^2}{2}\text{Wh} = (220 + 50)\text{ Wh} = 270\text{ Wh}$

16. A light beam is incident from air on a reflecting surface so that the reflected ray is found to be completely polarized when the angle of incidence is 48°. If some of the incident light (at an angle of 48°) passes into the material below the surface, what is the angle of refraction?
(a) 39° (b) 42° (c) 43° (d) 38°

Solution: (b); $r = (90° - 48°) = 42°$

17. In one cycle, a heat engine absorbs 500 J from a high temperature reservoir and expels 300 J to a low temperature reservoir. If the efficiency of this engine is 60% of the efficiency of a Carnot engine, what is the ratio of the low temperature to the high temperature in the Carnot engine?
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

Solution: (a); $\eta_1 = 1 - \frac{300}{500} = \frac{2}{5} \therefore \eta_{heat} = \eta_{carnot} \times 0.6$

$\Rightarrow \eta_{carnot} = \frac{\eta_{heat}}{0.6} = \frac{2}{5} \times \frac{5}{3} = \frac{2}{3} \Rightarrow 1 - \frac{T_e}{T_n} = \frac{2}{3} \Rightarrow \frac{T_e}{T_n} = \frac{1}{3}$

18. What amount of energy will be released if a mass of 1 a.m.u is completely converted into energy? **[Ans: a]**
(a) 934.03 MeV (b) 994.03 MeV (c) 834.03 MeV (d) 894.03 MeV

19. If $\varphi = 2xy^4 - x^2z$, then determine $\vec{\nabla}\varphi$ at point $(2, -1, 2)$.
(a) $-(16\hat{i} + 16\hat{j} + 4\hat{k})$ (b) $-(6\hat{i} + 4\hat{j} + 4\hat{k})$ (c) $-(6\hat{i} + 6\hat{j} + 4\hat{k})$ (d) $-(6\hat{i} + 16\hat{j} + 4\hat{k})$

Solution: (d); $\phi = 2xy^4 - x^2z$

$\vec{\nabla}\phi = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)(2xy^4 - x^2z) = \frac{\delta\phi}{\delta x}\hat{i} + \frac{\delta\phi}{\delta y}\hat{j} + \frac{\delta\phi}{\delta z}\hat{k} = (2y^4 - 2zx)\hat{i} + (2.4x.y^3)\hat{j} + (-x^2)\hat{k}$

$\vec{\nabla}\phi(2, -1, 2) = \{2(-1)^4 - 2.2.2\}\hat{i} + \{2.4.2(-1)^3\}\hat{j} + (-2^2)\hat{k}$

$= \{2 - 8\}\hat{i} + \{-16\}\hat{j} - 4\hat{k} = -6\hat{i} - 16\hat{j} - 4\hat{k} = -(6\hat{i} + 16\hat{j} + 4\hat{k})$

20. An electron of mass of 9.1×10^{-31} kg is revolving around a nucleus in a circular path of a radius of 0.53×10^{-10} m. Calculate the angular velocity of the electron.
(a) 4.13×10^{16} rad s^{-1} (b) 4.13×10^{15} rad s^{-1} (c) 4.13×10^{14} rad s^{-1} (d) 5.13×10^{17} rad s^{-1}

Solution: (a); $\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} = m\omega^2 r \Rightarrow 9 \times 10^9 \times \frac{1.6\times10^{-19}\times1.6\times10^{-19}}{(0.53\times10^{-10})^3} = 9.11 \times 10^{-31} \times (\omega^2)$

$\Rightarrow \omega = 4.13 \times 10^{16}\ \text{rads}^{-1}$

21. The mass of a metallic sphere is 6 g, it is fastened at one end of a thread of length of 3 m and is rotated 4 times per second. What is its angular momentum?

(a) $2.356\ kgm^2s^{-1}$ (b) $1.356\ kgm^2s^{-1}$ (c) $1.984\ kgm^2s^{-1}$ (d) $2.784\ kgm^2s^{-1}$

Solution: (b); $L = I\omega = mvr = m\omega r^2 = 6 \times 10^{-3} \times (2\pi \times 4) \times 3^2\ kgm^2s^{-1} = 1.357\ kgm^2s^{-1}$

22. A truck of mass 900 kg moves with a velocity of 60 km per hour. The truck is stopped at a distance of 50 m by applying brake. If the frictional force of the ground is 200 N, then calculate the magnitude of force due to brake.

(a) 2100 N (b) 2500 N (c) 2300 N (d) 2700 N

Solution: (c); $(F_{break} + F_k)\, d = \frac{1}{2} \times 900 \times \left(\frac{60}{3.6}\right)^2 \Rightarrow F_{break} = 2300\ N$

23. An engine pumps 1000 kg of water per minute from a well of a depth of 100 m. If the 42% efficiency of the engine is lost, find the horsepower of the engine.

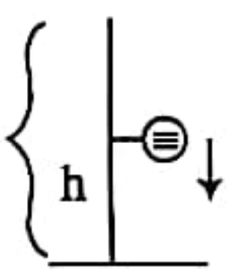
(a) 37.75 H.P. (b) 47.75 H.P. (c) 33.75 H.P. (d) 43.75 H.P.

Solution: (a); $P_{in} = \frac{P_{out}}{\eta} = \frac{\frac{1000 \times 9.8 \times 100}{60}}{1-0.42}\ W = 2.81609 \times 10^4 W = 37.75\ HP$

24. A body is allowed to fall freely from a height of 30 m. From the ground, where will its kinetic energy be twice the potential energy?

(a) 15 m (b) 12 m (c) 8 m (d) 10 m

Solution: (d); $2E_p = E_k \Rightarrow 2mgh = \frac{1}{2}mv^2 \Rightarrow 2h = 30 - h \Rightarrow 3h = 30 \Rightarrow h = 10\ m$



25. In an experiment, the value of $\frac{L}{T^2}$ at a place was found $0.25\ ms^{-2}$. Calculate the value of g at that place.

(a) $9.87\ ms^{-2}$ (b) $9.81\ ms^{-2}$ (c) $9.83\ ms^{-2}$ (d) $9.85\ ms^{-2}$

Solution: (a); $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow \frac{T^2}{4\pi^2} = \frac{L}{g} \Rightarrow g = \frac{4\pi^2 L}{T^2} = 4(3.1416)^2 \times 0.25\ ms^{-2} = 9.8696\ ms^{-2} \approx 9.87\ ms^{-2}$

26. Two pendulums having 2 sec time period, one is on the earth and the other is on the moon have the length ratio of 81:16. If the value of g on the earth's surface is $9.81\ ms^{-2}$, find the value of g on the moon's surface.

(a) $1.538\ ms^{-2}$ (b) $1.638\ ms^{-2}$ (c) $1.938\ ms^{-2}$ (d) $1.338\ ms^{-2}$

Solution: (c); $T_1 = T_2 \Rightarrow 2\pi\sqrt{\frac{L_1}{g_1}} = 2\pi\sqrt{\frac{L_2}{g_2}} \Rightarrow \frac{L_1}{g_1} = \frac{L_2}{g_2} \Rightarrow g_2 = g_1 \times \frac{L_2}{L_1} = 9.81 \times \frac{16}{81}\ ms^{-2} = 1.938\ ms^{-2}$

27. Density of the carbon-di-oxide gas is $1.98\ kgm^{-3}$ at 0°C temperature and at $1.0 \times 10^5\ Nm^{-2}$ pressure. Calculate the root mean square velocity of the molecules of that gas at 30°C temperature and at same pressure.

(a) $414.08\ ms^{-1}$ (b) $420.08\ ms^{-1}$ (c) $424.08\ ms^{-1}$ (d) $410.08\ ms^{-1}$

Solution: (d); $c_1 = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.98}}\ ms^{-1} = 389.249\ ms^{-1} \therefore c_2 = c_1 \times \sqrt{\frac{T_2}{T_1}} = 389.25\sqrt{\frac{303}{273}} = 410.08\ ms^{-1}$

28. If $R = 8.31\ Jkg^{-1}mol^{-1}$, then find the volume of 20 g oxygen at 27°C and 72 cm-Hg pressure.

(a) $16.24 \times 10^{-3}\ m^3$ (b) $26.24 \times 10^{-3}\ m^3$ (c) $16.24 \times 10^{-2}\ m^3$ (d) $26.24 \times 10^{-2}\ m^3$

Solution: (a); $PV = nRT \Rightarrow V = \frac{nRT}{P} = \frac{\frac{20}{32} \times 8.314 \times 300}{\frac{72}{76} \times 1.01325 \times 10^5}\ m^3 = 16.24 \times 10^{-3}\ m^3$

29. One afternoon, Jamil and his friends were gossiping beside a lake. All on a sudden, Jamil noticed that a bubble from the bottom of the lake of transparent water was coming out on the surface of the water. After coming to the surface, the bubble took a large size. Size of the bubble on the surface was 5 times, and atmospheric pressure was 10^5 Nm^{-2}. [Density of water, $\rho = 1000$ kgm^{-3}]. Calculate the depth of the lake.
(a) 38.82 m (b) 40.82 m (c) 42.82 m (d) 44.82 m
Solution: (b); $\frac{P_a+P_w}{P_a} = 5 \Rightarrow P_a + P_w = 5P_a \Rightarrow P_w = 4P_a \Rightarrow h\rho g = 4P_a \Rightarrow h = \frac{4p_a}{\rho g} = \frac{4\times10^5}{10^3\times9.8}$ m $= 40.82$ m

30. From what height a piece of ice is allowed to fall due to the action of gravity so that 10% of the ice will be melted by the heat produced? Consider that all the mechanical energy has been converted into heat.
(a) 5.6×10^5 m (b) 3.4×10^5 m (c) 5.6×10^4 m (d) 3.4×10^4 m
Solution: (b); $mgh \times 0.1 = ml_f \Rightarrow h = \frac{336000}{0.1\times9.8}$ m $= 3.42857 \times 10^5$ m

31. There are 10 bulbs of 100 W, 5 bulbs of 60 W and a heater of 3 kW in a house. If the lamps are illuminated for 6 hours and the heater is used for 2 hours daily, how many units of electric energy will be used in the month of January in that house?
(a) 325.5 kWh (b) 455.8 kWh (c) 527.8 kWh (d) 427.8 kWh
Solution: (d); $E = (10 \times 0.1 \times 6 + 5 \times 0.06 \times 6 + 3 \times 2) \times 31$ kWh $= 427.8$ kWh

32. The third spectral line is produced by a plane diffraction grating subtends an angle of diffraction of 30°. If there are 3000×10^2 lines per meter length, find the wavelength of light.
(a) 5556 Å (b) 6556 Å (c) 6655 Å (d) 7557 Å
Solution: (a); $\frac{1}{N}\sin\theta = 3\lambda \Rightarrow \frac{1}{3\times10^5}\sin 30° = 3\lambda \Rightarrow \lambda = 5556A°$

33. Man can tolerate minimum light of intensity of 10^{-10} Wm^{-2}. If the area of the eye ball is 0.4 cm^2 and frequency of light is 6×10^{14} Hz, then how many photons will enter into eyes per second? (Planck's constant, $h = 6.63 \times 10^{-34}$ Js).
(a) 10^5 (b) 10^4 (c) 10^6 (d) 10^3
Solution: (b); $I \times A \times t = nhf \Rightarrow 10^{-10} \times 0.4 \times 10^{-4} \times 1 = n \times 6 \times 10^{14} \times 6.63 \times 10^{-34} \Rightarrow n = 10^4$

34. The following truth table is of which gate? **[Ans: c]**

A	B	X
0	1	1
1	0	1
1	1	0
0	0	1

(a) AND gate (b) NOR gate (c) NAND gate (d) OR gate

Old Syllabus

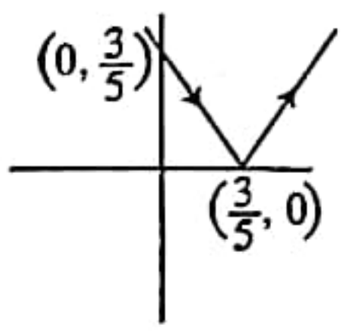
35. The light from a 10mW laser is concentrated on a 2 mm^2 area. What is the maximum value of corresponding sinusoidal electric field? ($\varepsilon_0 = 8.854 \times 10^{-12}$ Fm^{-1} and $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}).
(a) 1.25 kVm^{-1} (b) 1.65 kVm^{-1} (c) 2.24 kVm^{-1} (d) 1.94 kVm^{-1}
Solution: (d); We know, $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ So, Intensity, $I = S_{avg} = \frac{E_{max}.B_{max}}{2\mu_0} = \frac{(E_{max})^2}{2\mu_0 c}$
$\therefore E_{max} = \sqrt{2I.\mu_0 c} = \sqrt{2\left(\frac{10\times10^{-3}}{2\times10^{-6}}\right).\mu_0.c} = 1941.63$ $Vm^{-1} = 1.941$ kVm^{-1}

Mathematics: MCQ ($35 \times 1 = 35$)

Short Syllabus

36. If a thin light-ray falls upon the X-axis along the line $5x + 5y = 3$ and gets fully reflected, what will be equation of the line of the reflected ray?

(a) $3x - 3y = 7$ (b) $5x - 5y = 7$ (c) $3x - 3y = 5$ (d) $5x - 5y = 3$



Solution: (d); The equation of the straight line: $5x + 5y = 3$

$\Rightarrow \frac{x}{\frac{3}{5}} + \frac{y}{\frac{3}{5}} = 1$ and $m = -\frac{\frac{3}{5}}{\frac{3}{5}} = -1$

$\therefore$ Equation of the reflected straight line along x-axis: $y - 0 = -m\left(x - \frac{3}{5}\right)$

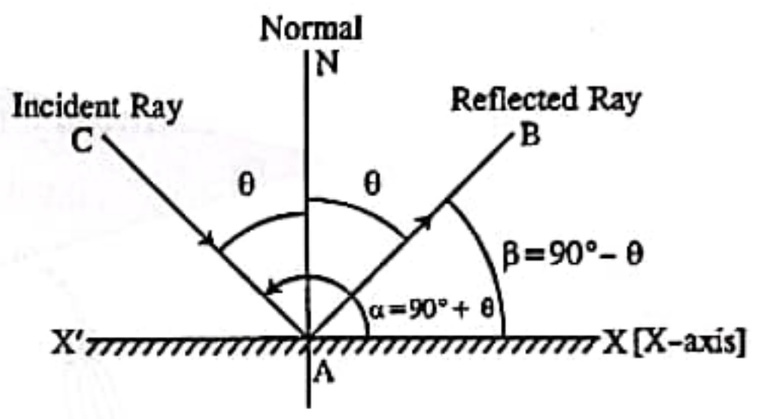
$\Rightarrow y = 1\left(x - \frac{3}{5}\right) \Rightarrow y = x - \frac{3}{5} \Rightarrow 5y = 5x - 3 \Rightarrow 5x - 5y = 3$

Note: Slope of the incident ray, $m = \tan\alpha = \tan(90° + \theta)$

$\therefore m = -\cot\theta$ Now Slope of the reflected ray,

$m_1 = \tan\beta = \tan(90° - \theta) \Rightarrow m_1 = \cot\theta = -(-\cot\theta)$

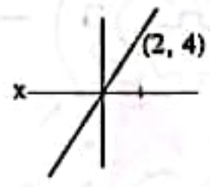
$\therefore \boxed{m_1 = -m}$



So, if a light ray falls on x-axis along a straight line with slope 'm' and gets reflected then the slope of the reflected straight line is '$-m$'.

37. A line intersects another perpendicular line at (2, 4) that goes through the origin. Find the equation of the first line.

(a) $x + 2y = 10$ (b) $7x - y = 10$ (c) $2x - y = 0$ (d) $x + y = 6$



Solution: (a); Equation of second line, $y = \frac{4}{2}x \Rightarrow 2x - y = 0$

The equation of first line: $x + 2y = 2 + 2 \times 4 \Rightarrow x + 2y = 10$

38. Given $\frac{\cos^{-1}x}{\sin^{-1}x} < 1$, find the most appropriate value of x.

(a) $1 \geq x > \frac{1}{\sqrt{2}}$ (b) $x > \frac{1}{\sqrt{2}}$ (c) $x < \frac{1}{\sqrt{2}}$ (d) $-1 \leq x < \frac{1}{\sqrt{2}}$

Solution: (a); $\frac{\cos^{-1}x}{\sin^{-1}x} < 1 \Rightarrow \cos^{-1}x < \sin^{-1}x \Rightarrow \sin^{-1}\sqrt{1-x^2} < \sin^{-1}x \Rightarrow \sqrt{1-x^2} < x$

$\Rightarrow 1 - x^2 < x^2 \Rightarrow 2x^2 > 1 \Rightarrow x^2 > \frac{1}{2} \therefore x > \frac{1}{\sqrt{2}}$

But for $\cos^{-1}x$ & $\sin^{-1}x$; $-1 \leq x \leq 1$

As $x > \frac{1}{\sqrt{2}}$ therefore, $\frac{1}{\sqrt{2}} < x \leq 1$; Again $\sin^{-1}x \neq 0 \therefore x \neq 0$

For $-1 \leq x < 0$; $\frac{\cos^{-1}x}{\sin^{-1}x} < 0[-ve] \therefore \frac{\cos^{-1}x}{\sin^{-1}x} < 1$

$\therefore$ Appropriate value of x is $-1 \leq x < 0$ or $\frac{1}{\sqrt{2}} < x \leq 1$

39. The sum of all angles of a polygon is 1080°. How many sides does the polygon have?

(a) 9 (b) 8 (c) 7 (d) 6

Solution: (b); $(n - 2) \times 180° = 1080° \Rightarrow n - 2 = 6 \therefore n = 8$

40. The roots of the equation $x^3 + px^2 + qx + r = 0$ are a, b and c. Evaluate $\sum(a + b)^2$.

(a) $2(p^2 - q)$ (b) $2(p^2 - 3q^3)$ (c) p^2 (d) $2(p^2 + 3q^3)$

Solution: (a); $x^3 + px^2 + qx + r = 0$; $a + b + c = -p$; $ab + bc + ca = q$ and $abc = -r$

Now, $\sum(a + b)^2 = (a + b)^2 + (b + c)^2 + (c + a)^2 = 2(a^2 + b^2 + c^2 + ab + bc + ca)$

$= 2\{(a + b + c)^2 - 2ab - 2bc - 2ca + ab + bc + ca\} = 2\{(a + b + c)^2 - (ab + bc + ca)\} = 2(p^2 - q)$

41. Find the cubic equation whose roots are 2 and $1 + 2i$.

(a) $x^3 - 4x^2 + 9x = 6$ (b) $x^3 + 4x^2 + x = 10$ (c) $x^3 - 4x^2 + x = 10$ (d) $x^3 - 4x^2 + 9x = 10$

Solution: (d); Roots are $2, 1 + 2i, 1 - 2i$; Now, $2 + 1 + 2i + 1 - 2i = 4$

$\Rightarrow 2(1 + 2i) + 2(1 - 2i) + (1 + 2i)(1 - 2i) = 2 + 4i + 2 - 4i + 1 + 2^2 = 9$

$\Rightarrow 2(1 + 2i)(1 - 2i) = 2(1 + 2^2) = 10$

$\therefore x^3 - 4x^2 + 9x - 10 = 0 \therefore x^3 - 4x^2 + 9x = 10$ **(Ans.)**

Alternative: Here, $x = 1 + 2i \Rightarrow x - 1 = 2i \Rightarrow x^2 - 2x + 1 = 4i^2 = -4 \therefore x^2 - 2x + 5 = 0$ which is a quadratic equation having roots $1 + 2i$ and $1 - 2i$

Now, Another root is $2 \therefore$ Cubic equation, $(x - 2)(x^2 - 2x + 5) = 0$

$\Rightarrow x^3 - 2x^2 + 5x - 2x^2 + 4x - 10 = 0 \therefore x^3 - 4x^2 + 9x = 10$ **(Ans.)**

42. For the ellipse $5x^2 + 4y^2 = 20$, find the equation of the directrix.

(a) $y = \pm 5$ (b) $x = \pm 5$ (c) $y = \pm\sqrt{5}$ (d) $x = \pm\sqrt{5}$

Solution: (a); $\frac{x^2}{4} + \frac{y^2}{5} = 1 \therefore a = 2, b = \sqrt{5}\ (b > a)$; $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{5}} = \frac{1}{\sqrt{5}}$;

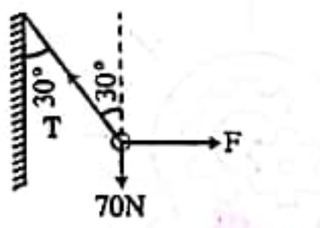
Equation of the directrix, $y = \pm be = \pm\sqrt{5} \times \sqrt{5} = \pm 5$

43. Suppose a solid ball weighs 70 N and is hanging from the wall with the help of a string. A horizontal force is applied on the ball to make an angle of 30° away from the wall. What is the magnitude of the force F?

(a) 50.4 N (b) 65.5 N (c) 80.8 N (d) 40.4 N

Solution: (d); $T\cos 30° = 70 \Rightarrow T \times \frac{\sqrt{3}}{2} = 70 \Rightarrow T = \frac{140}{\sqrt{3}}$ N

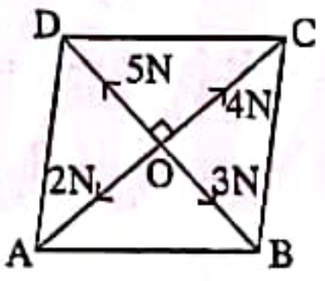
Again, $F = T\sin 30° = \frac{140}{\sqrt{3}} \times \frac{1}{2}$ N $= \frac{70}{\sqrt{3}}$ N $= 40.4145$ N ≈ 40.4N



44. Suppose the two diagonals of a rhombus ABCD intersect at point O. Four forces 2 N, 3 N, 4 N and 5 N are applied along the sides OA, OB, OC and OD respectively. What is the resultant force?

(a) 5 N (b) $2\sqrt{2}$ N (c) $\sqrt{2}$ N (d) $\sqrt{5}$ N

Solution: (b); $R = \sqrt{(5-3)^2 + (4-2)^2}$ N $= \sqrt{2^2 + 2^2}$ N $= 2\sqrt{2}$ N



45. The equation of an ellipse is $4x^2 + 9y^2 = 36$. The area (in square unit) bounded by the ellipse is-

(a) 6π (b) 2π (c) 8π (d) 12π

Solution: (a); $\frac{x^2}{9} + \frac{y^2}{4} = 1 \therefore a = 3, b = 2 \therefore$ Area $= \pi ab = \pi(3)(2)$ sq. units $= 6\pi$ sq. units

47. If $A + B + C = \frac{\pi}{2}$ and $\sin B \sin C = -\sin A$ then what is the value of $\cot A + \cot B + \cot C$?

(a) -1 (b) 0 (c) 1 (d) None of these

Solution: (b); $A + B + C = \frac{\pi}{2}$; $\sin B \sin C = -\sin A \Rightarrow \sin B \sin C = -\sin\left(\frac{\pi}{2} - (B + C)\right)$

$\Rightarrow \sin B \sin C = -\cos(B + C) \Rightarrow \sin B \sin C = -\cos B \cos C + \sin B \sin C \Rightarrow \cos B \cos C = 0$ ..... (i)

Again, $A + B + C = \frac{\pi}{2} \Rightarrow B + C = \frac{\pi}{2} - A \Rightarrow \cot(B + C) = \cot\left(\frac{\pi}{2} - A\right) \Rightarrow \frac{\cot B \cot C - 1}{\cot B + \cot C} = \tan A = \frac{1}{\cot A}$

$\Rightarrow \cot B \cot C \cot A - \cot A = \cot B + \cot C$

$\Rightarrow \cot A + \cot B + \cot C = \cot A \cot B \cot C = \frac{\cos A}{\sin A} \cdot \frac{\cos B \cos C}{\sin B \sin C} = \frac{\cos A}{\sin A} \cdot 0 = 0$ [from (i)]

48. If $\vec{F} = 4\hat{i} + 3\hat{j} + \hat{k}$ is the force and $\vec{S} = \hat{i} + 2\hat{j} + 5\hat{k}$ is the displacement, then what is the amount of work?

(a) 15 unit (b) 12 unit (c) 10 unit (d) 19 unit

Solution: (a); $W = \vec{F}.\vec{S} = (4\hat{i} + 3\hat{j} + \hat{k}).(\hat{i} + 2\hat{j} + 5\hat{k}) = 4.1 + 3.2 + 1.5 = 4 + 6 + 5$ unit $= 15$ units

49. If $y = (m - x)^{-1}$, then evaluate $y_n = \frac{d^n y}{dx^n}$.

(a) $\frac{n!}{(m-x)^n}$ (b) $\frac{n!}{(m-x)^{n+1}}$ (c) $\frac{(-1)^n n!}{(m-x)^n}$ (d) $\frac{(-1)^{n+1} n!}{(m-x)^{n+1}}$

Solution: (b); We know, if $y = (ax + b)^{-1}$ then, $y_n = \frac{(-1)^n . a^n n!}{(ax+b)^{n+1}}$ then, if $y = (m - x)^{-1}$

then, $y_n = \frac{(-1)^n (-1)^n . n!}{(m-x)^{n+1}} = \frac{(-1)^{2n} . n!}{(m-x)^{n+1}} = \frac{n!}{(m-x)^{n+1}} [\because (-1)^{2n} = 1]$

50. Find the inverse of $A = \begin{bmatrix} 4 & 5 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{bmatrix}$.

(a) $\begin{bmatrix} \frac{1}{4} & \frac{1}{5} & 0 & 0 \\ \frac{1}{3} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{4} & -5 & 0 & 0 \\ -3 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -2 \\ 0 & 0 & -4 & \frac{1}{3} \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -3 & 0 & 0 \\ -5 & 4 & 0 & 0 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & -2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -5 & 0 & 0 \\ -3 & 4 & 0 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -4 & 3 \end{bmatrix}$

Solution: (d); [Using Calculator]

52. $\cos^{-1} \cos \frac{4\pi}{3}$ is equal to-

(a) $\frac{4\pi}{3}$ (b) π (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

Solution: (c); $\cos^{-1}\left(\cos \frac{4\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

53. Given the matrix, $A = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$, what is the resultant matrix A^n?

(a) $\begin{bmatrix} 1 & n^n \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$ (d) $\begin{bmatrix} n & n \\ 0 & 1 \end{bmatrix}$

Solution: (b); $A = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ Using calculator, $A^2 = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$ [let $n = 3$]; $A^3 = \begin{bmatrix} 1 & 3n \\ 0 & 1 \end{bmatrix}$; $A^4 = \begin{bmatrix} 1 & 4n \\ 0 & 1 \end{bmatrix}$

$\therefore A^n = \begin{bmatrix} 1 & n.n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

54. If the roots of the equation $6x^2 - 5x + 1 = 0$ are a and b; then the equation with roots $\frac{1}{a}$ and $\frac{1}{b}$ is-

(a) $x^2 - 5x + 7 = 0$ (b) $x^2 - 4x + 3 = 0$ (c) $x^2 - 11x + 30 = 0$ (d) $x^2 - 5x + 6 = 0$

Solution: (d); One root of $6x^2 - 5x + 1 = 0$ is a $\therefore 6a^2 - 5a + 1 = 0$ ..... (i)

$\therefore$ Now, one root of the new equation $x = \frac{1}{a} \Rightarrow a = \frac{1}{x}$

Putting the value of a in.... (i) $\Rightarrow \frac{6}{x^2} - \frac{5}{x} + 1 = 0 \Rightarrow 6 - 5x + x^2 = 0 \Rightarrow x^2 - 5x + 6 = 0$

Shortcut: Putting $\frac{1}{x}$ instead of $x \Rightarrow 6\left(\frac{1}{x}\right)^2 - 5\left(\frac{1}{x}\right) + 1 = 0 \therefore x^2 - 5x + 6 = 0$

55. The determinants of the reverse identity matrices are defined as follow:

$|J_1| = |1| = +1, |J_2| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1, |J_3| = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -1, \cdots$ Find the determinant of J_{100}.

(a) $+1$ (b) 0 (c) -1 (d) None of the others

Solution: (a);

$|J_1| = 1$
$|J_5| = 1$
$\cdots\cdots\cdots\cdots\cdots\cdots$
$|J_{4n+1}| = 1$

$|J_2| = -1$
$|J_6| = -1$
$\cdots\cdots\cdots\cdots\cdots\cdots$
$|J_{4n+2}| = -1$

$|J_3| = -1$
$|J_7| = -1$
$\cdots\cdots\cdots\cdots\cdots\cdots$
$|J_{4n+3}| = -1$

$|J_4| = 1$
$|J_8| = 1$
$\cdots\cdots\cdots\cdots\cdots\cdots$
$|J_{4n}| = 1$

$\therefore |J_{100}| = |J_{4\times25}| = 1$

57. Given $A = \begin{bmatrix} x+y & -6 \\ ab & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & a+b \\ -9 & x+y \end{bmatrix}$, where $A^T + B^T = I$. What are the values of a and b?

(a) $3, -3$ (b) $-3, -3$ (c) $3, 3$ (d) $-3, 3$

Solution: (c); $A^T + B^T = \begin{bmatrix} x+y & ab \\ -6 & 7 \end{bmatrix} + \begin{bmatrix} -3 & -9 \\ a+b & x+y \end{bmatrix} = \begin{bmatrix} x+y-3 & ab-9 \\ -6+a+b & 7+x+y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore a + b - 6 = 0; ab - 9 = 0$

$\therefore a + b = 6; \quad ab = 9$ Solving we get, $a = b = 3$

58. Partial fraction of $\frac{5x^2-4}{x^2(x-2)}$ is-

(a) $\frac{3}{x} + \frac{1}{x^2} + \frac{4}{x-2}$ (b) $\frac{3}{x^2} + \frac{4}{x-2}$ (c) $\frac{2}{x} + \frac{1}{x^2} + \frac{4}{x-2}$ (d) $\frac{1}{x} + \frac{2}{x^2} + \frac{4}{x-2}$

Solution: (d); $\frac{5x^2-4}{x^2(x-2)} \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-2} \ldots\ldots (1)$

$\Rightarrow 5x^2 - 4 \equiv A(x-2) + Bx(x-2) + Cx^2 \ldots\ldots (2)$

Putting $x = 0$ we get, $-4 = -2A \therefore A = 2$

Putting $x = 2$ we get, $16 = 4C \therefore C = 4$

$(2) \Rightarrow 5x^2 - 4 = 2(x-2) + Bx(x-2) + 4x^2 \ldots\ldots (3)$

Putting $x = 1$ we get, $1 = -2 - B + 4 \therefore B = 1 \therefore (1) \Rightarrow \frac{5x^2-4}{x^2(x-2)} = \frac{2}{x^2} + \frac{1}{x} + \frac{4}{x-2} = \frac{1}{x} + \frac{2}{x^2} + \frac{4}{x-2}$

Alternate: Option Test.

59. Given $y = \cos x^{\cos x^{\cos x^{\cdots\infty}}}$, then evaluate $\frac{dy}{dx}$.

(a) $\frac{-y^2 \tan x}{y \ln \cos x - 1}$ (b) $\frac{y^2 \tan x}{y \ln \cos x - 1}$ (c) $\frac{-y^3 \cot x}{1 - y \ln \cos x}$ (d) $\frac{y^2 \cot x}{1 - y \ln \cos x}$

Solution: (b); $y = (\cos x)^{\cos x^{\cos x^{\cdots\cdots\cdots\infty}}}$

$\Rightarrow y = (\cos x)^y \Rightarrow \ln y = y \ln(\cos x) \Rightarrow \frac{1}{y} \times \frac{dy}{dx} = \frac{y}{\cos x}(-\sin x) + \ln(\cos x)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \ln(\cos x)\right) = -y \tan x \Rightarrow \frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \ln(\cos x)} = \frac{y^2 \tan x}{y \ln(\cos x) - 1}$

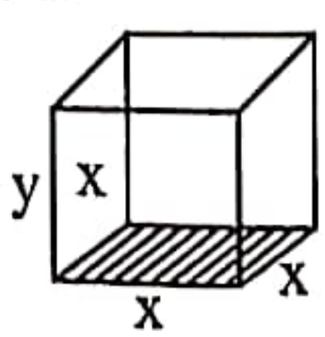
60. Suppose you want to build a 1125 ft^3 swimming pool with a square base of x ft on each side and having y ft depth. If the cost of construction is defined as $c = 5x^2 + 30xy$, then what values of x and y will minimize the cost?

(a) 5 ft, 15 ft (b) 5 ft, 45 ft (c) 15 ft, 5 ft (d) None of the others

Solution: (c); $V = x^2y \Rightarrow y = \frac{1125}{x^2}$

$c = 5x^2 + 30x.\frac{1125}{x^2} = 5x^2 + \frac{33750}{x}$

$\Rightarrow \frac{dc}{dx} = 10x - \frac{33750}{x^2}$ for max and min $\frac{dc}{dx} = 0$



$\Rightarrow 10x - \frac{33750}{x^2} = 0 \Rightarrow 10x^3 - 33750 = 0 \Rightarrow x^3 - 3375 = 0 \Rightarrow x = 15$ ft Now, $y = \frac{1125}{(15)^2}$ ft $= 5$ ft

61. General solution of the equation $2(\cos x + \sec x) = 5$ is-

(a) $n\pi \pm \frac{\pi}{3}$ (b) $2n\pi \pm \frac{\pi}{3}$ (c) $2n\pi \pm \frac{\pi}{6}$ (d) $n\pi \pm \frac{\pi}{6}$

Solution: (b); $2(\cos x + \sec x) = 5 \Rightarrow 2\left(\cos x + \frac{1}{\cos x}\right) = 5 \Rightarrow 2\left(\frac{\cos^2 x+1}{\cos x}\right) = 5$

$\Rightarrow 2\cos^2 x + 2 = 5\cos x$

$\Rightarrow 2\cos^2 x - 5\cos x + 2 = 0 \Rightarrow \cos x = \frac{1}{2}, 2$; But $\cos x \neq 2 \therefore \cos x = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow x = 2n\pi \pm \frac{\pi}{3} [N \in \mathbb{Z}]$

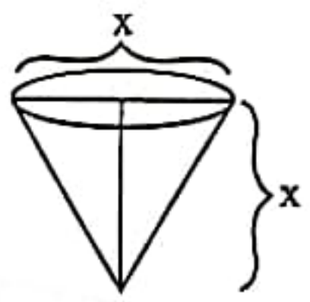
62. The volume of a cone-shaped metallic object is increasing at a rate of 20 ft^3/min. Both the base-diameter and height of the object are always equal. How fast is the height of the object increasing when it is 10 ft tall?

(a) 0.255 ft/min (b) 0.064 ft/min (c) 0.085 ft/min (d) 0.322 ft/min

Solution: (a); $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{x}{2}\right)^2 \times x = \frac{1}{12}\pi x^3$

$\frac{dV}{dt} = \frac{1}{12}\pi(3x^2)\frac{dx}{dt} \Rightarrow 20 = \frac{1}{12}\pi(3 \times 10^2)\frac{dx}{dt}$ [when $x = 10$ ft]

$\therefore \frac{dx}{dt} = 0.2546$ ft/min ≈ 0.255 ft/min



63. Find the equation of the tangent of the curve $xy = 3$ at the point $\left(4, \frac{3}{4}\right)$.

(a) $16x - 3y = 24$ (b) $3x - 16y = 24$ (c) $3x + 16y = 24$ (d) $16x + 3y = 24$

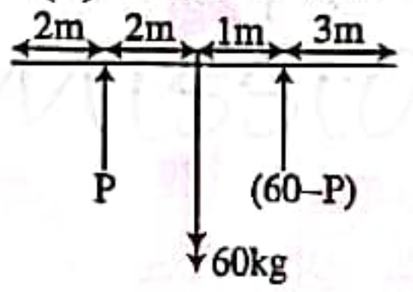
Solution: (c); $xy = 3 \Rightarrow y = \frac{3}{x} \Rightarrow \frac{dy}{dx} = -\frac{3}{x^2} \Rightarrow \frac{dy}{dx}\left(4, \frac{3}{4}\right) = -\frac{3}{4^2} = -\frac{3}{16}$

$\therefore$ Eq^n of tangent, $y - \frac{3}{4} = -\frac{3}{16}(x - 4) \Rightarrow 16y - 12 = -3(x - 4) \Rightarrow 16y - 12 = -3x + 12 \Rightarrow 3x + 16y = 24$

64. Two men are carrying a straight uniform bar of length 8 m and weight 60 kg. One man supports it at a distance of 2 m from one end and the other man at a distance 3 m from the other end. What weight does each man bear?

(a) 20 kg, 40 kg (b) 30 kg, 30 kg (c) 45 kg, 15 kg (d) None of these

Solution: (a); $P \times 2 = (60 - P) \times 1 \Rightarrow 2P = 60 - P \Rightarrow P = 20$ kg - wt



$\therefore 60 - p = 60 - 20 = 40$ kg - wt

65. If $S = 5 + 10t + t^3$ then find acceleration after 2 sec (t is in sec and S is in m).

(a) $-12\ ms^{-2}$ (b) $6\ ms^{-2}$ (c) $10\ ms^{-2}$ (d) None of these

Solution: (d); $S = 5 + 10t + t^3 \Rightarrow v = \frac{ds}{dt} = 10 + 3t^2 \Rightarrow a = \frac{dv}{dt} = 6t$; at $t = 2$sec; $a = 6 \times 2 = 12\ ms^{-2}$

66. Evaluate $\int \frac{x}{(1-x)^2} dx$

(a) $\frac{1}{1-x} + \ln(1 - x) + C$ (b) $\frac{1}{1-x} - \ln(1 - x) + C$

(c) $-\frac{1}{1-x} - \ln(1 - x) + C$ (d) $-\frac{1}{1-x} + \ln(1 - x) + C$

Solution: (a); $\int \frac{x}{(1-x)^2} dx = -\int \frac{(1-x)-1}{(1-x)^2} dx = \int \left\{\frac{1}{1-x} - \frac{1}{(1-x)^2}\right\} dx = -\left[\frac{\ln|1-x|}{-1} - \left(-\frac{1}{(1-x)(-1)}\right)\right] + C$

$= \frac{1}{1-x} + \ln|1 - x| + C$

67. Evaluate $\int e^x \left\{\frac{1}{1-x} + \frac{1}{(1-x)^2}\right\} dx$

(a) $\frac{e^x}{1-x} + C$ (b) $\frac{e^x}{x-1} + C$ (c) $\frac{e^x}{(1-x)^2} + C$ (d) $\frac{-e^x}{(1-x)^2} + C$

Solution: (a); $\int e^x \left\{\frac{1}{1-x} + \frac{1}{(1-x)^2}\right\} dx = e^x \frac{1}{1-x} + C$ [Here, $f(x) = \frac{1}{1-x} \Rightarrow f'(x) = (-1)(1 - x)^{-2}(-1) = \frac{1}{(1-x)^2}$]

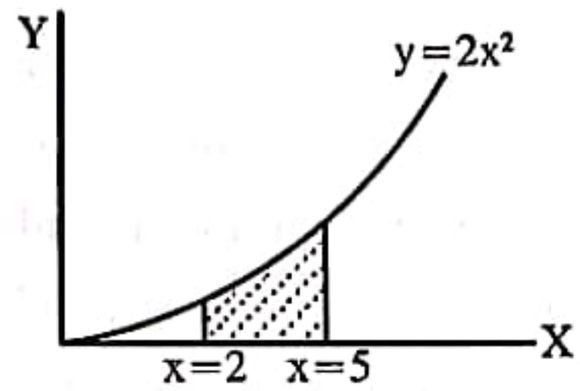
$[\therefore \int e^x [f(x) + f'(x)]dx = e^x f(x) + C]$

68. A particle falls freely from a tower for 4 seconds. What is the distance it crosses in the last 2 seconds?

(a) 128 ft (b) 16 ft (c) 96 ft (d) 192 ft

Solution: (d); $d = \frac{1}{2}g(4^2 - 2^2)$ ft $= \frac{1}{2} \times 32(16 - 4)$ ft $= 16 \times 12$ ft $= 192$ ft[Note: in f. p. s system $g = 32$ fts^{-2}]

70. What is the area of the shaded region of the following figure?



(a) 39 sq. unit (b) 13 sq. unit (c) 78 sq. unit (d) None of these

Solution: (c); Area $= \int_2^5 2x^2 dx = \left[\frac{2x^3}{3}\right]_2^5 = \frac{2}{3}[125 - 8]$ sq. unit $= 78$ sq. unit

Extra Syllabus

46. A bag contains 10 blue and 15 red balls. If a boy draws 2 balls at random consecutively, which one is the probability of both be of same color?

(a) $\frac{1}{2}$ (b) $\frac{1}{11}$ (c) $\frac{10}{99}$ (d) $\frac{4}{33}$

Solution: (a); $P = \frac{^{10}C_2}{^{25}C_2} + \frac{^{15}C_2}{^{25}C_2} = \frac{1}{2}$

56. For which value of x, the 21st and 22nd terms are equal in the expansion of $(1 + x)^{44}$?

(a) $\frac{6}{7}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{7}$

Solution: (c); $(1 + x)^{44} \therefore 21^{st}$ term $= {}^{44}C_{20}\, x^{20}$ and 22^{nd} term $= {}^{44}C_{21}\, x^{21} \therefore {}^{44}C_{20}\, x^{20} = {}^{44}C_{21}\, x^{21} \therefore x = \frac{7}{8}$

Alternate: As T_{21} and T_{22} are equation then $\frac{T_{22}}{T_{21}} = 1$

$\frac{T_{22}}{T_{21}} = \frac{T_{21+1}}{T_{21}} = \frac{44-21+1}{21} \times \frac{x}{1} = 1 \quad \Rightarrow \frac{24}{21}x = 1 \therefore x = \frac{21}{24} = \frac{7}{8}$

Old Syllabus

51. The region between the curve $y = \sqrt{x}$ and the X-axis is revolved about the X-axis to generate a solid. Find its volume.

(a) $\frac{8\pi}{3}$ (b) $\frac{16\pi}{3}$ (c) 16π (d) 8π

Solution: (Blank); limit is not given

69. Suppose a pyramid has a square base of length 3 m in each side and a height of 5 m. What is the volume of that pyramid?

(a) 9 m^3 (b) 15 m^3 (c) 27 m^3 (d) 45 m^3

Solution: (b); $V = \frac{1}{3}a^2h = \frac{1}{3}(3)^2 \times 5$ m^3 $= (3 \times 5)$ m^3

$= 15$ m^3 [a = length of each sides of the bose sqare; h = height]

English: $(15 \times 1 = 15)$

Read the following passage carefully and pick up the correct answer (Question 71-73)

The "free software movement" is a social and political campaign which states that users may run, modify, and redistribute as they please. Since the 1990s, however, a remarkable division has existed within the movement. Supporters of the original "free software" philosophy argue that proprietary software is unethical as computers can be considered as tools of positive social change and restricting access to such tools is against the public interest. A competing philosophy, called "open source," does not view the issue as a moral one and does not oppose cooperation with commercial developers of proprietary software. The open-source philosophy regards public, collaborative authorship of code as an efficient model of software development.

The first computers ran software that was developed through open collaboration between corporate researchers and academics. However, as computers became more complex, the costs of developing software increased, and companies began to charge license fees and prohibit users from modifying the programs. In response, some users started to develop alternatives to commercial software and openly share the source code, first through online bulletin board systems and, then by other means as the Internet developed. In 1997, an essay on the free software movement titled "The Cathedral and the Bazaar" inspired Netscape Communications Corporation to release its web browser as free software, marking the beginning of open-source collaboration between commercial developers and users. Some early proponents of free software argued against the newly formed Open-Source Initiative, contending that the organization's narrow focus on releasing the source code for software ignored the greater issue of campaigning for truly free software and threatened to obscure distinctions among truly free, partially free, and truly proprietary software.

71. According to the passage which one of the following statements is not true? **[Ans: d]**
(a) Free Software supporters considers selling a software is unscrupulous.
(b) Free Software supporters believe that computers should bring optimism in the society.
(c) Open-Source philosophy does not endorse joint-venture coding with the commercial programmers.
(d) Open-Source philosophy encourages proprietary software.

72. The author implies that which of the following occurred after Netscape Corporation released its web browser as open-source software? **[Ans: c]**
(a) Free software advocates focused increasingly on technical questions.
(b) The original free software philosophy was embraced by many commercial software developers.
(c) The free software movement softened its criticism of commercial software corporations.
(d) Not all free software advocates agreed that Netscape's browser fit the goals of their movement.

73. According to the author, which of the following was true of software development prior to 1997? **[Ans: a]**
(a) Software collaborations did not exist outside of purely academic settings.
(b) Corporations had collaborated with outside programmers but increasingly released proprietary software.
(c) Free and restrictive licenses were equally common among software products.
(d) The most common view among free software advocates was that user freedom was more important than the efficiency of software development.

74. Motorcycles having larger, more powerful engines normally accelerate faster and achieve higher speeds than motorcycles with smaller engines. However, the majority of motorcycle crashes involve motorcycles with smaller engines. Hence, to enhance the safety for motorcycle riders, we should encourage them to ride motorcycles with larger, more powerful engines. **[Ans: d]**

Which of the following statement, if true, most seriously weakens the argument above?

(a) More experienced and knowledgeable riders are more likely to ride motorcycles that have larger engines than motorcycles that have smaller engines.

(b) Motorcycles that have smaller engines tend to be lighter and therefore easier to maneuver at low speed than motorcycles with larger engines.

(c) The average speed at which motorcycles were traveling just prior to crashes is relatively low, approximately 30 miles per hour.

(d) Approximately 75 percent of crashes involving motorcycles also involve another vehicle, most often a passenger automobile.

75. A drug having higher efficacy in treating many types of infection can, at present, be obtained only from the bark of the ibora, a tree that is very rare. It takes the bark of 500 trees to make 100 gram of the drug. It follows, therefore, that continued production of the drug must lead to the ibora's extinction from the planet.

Which of the following, if true, most seriously weakens the argument above? **[Ans: d]**

(a) The drug made from ibora bark is dispensed to doctors from a central authority.

(b) The drug made from ibora bark is expensive to produce.

(c) The leaves of the ibora are used in a number of medical products.

(d) The ibora can be propagated from cuttings and grown under cultivation.

76. Overweight is the root cause of heart disease, stroke, and high blood pressure. Studies have shown that high-protein diets help people lose weight who have not been able to lose weight in any other way. Therefore, high protein diets can be an important part of a healthy lifestyle. **[Ans: c]**

Which of the following, if true, most undermines the conclusion expressed above about high-protein diets?

(a) High-protein diets are easier than ordinary diets for most dieters to follow because they allow dieters to eat filling foods like meat and eggs.

(b) Although many dieters initially show rapid weight loss on high-protein diets, most of them regain the weight as soon as they go off the diet.

(c) High-protein diets have been linked to heightened rates of high cholesterol, stomach cancer. and kidney failure.

(d) Society should be more understanding of overweight people, and should not pressure them to pursue extreme diets.

77. The use of radar detectors in commercial vehicles was banned because commercial truck and bus drivers were using these devices to drive faster than the posted speed limit without fear of being arrested. Since drivers of non-commercial vehicles also use radar detectors and since speeding decreases safety for any vehicle, use of radar detectors should also be banned in non-commercial vehicles to increase safety. Which of the following, if true, most strongly supports the argument above? **[Ans: c]**

(a) The average non-commercial vehicle driver is involved in less long-distance driving than is the average commercial vehicle driver.

(b) In many places airplanes or helicopters are used rather than radar to locate vehicles traveling faster than the posted speed limit.

(c) The ban on radar detectors in commercial vehicles has been effective in deterring them from speeding.

(d) Traffic accidents involving a truck or bus generally pose a greater threat of injury or death than do other accidents.

78. Passengers must exit airplanes rapidly after accidents because gases released following accidents are toxic to humans and often explode soon after being released. In order to prevent passenger deaths from gas inhalation, safety officials recommend that passengers be provided with smoke hoods that prevent inhalation of the gases. Which of the following, if true, constitutes the strongest reason to require implementation of the safety officials' recommendation? **[Ans: b]**

(a) Test evacuations showed that putting on the smoke hoods added considerably to the overall time it took passengers to leave the cabin.

(b) In many airplane accidents, passengers who were able to reach emergency exits were overcome by toxic gases before they could exit the airplane.

(c) Although the smoke hoods protect passengers from the toxic gases, they can do nothing to prevent the gases from igniting.

(d) Some experienced flyers fail to pay attention to the safety instructions given on every commercial flight before takeoff.

Choose the most appropriate word(s)/phrase to fill in the blank in the given sentence (Question 79 to 82)

79. A number of scientific theories that are —— today, were once treated with —— by the scientific establishment. **[Ans: c]**

(a) Dismissed, contempt (b) Accepted, approbation

(c) Unchallenged, disdain (d) Unrivalled, reverence

80. I am impressed at the length he has gone through to create this detailed report in which he —— the text at several places. **[Ans: b]**

(a) annotated (b) referred (c) interpreted (d) inferred

81. While filling up the survey, he was very open and —— in revealing his feelings about the organization's compensation program. **[Ans: c]**

(a) astute (b) incredulous (c) candid (d) virile

82. Like people living near an active volcano, many of us are —— about the possibility of atomic warfare and its attendant destruction. **[Ans: b]**

(a) unconcerned (b) worried (c) irritated (d) excited

Choose the word which is most nearly SIMILAR in meaning to the word in capital letters. (Question 83 to 85)

83. INDOMITABLE **[Ans: d]**

(a) adequate (b) improper (c) doctrine (d) unconquerable

84. DIVULGE **[Ans: b]**

(a) divert (b) reveal (c) docile (d) deprive

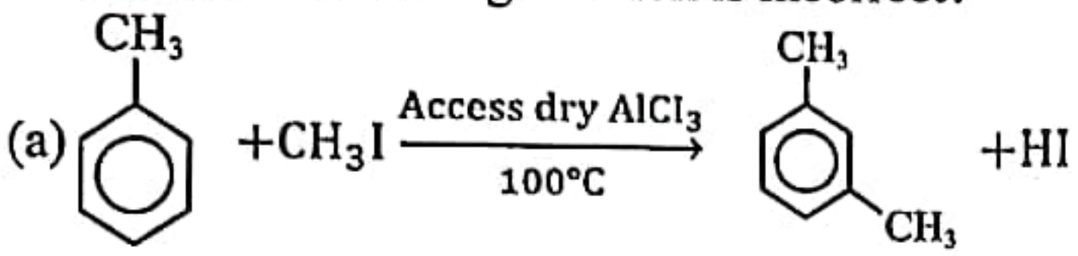
85. ACUMEN **[Ans: a]**

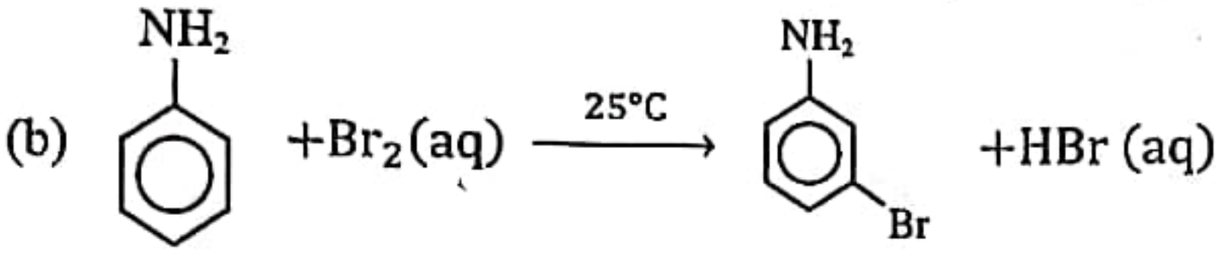
(a) keenness (b) brilliance (c) swiftness (d) greediness

Chemistry: MCQ ($15 \times 1 = 15$)

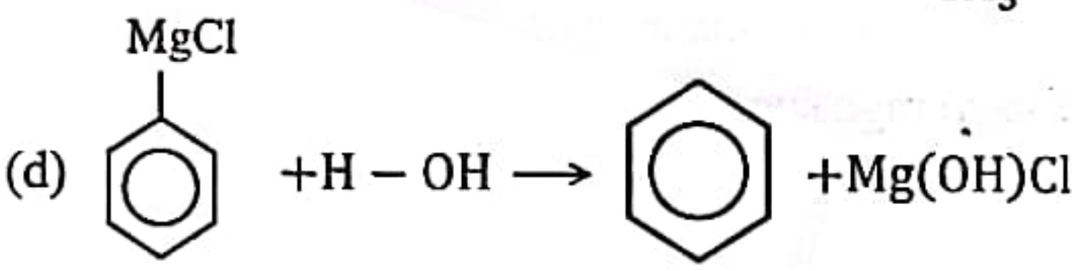
Short Syllabus

86. Which of the following reaction is incorrect?

(a) 

(b) 

(c) $H_3C-\underset{}{\overset{H}{\overset{|}{C}}}=O + CH_3MgBr \longrightarrow H_3C-\underset{CH_3}{\overset{H}{\overset{|}{\underset{|}{C}}}}-OMgBr \xrightarrow{H_2O} H_3C-\underset{CH_3}{\overset{H}{\overset{|}{\underset{|}{C}}}}-OH + Mg(OH)Br$

(d) 

Solution: (b); $-NH_2$ is ortho para indicating group.

87. Which of the following is true? **[Ans: a]**

(i) The order of reactivity in S_N2 is $CH_3X > 1° > 2° > 3°$

(ii) The order of reactivity in E_2 is $CH_3X > 1° > 2° > 3°$

(iii) Strong base favors the E_2 and S_N2 reactions.

(iv) More polar solvent favors S_N1 reaction but less polar solvent E_1 reaction.

(a) i (b) ii (c) iii (d) iv

88. How many structural isomers would the compound C_6H_{14} have? **[Ans: b]**

(a) 3 (b) 5 (c) 7 (d) 9

89. In which reaction is the underlined substance acting as a base?

(a) $HNO_3 + \underline{H_2SO_4} \longrightarrow H_2NO_3^+ + HSO_4^-$ (b) $HSiO_3^- + \underline{HCN} \longrightarrow CN^- + H_2O + SiO_2$

(c) $HNO_2 + \underline{HCO_3^-} \longrightarrow H_2O + CO_2 + NO_2^-$ (d) $C_6H_5O^- + \underline{CH_2ClCO_2H} \longrightarrow C_6H_5OH + CH_2ClCO_2^-$

Solution: (c); $HNO_2 + \underline{HCO_3^-} \rightarrow NO_2^- + H_2CO_3 \rightarrow H_2O + CO_2 + NO_2^-$

90. 80 mL gas is collected over water at 17°C and 750 mm-Hg pressure. The same amount of gas occupies a volume of 72 mL at STP in dry condition. Calculate the vapor pressure at 17°C .

(a) 726.6 mm − Hg (b) 32.41 mm − Hg (c) 147.6 mm − Hg (d) 23.41 mm − Hg

Solution: (b); $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow P_1 = 726.6$ mm Hg

$\Delta P = (750 - 726.6)$mm Hg $= 23.41$ mm Hg

92. What will be the values of b and d in the balanced chemical equation?

$a\ MnO_4^-(aq) + b\ C_2O_4^{2-}(aq) + c\ H^+(aq) \rightarrow d\ Mn^{2+}(aq) + e\ CO_2(aq) + f\ H_2O(l)$

(a) 2, 5 (b) 5, 2 (c) 2, 2 (d) 5, 5

Solution: (b); $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

93. Calculate the solubility of $NiCO_3$ in grams per litre. The value of Ksp for $NiCO_3 = 1.4 \times 10^{-7}$, where 118.7 g/mol is the molecular mass of $NiCO_3$.
(a) $1.96 \times 10^{-14}\ gL^{-1}$ (b) $3.7 \times 10^{-4}\ gL^{-1}$ (c) $0.044\ gL^{-1}$ (d) $0.118\ gL^{-1}$
Solution: (c); $S(mol\ L^{-1}) = \sqrt{1.4 \times 10^{-7}} \therefore S(g\ L^{-1}) = \sqrt{1.4 \times 10^{-7}} \times 118.7 = 0.044\ gL^{-1}$

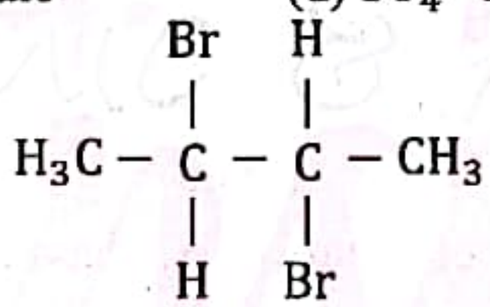
95. When few drops of $Ba(NO_3)_2$ solution is added to a test-tube containing the solution of molecules AB, white precipitate is formed. This precipitate remains insoluble on addition of hot dilute HCl. What would be B? **[Ans: b]**
(a) CO_3^{2-} (b) SO_4^{2-} (c) PO_4^{3-} (d) I^-

96. Consider a cell consisting of $Zn(s)|Zn^{2+}(1.00 \times 10^{-5}M)||Ag^+(0.10M)|\ Ag(s)$ and standard reduction potentials of the cell are $E^o_{Zn(red)} = -0.76V$ and $E^o_{Ag(red)} = 0.80\ V$ at 25°C. For this cell the following parameters age given, **[Ans: d]**
(i) The cell reaction is $Zn(s) + 2Ag^+(aq) \rightarrow Zn^{2+}(aq) + 2Ag(s)$.
(ii) $E_{cell} = 1.647\ V$. (iii) $\Delta G^o = -3.178 \times 10^5 J$.
Which of the following is more appropriate?
(a) (i) and (ii) (b) (ii) and (iii) (c) (i) and (iii) (d) (i), (ii) and (iii)

97. Which is the topmost layer of atmosphere? **[Ans: a]**
(a) Exosphere (b) Ionosphere (c) Thermosphere (d) None of these

98. According to the kinetic theory of the gas, **[Ans: a]**
(i) RMS speed, $c = \sqrt{\frac{3RT}{M}}$ (ii) Average speed, $\bar{c} = \sqrt{\frac{8RT}{\pi M}}$
(iii) Most probable speed, $\alpha = \sqrt{\frac{2RT}{3M}}$
Which of the following is more appropriate?
(a) (i) and (ii) (b) (ii) and (iii) (c) (i) and (iii) (d) (i), (ii) and (iii)

99. Which of the following is correct? **[Ans: c]**
(a) PI_5 is a trigonal pyramidal molecule (b) XeF_4 is a tetrahedral molecule
(c) ICl_5 is a square pyramidal molecule (d) SO_4^{2-} is a square planner molecule

100. Structure of 2, 3-dibromobutane is-  **[Ans: d]**
This structure
(i) has two chiral centers (ii) shows two optically active isomers
(iii) shows one optically inactive isomer
Which of the following is more appropriate?
(a) (i) and (ii) (b) (ii) and (iii) (c) (i) and (iii) (d) (i), (ii) and (iii)

Extra Syllabus

91. Which statement about exothermic and endothermic reactions is correct? **[Ans: b]**
(a) In an endothermic reaction, energy is used to break bonds but no energy is released when bonds form.
(b) In an endothermic reaction, energy is released when bound form but more energy is used to break bonds.
(c) In an exothermic reaction, energy is released both by breaking and by forming bonds.
(d) In an exothermic reaction, energy is released when bonds form but no energy is needed to break bonds.

94. The basis of MRI experiment to detect tumors, strokes, and bleeds **[Ans: d]**
(i) Proton (H^+) resonance occurs due to absorption of radiofrequency.
(ii) Affected cell contains more proton which produce more proton (H^+) resonance signal.
(iii) Mutual interaction occurs between protons and electrons due to application of radiofrequency.
Which of the following is more appropriate?
(a) (i) and (ii) (b) (ii) and (iii) (c) (i) and (iii) (d) (i), (ii) and (iii)